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Vibration Control of Seat Suspension using H_∞ Reliable Control

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Abstract: This paper is concerned with the problem of robust H_∞ reliable load-dependent control design for a class of semi-active seat suspension systems. A four degree-of-freedom human body model is considered in order to investigate the control strategy more precisely. It is assumed that the human body mass resides in an interval and can be measured online. The load-dependent approach is based on a parameter-dependent Lyapunov function. Thus the proposed method is capable of reducing the conservatism and improving the closed-loop performance. Actuator faults are also considered in the controller design process and the resulting control systems are reliable because they provide guaranteed asymptotic stability and H_∞ performance in spite of the possible actuator failures. The usefulness and advantages of the controller design methodology are demonstrated through a design example.

Key words: Actuator faults, human body model, load dependent, reliable control, seat suspension.

1. INTRODUCTION

The demand for improved ride comfort is motivated by the increasing number of vehicles, prolonged driving hours and exposure to severe environments such as rough road conditions and complex ride behaviors. The impact of vertical vibration on the occurrence of back pain has been discussed in many studies (Choi et al., 2003; Guo et al., 2004; Hostens et al., 2004; Rakheja et al., 1994). A driver exposed for a long time to vertical vibrations tends to feel uncomfortable and painful in their back. A primary approach to improve drivers' comfort is to attenuate the vibration transmission from the chassis to the driver. Moreover, among the majority of commercial vehicles, the seat suspension system is the only mechanism utilized to control the transmission of low-frequency and high-amplitude ride vibration, which has lead to the considerable number of studies on numerous types of seat suspension systems (Cheok et al., 1989; Choi and Han, 2003; Guo et al., 2005; Wan and Schimmels, 1997).

However, most existing results have concentrated on vibration control of a rigid dummy mass on the seat, and it is obviously not precise enough to simplify the complicated human

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body as a rigid mass without any biodynamics included. Therefore, a mathematical seated human body model is necessary to conduct research into ride comfort and safety improvements.

A seated human body model, which is a sophisticated dynamic system with complex mechanical properties, is very much required in order to gain a good insight into the vibration attenuation performance. Recently, biodynamics of humans in a seated posture have been drawing considerable attention, and a great amount of mathematical seated human body models exposed to vibration have been developed (Liang and Chiang, 2008; Wei and Griffin, 1998), varying from one DOF to 11 DOF. This paper selects a four DOF human body model that captures the essential dynamics of a seated human exposed to vertical vibration to obtain a good tradeoff between facility and accuracy.

Meanwhile, the human body mass serves as an uncertain factor, which changes along with different drivers. As a result, the robust multi-objective control technique becomes a major approach in suspension control and many important results have been reported so far (Du et al., 2003, 2005; Gao et al., 2007, 2005; Wang and Unbehauen, 1999; Wang et al., 2002). It is worth mentioning that the solutions given in the quadratic framework tend to produce more conservative results since the same Lyapunov function must be used for the entire uncertainty domain. Moreover, it is noted that the gain matrices for the designed controllers remain constant for varying uncertain parameters. However, for active seat suspension systems with parameters such as the driver's mass, which can be measured online conveniently, it is a waste to ignore the available online information that can be utilized in the realization of control strategy because less conservative results can be achieved via a load-dependent method using the online measurable human body mass (Gao et al., 2006).

Furthermore, due to the growing complexity of automated control systems and actuators, various faults are likely to be encountered, especially actuator faults. This motivates interest in fault-tolerant control system design, and the objective is to prevent the faults in the control loop from causing an overall system failure. During the past few decades, much research has been done into the area of reliable and fault-tolerant control of dynamic systems and a great number of theoretic results have also been presented, see e.g. Shi et al. (2003), Shi et al. (2006), Wang and Qiao (2004), Wang et al. (2008), Wang et al. (2006), Wang et al. (1999), and Yang et al. (1998). In particular, a methodology for the design of reliable linear control systems is proposed in Veillette et al. (1992) such that the resulting closed-loop system has guaranteed stability and H_∞ performance even when some control components suffer from outages. The authors in Yang et al. (2001) present a more practical actuator fault model than outage, and give an algebra Riccati equation (ARE) method for output feedback controller design.

In this paper, a design method for robust H_∞ reliable state-feedback load-dependent control against actuator faults is presented for a class of semi-active seat suspension systems with parameter uncertainties. A four DOF vibration model of the human body is employed and combined with the seat to investigate the seat suspension system more precisely. The mathematical seat suspension model with parameter uncertainties and possible actuator faults is then established. A robust H_∞ reliable state-feedback controller design method is developed via a parameter-dependent Lyapunov approach. Next, the existence conditions for an admissible controller are formulated in the form of linear matrix inequalities (LMIs), and the controller design problem is cast into a convex optimization problem subject to LMI constraints. The corresponding closed-loop system with the constructed robust H_∞ reliable load-

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dependent controller is asymptotically stable with a guaranteed H_∞ performance in spite of the possible actuator faults. Simulation results of a design example in both frequency and time domains are given to demonstrate the effectiveness of the proposed controller design method.

The main contribution of this paper is the introduction of a robust H_∞ reliable load-dependent controller design approach, which is capable of guaranteeing the asymptotical stability of the closed-loop seat suspension system in the case of actuator faults and parameter uncertainties. Thus, it is very practical and has large application potentials.

The rest of this paper is organized as follows. The problem of multi-objective robust H_∞ reliable load-dependent controller design is formulated mathematically for a semi-active seat suspension system with a human body model in Section 2. The main results, including stability and performance analysis, are presented in Section 3. A design example demonstrating the effectiveness and advantages of the proposed methodology is given in Section 4 and some concluding remarks are given in Section 5.

Notation. The notation used throughout the paper is fairly standard. The superscript T stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|_\infty$ denotes the H_∞ norm for matrices; the notation $P > 0$ (≥ 0) is used to denote a symmetric and positive definite (semi-definite) matrix. In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is readily induced by symmetry and diag $\{\ldots\}$ stands for a block-diagonal matrix. sym(A) is used to denote $(A + A^T)$ for simplicity. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The space of square-integrable vector functions over $[0, \infty)$ is denoted by $L_2[0, \infty)$, and for $w = \{w(t)\} \in L_2[0, \infty)$, its norm is denoted by $\|w\|_2 = \sqrt{\int_{t=0}^{\infty} |w(t)|^2 \, dt}$.

2. PROBLEM FORMULATION

The seat-driver model consisting of the seat suspension and human body shown in Figure 1 is employed here for the design of control laws. The road excitation input is transmitted to the cabin floor. It is assumed that only the vertical motion of the vehicle exists for simplification and both pitching and rolling motions are neglected in this paper. The related parameters are defined in Table 1.

This linear model of a seated human body was established by Boileau and Rakheja (1998), based on experimental results on live human bodies. Human bodies are composed of four mass segments, i.e. head, upper torso, lower torso, and thighs; arms and legs are combined with the upper torso and thighs, respectively. An optimization procedure has been performed with human body anthropometric and biodynamical data as constraints, and with measurements on seat-to-head transmissibility, driving-point mechanical impedance, and apparent mass as objective functions in the interest of obtaining physical parameters, i.e. m_i, c_i, and k_i, $i = 3, 4, 5, 6$. The mass of the lower legs and feet is not included in this representation, due to their negligible contributions to the biodynamic response of the seated body. Moreover, z_1–z_6 are the displacements of the corresponding masses, respectively, z_0 is the road displacement input, and u is the active control input of the seat suspension system.

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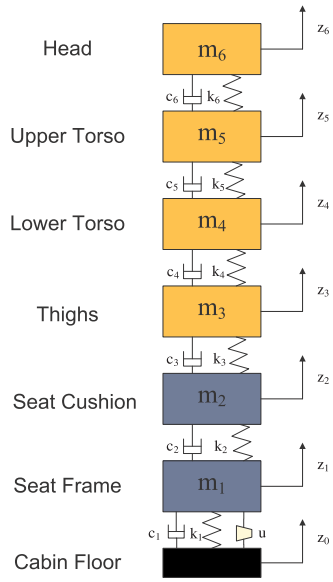


Figure 1. Vibration model of the seat suspension system with the human body.

Table 1. The parameters of the seat–human suspension system.

c_1	damping of seat suspension	k_1	stiffness of seat suspension
c_2	damping of seat cushion	k_2	stiffness of seat cushion
c_3	damping of buttocks & thighs	k_3	stiffness of buttocks & thighs
c_4	damping of lumbar spine	k_4	stiffness of lumbar spine
c_5	damping of thoracic spine	k_5	stiffness of thoracic spine
c_6	damping of cervical spine	k_6	stiffness of cervical spine

The dynamic equations of the seat suspension with a human body are given by

$$\begin{aligned}
m_1\ddot{z}_1 &= -c_1(\dot{z}_1-\dot{z}_0)-k_1(z_1-z_0)+c_2(\dot{z}_2-\dot{z}_1)+k_2(z_2-z_1)-u,\\
m_2\ddot{z}_2 &= -c_2(\dot{z}_2-\dot{z}_1)-k_2(z_2-z_1)+c_3(\dot{z}_3-\dot{z}_2)+k_3(z_3-z_2),\\
m_3\ddot{z}_3 &= -c_3(\dot{z}_3-\dot{z}_2)-k_3(z_3-z_2)+c_4(\dot{z}_4-\dot{z}_3)+k_4(z_4-z_3),\\
m_4\ddot{z}_4 &= -c_4(\dot{z}_4-\dot{z}_3)-k_4(z_4-z_3)+c_5(\dot{z}_5-\dot{z}_4)+k_5(z_5-z_4),\\
m_5\ddot{z}_5 &= -c_5(\dot{z}_5-\dot{z}_4)-k_5(z_5-z_4)+c_6(\dot{z}_6-\dot{z}_5)+k_6(z_6-z_5),\\
m_6\ddot{z}_6 &= -c_6(\dot{z}_6-\dot{z}_5)-k_6(z_6-z_5).
\end{aligned} \tag{1}$$

As mentioned above, the driver body mass usually changes with different drivers. Throughout the paper, it is assumed that the driver body mass resides in an interval and can be measured online,

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$$m_i = m_{i0}(1+\lambda), \quad i = 3,4,5,6, \tag{2}$$

where m_{i0} is the nominal body mass of the corresponding segments, and $|\lambda| \le \lambda_0$ is a scalar introduced to describe the interval that the driver body mass is located within. For convenience, denote $m_{i1} = m_{i0}(1+\lambda)\big|_{\lambda=-\lambda_0}$ and $m_{i2} = m_{i0}(1+\lambda)\big|_{\lambda=\lambda_0}$ as the lower and upper bounds. It is worth noting that by equation 2 we assume that the four mass segments (head, upper torso, lower torso, and thighs) vary proportionally with different drivers, which is quite common in practical applications.

Choose the following set of state variables:

$$\begin{aligned}
&x_1(t) = z_1(t) - z_0(t), \quad x_2(t) = \dot{z}_1(t), \quad x_3(t) = z_2(t) - z_1(t), \quad x_4(t) = \dot{z}_2(t),\\
&x_5(t) = z_3(t) - z_2(t), \quad x_6(t) = \dot{z}_3(t), \quad x_7(t) = z_4(t) - z_3(t), \quad x_8(t) = \dot{z}_4(t),\\
&x_9(t) = z_5(t) - z_4(t), \quad x_{10}(t) = \dot{z}_5(t), \quad x_{11}(t) = z_6(t) - z_5(t), \quad x_{12}(t) = \dot{z}_6(t),
\end{aligned} \tag{3}$$

where $x_i(t)$ $(i = 1,3,5,7,9,11)$ is the deflection of the corresponding spring (k_1 to k_6) and $x_i(t)$ $(i = 2,4,6,8,10,12)$ is the speed of the corresponding mass (m_1 to m_6).

Define the state vector as

$$x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t) \;\; x_4(t) \;\; x_5(t) \;\; x_6(t) \;\; x_7(t) \;\; x_8(t) \;\; x_9(t) \;\; x_{10}(t) \;\; x_{11}(t) \;\; x_{12}(t)]^T,$$

and

$$\omega(t) = \dot{z}_0(t), \tag{4}$$

where $\omega(t)$ represents the disturbance caused by road roughness.

Then the dynamic equations in equation 1 can be written in the following state-space form:

$$\dot{x}(t) = A(\lambda)x(t) + B(\lambda)u(t) + B_w(\lambda)\omega(t), \tag{5}$$

where

$$A(\lambda) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a_{86} & a_{87} & a_{88} & a_{89} & a_{8,10} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{10,8} & a_{10,9} & a_{10,10} & a_{10,11} & a_{10,12}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{12,10} & a_{12,11} & a_{12,12}
\end{bmatrix},$$

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with

$$a_{21} = -\frac{k_1}{m_1}, \quad a_{22} = -\frac{c_1+c_2}{m_1}, \quad a_{23} = \frac{k_2}{m_1}, \quad a_{24} = \frac{c_2}{m_1},$$

$$a_{42} = \frac{c_2}{m_2}, \quad a_{43} = -\frac{k_2}{m_2}, \quad a_{44} = -\frac{c_2+c_3}{m_2}, \quad a_{45} = \frac{k_3}{m_2}, \quad a_{46} = \frac{c_3}{m_2},$$

$$a_{64} = \frac{c_3}{m_3}, \quad a_{65} = -\frac{k_3}{m_3}, \quad a_{66} = -\frac{c_3+c_4}{m_3}, \quad a_{67} = \frac{k_4}{m_3}, \quad a_{68} = \frac{c_4}{m_3},$$

$$a_{86} = \frac{c_4}{m_4}, \quad a_{87} = -\frac{k_4}{m_4}, \quad a_{88} = -\frac{c_4+c_5}{m_4}, \quad a_{89} = \frac{k_5}{m_4}, \quad a_{8,10} = \frac{c_5}{m_4},$$

$$a_{10,8} = \frac{c_5}{m_5}, \quad a_{10,9} = -\frac{k_5}{m_5}, \quad a_{10,10} = -\frac{c_5+c_6}{m_5}, \quad a_{10,11} = \frac{k_6}{m_5}, \quad a_{10,12} = \frac{c_6}{m_5},$$

$$a_{12,10} = \frac{c_6}{m_6}, \quad a_{12,11} = -\frac{k_6}{m_6}, \quad a_{12,12} = -\frac{c_6}{m_6},$$

and

$$B(\lambda) = \begin{bmatrix} 0 & -\frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$B_w(\lambda) = \begin{bmatrix} -1 & \frac{c_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \tag{6}$$

Before designing the robust H_∞ reliable state-feedback load-dependent control law for the seat suspension system, we need to consider the following three requirements first:

(1) Ride comfort. Ride comfort can be quantified by the human head acceleration in the vertical direction; therefore, it is chosen as the first control output, i.e. $z_{o1}(t) = \ddot{z}_6(t)$. In order to design the desired semi-active seat suspension to perform in a sufficiently wide range of shock and vibration environments, the H_∞ norm is employed to measure the performance since the H_∞ norm of a linear time-invariant (LTI) system is equivalent to the energy-to-energy gain and its value actually gives an upper bound on the root-mean-square (rms) gain. Hence, our primary goal is to minimize the H_∞ norm of the transfer function $T_{z_{o1}w}$ from the disturbance $w(t)$ to the control output $z_{o1}(t)$ to improve the ride comfort performance, that is,

$$\min \left\| T_{z_{o1}w} \right\|_\infty. \tag{7}$$

(2) Suspension deflection limitation. In order to avoid vehicle component damage and ride comfort degradation, the seat suspension controller must be capable of preventing the suspension from hitting its travel limit. Therefore, we need to take the suspension stroke limitation into consideration, i.e.

$$|z_1(t) - z_0(t)| \le z_{\max}, \tag{8}$$

where $z_{\max}$ is defined as the maximum possible suspension deflection.

(3) Maximum actuator control force. Due to the limited power of the actuator, the active control force provided for the seat suspension system should be confined to a prescribed range, i.e.

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$$|u(t)| \le u_{\max}, \tag{9}$$

where $u_{\max}$ is defined as the maximum possible actuator control force.

It is obvious that only the first requirement needs to be minimized because the latter two are actually hard constraints. In other words, the strategy in suspension system control law design is to minimize the H_∞ norm of the transfer function $T_{z_{o1}w}$ from the disturbance $w(t)$ to the control output $z_{o1}(t)$ while holding the other two requirements satisfied.

According to the above three specifications, the following output variables are defined:

$$\begin{aligned} z_{o1}(t) &= \ddot{z}_6(t), \\ z_{o2}(t) &= (z_1(t) - z_0(t))/z_{\max}. \end{aligned} \tag{10}$$

Therefore, the vehicle seat suspension system can be described by the following state-space equation:

$$\begin{aligned} \dot{x}(t) &= A(\lambda)x(t) + B(\lambda)u(t) + B_w(\lambda)\omega(t), \\ z_{o1}(t) &= C_1(\lambda)x(t), \\ z_{o2}(t) &= C_2(\lambda)x(t), \end{aligned} \tag{11}$$

where $A(\lambda)$, $B(\lambda)$, and $B_w(\lambda)$ are already defined in equation 6, and

$$\begin{aligned} C_1(\lambda) &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_6/m_6 & -k_6/m_6 & -c_6/m_6 \end{bmatrix}, \\ C_2(\lambda) &= \begin{bmatrix} 1/z_{\max} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{12}$$

Next, by considering the actuator fault, we introduce a state-feedback controller of the form

$$u^f(t) = M_a K_a(\lambda)x(t), \tag{13}$$

where $K_a(\lambda)$ is the actuator fault-tolerant feedback control gain matrix to be determined; the actuator fault matrix $M_a = \text{diag}\{m_{a1}, m_{a2}, \ldots, m_{aq}\}$, where $\check{m}_{ai} \le m_{ai}(t) \le \hat{m}_{ai}$ $(i = 1, 2, \ldots, q)$, and $\check{m}_{ai}$, $\hat{m}_{ai}$ are prescribed lower and upper bounds of the actuator fault; $m_{ai}(t)$ represents the possible fault of the corresponding actuator $u_i^f(t)$. There are three cases that stand for three different actuator conditions.

1. $\check{m}_{ai} = \hat{m}_{ai} = 0$, so $m_{ai}(t) = 0$, which means that the corresponding actuator $u_i^f(t)$ is complete failure.
2. $\check{m}_{ai} = \hat{m}_{ai} = 1$, so we have $m_{ai}(t) = 1$, which represents the case of no fault in the very actuator $u_i^f(t)$.
3. $0 < \check{m}_{ai} < \hat{m}_{ai} < 1$, which indicates that there exists a partial fault in the corresponding actuator $u_i^f(t)$.

Then, the closed-loop system with the state-feedback controller 13 can be written as

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$$\begin{aligned}\dot{x}(t) &= (A(\lambda) + B(\lambda)K(\lambda))x(t) + B_w(\lambda)\omega(t),\\ z_{o1}(t) &= C_1(\lambda)x(t),\end{aligned} \tag{14}$$

where $K(\lambda) = M_a K_a(\lambda)$. It is not difficult to see that the system matrices $A(\lambda)$, $B(\lambda)$, $B_w(\lambda)$, $C_1(\lambda)$, and $C_2(\lambda)$, which are dependent on the human body mass, can be expressed as functions of λ, that is,

$$\begin{aligned}(A, B, B_w, C_1, C_2)(\lambda) &= \sum_{i=1}^{2} \alpha_i(\lambda)(A, B, B_w, C_1, C_2)_i,\\ \alpha_1(\lambda) + \alpha_2(\lambda) &= 1,\ \alpha_i(\lambda) \geq 0,\ i = 1, 2,\end{aligned} \tag{15}$$

where

$$\begin{aligned}(A, B, B_w, C_1, C_2)_1 &= (A(\lambda), B(\lambda), B_w(\lambda), C_1(\lambda), C_2(\lambda))\big|_{\lambda=-\lambda_0},\\ (A, B, B_w, C_1, C_2)_2 &= (A(\lambda), B(\lambda), B_w(\lambda), C_1(\lambda), C_2(\lambda))\big|_{\lambda=\lambda_0}.\end{aligned}$$

Moreover, the relationship between the vector $\alpha(\lambda) \triangleq (\alpha_1(\lambda), \alpha_2(\lambda))$ and the online measurable body mass m_i, $i = 3, 4, 5, 6$ is given by

$$\begin{aligned}\alpha_1(\lambda) &= \left(\frac{1}{m_i} - \frac{1}{m_{i2}}\right) \Big/ \left(\frac{1}{m_{i1}} - \frac{1}{m_{i2}}\right),\\ \alpha_2(\lambda) &= \left(\frac{1}{m_{i1}} - \frac{1}{m_i}\right) \Big/ \left(\frac{1}{m_{i1}} - \frac{1}{m_{i2}}\right).\end{aligned} \tag{16}$$

In this paper, the problem of robust H_∞ reliable seat suspension controller design is to find a state-feedback control law

$$u(t) = K(\lambda)x(t), \tag{17}$$

such that the following requirements are satisfied:

(1) the closed-loop system is asymptotically stable;
(2) under zero initial condition, the performance $\left\|T_{z_{o1}w}\right\|_\infty < \gamma$ is minimized subject to $|z_1(t) - z_0(t)| \leq z_{\max}$ and $|u(t)| \leq u_{\max}$, where $T_{z_{o1}w}$ denotes the closed-loop transfer function from the road disturbance $w(t)$ to the control output $z_{o1}(t)$.

3. ROBUST H_∞ RELIABLE CONTROLLER DESIGN

In this section, a robust H_∞ reliable state-feedback controller is derived for the seat suspension system so that the closed-loop system in equation 14 is asymptotically stable and can also ensure a prescribed gain from disturbance ω to performance output z_{o1} while keep-

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ing the suspension stroke and maximum control force constraints satisfied. We first give the following lemma which will be used in later development.

Lemma 1. *For a time-varying diagonal matrix* $\Phi(t) = \text{diag}\{\sigma_1(t), \sigma_2(t), \ldots, \sigma_p(t)\}$ *and two matrices* R *and* S *with appropriate dimensions, if* $|\Phi(t)| \le V$, *where* $V > 0$ *is a known diagonal matrix, then for any scalar* $\varepsilon > 0$, *it is true that*

$$R\Phi S + S^T\Phi^T R^T \le \varepsilon R V R^T + \varepsilon^{-1} S^T V S.$$

Next, we introduce the following matrices which will be used in our later development.

$$\begin{aligned} M_{a0} &= \text{diag}\{m_{a01}, m_{a02}, \ldots, m_{a0q}\}, \\ L_a &= \text{diag}\{l_{a1}, l_{a2}, \ldots, l_{aq}\}, \\ J_a &= \text{diag}\{j_{a1}, j_{a2}, \ldots, j_{aq}\}, \end{aligned}$$

where $m_{a0i} = (\check{m}_{ai} + \hat{m}_{ai})/2$, $l_{ai} = [m_{ai}(t) - m_{a0i}]/m_{a0i}$ and $j_{ai} = (\hat{m}_{ai} - \check{m}_{ai})/(\hat{m}_{ai} + \check{m}_{ai})$ with $i = 1, 2, \ldots, q$. Then, we have $M_a = M_{a0}(I + L_a)$ and $L_a^T L_a \le J_a^T J_a \le I$.

Then, we give the following theorem to solve the controller design problem for the seat suspension system.

Theorem 1. *Consider system 14 with the robust* H_∞ *reliable state-feedback controller in equation 17; the closed-loop system in equation 14 is asymptotically stable and satisfies* $\|z_{o1}\|_2 < \gamma \|w\|_2$ *for all* w *satisfying* $\|w\|_2^2 \le w_{\max} = \rho/\gamma^2$, *the suspension stroke constraint, and the maximum control force constraint under zero initial condition if there exist matrices* $\bar{P}(\lambda) > 0$ *and* $\bar{K}_a(\lambda)$ *satisfying*

$$\begin{bmatrix} \text{sym}(A(\lambda)\bar{P}(\lambda) + B(\lambda)M_{a0}\bar{K}_a(\lambda)) + \varepsilon_{a3}B(\lambda)J_aB(\lambda)^T & B_w(\lambda) & \bar{P}(\lambda)C_1^T(\lambda) & M_{a0}^T\bar{K}_a^T(\lambda) \\ * & -\gamma^2 & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\varepsilon_{a3}J_a^{-1} \end{bmatrix} < 0, \quad (18)$$

$$\begin{bmatrix} -I + \varepsilon_{a5}J_a & \sqrt{\rho}M_{a0}\bar{K}_a(\lambda) & 0 \\ * & -u_{\max}^2\bar{P}(\lambda) & \sqrt{\rho}\bar{K}_a^T(\lambda)M_{a0}^T \\ * & * & -\varepsilon_{a5}J_a^{-1} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -I & \sqrt{\rho}C_2 \\ * & -z_{\max}^2\bar{P}(\lambda) \end{bmatrix} < 0 \quad (19)$$

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and an admissible controller in the form of equation 17 is given by

$$K(\lambda) = M_a K_a(\lambda) = M_a \bar{K}_a(\lambda)\bar{P}^{-1}(\lambda). \tag{20}$$

Proof. See the Appendix. □

Remark 1. Note that the system 5 can be used to represent many important physical systems subject to inherent input delays, parameter uncertainties, exogenous disturbances, and possible actuator faults besides the seat suspension system. Thus, this controller design method has large application potentials.

Remark 2. $\left\|T_{z_{o1}w}\right\|_\infty$ is actually the largest gain across frequency in the singular value norm. The H_∞ norm measures the system input–output gain for finite energy or finite rms input signals. The constraint $\left\|T_{z_{o1}w}\right\|_\infty < \gamma$ can be interpreted as a disturbance rejection performance. This constraint is also useful to enforce robust stability (Scherer et al., 1997).

It can be noted that, for fixed λ, equations 18 and 19 are linear matrix inequalities over the matrix variables to be determined, which means that, for fixed λ, by solving the linear matrix inequalities in equations 18 and 19, we can find a fault-tolerant controller guaranteeing the required H_∞ performance.

According to the inner property of the polytopic human body mass uncertainties, we assume the matrix functions $\bar{P}(\lambda)$ and $\bar{K}_a(\lambda)$ have the following form:

$$\bar{P}(\lambda) = \sum_{i=1}^{2} \alpha_i(\lambda)\bar{P}_i, \quad \bar{K}_a(\lambda) = \sum_{i=1}^{2} \alpha_i(\lambda)\bar{K}_{ai}. \tag{21}$$

Therefore, equations 18 and 19 in Theorem 1 are equivalent to the following inequalities:

$$\begin{bmatrix} \mathrm{sym}(A_i\bar{P}_i + B_i M_{a0}\bar{K}_{ai}) + \varepsilon_{a3}B_i J_a B_i^T & B_{wi} & \bar{P}_i C_{1i}^T & M_{a0}^T\bar{K}_{ai}^T \\ * & -\gamma^2 & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\varepsilon_{a3}J_a^{-1} \end{bmatrix} < 0, \quad i = 1, 2,$$

$$\begin{bmatrix} \mathrm{sym}\,(\Omega) & B_{w1} + B_{w2} & \bar{P}_1 C_{12}^T + \bar{P}_2 C_{11}^T & M_{a0}^T\bar{K}_{a1}^T + M_{a0}^T\bar{K}_{a2}^T \\ * & -2\gamma^2 & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & -2\varepsilon_{a3}J_a^{-1} \end{bmatrix} < 0, \tag{22}$$

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Table 2. System parameters of the proposed seat suspension.

Mass (kg)		Damping coefficient (Ns/m)		Spring constant (N/m)	
m_1	15	c_1	830	k_1	31,000
m_2	1	c_2	200	k_2	18,000
m_{30}	12.78	c_3	2064	k_3	90,000
m_{40}	8.62	c_4	4585	k_4	162,800
m_{50}	28.49	c_5	4750	k_5	183,000
m_{60}	5.31	c_6	400	k_6	310,000

$$\begin{bmatrix} -I + \varepsilon_{a5} J_a & \sqrt{\rho} M_{a0} \bar{K}_{ai} & 0 \\ * & -u_{\max}^2 \bar{P}_i & \sqrt{\rho} \bar{K}_{ai}^T M_{a0}^T \\ * & * & -\varepsilon_{a5} J_a^{-1} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -I & \sqrt{\rho} C_2 \\ * & -z_{\max}^2 \bar{P}_i \end{bmatrix} < 0, \quad i = 1, 2, \tag{23}$$

where

$$\Omega = A_1 \bar{P}_2 + B_1 M_{a0} \bar{K}_{a2} + A_2 \bar{P}_1 + B_2 M_{a0} \bar{K}_{a1} + \varepsilon_{a3} (B_1 J_a B_2^T + B_2 J_a B_1^T).$$

Based on the above conditions, the robust H_∞ reliable load-dependent controller design can be accomplished by solving the following convex optimization problem:

$$\min \ \gamma \quad \text{under the constraints of 22, 23.} \tag{24}$$

The feedback gain matrix function for controller equation 17 can be given by

$$K(\lambda) = M_a \left(\sum_{i=1}^{2} \alpha_i(\lambda) \bar{K}_{ai} \right) \left(\sum_{i=1}^{2} \alpha_i(\lambda) \bar{P}_i \right)^{-1}. \tag{25}$$

Remark 3. The results can be extended to stochastic systems when we model the road excitation and actuator fault occurrences stochastically, which is quite natural in practice (Wang et al., 2001). The deduction methods and procedures are basically similar.

4. A DESIGN EXAMPLE

In order to evaluate the effectiveness and usefulness of the controller design method proposed in the above sections, an example is now introduced. The schematic and biodynamical parameters of the model are borrowed from Choi and Han (2007) and listed in Table 2. Assume the maximum suspension deflection $z_{\max} = 0.08$ m and the maximum control force $u_{\max} = 2000$ N. Furthermore, assume that the uncertain parameter λ satisfies $|\lambda| \leq \lambda_0 = 0.5$ and $\check{m}_{ai} = 0.1$, $\hat{m}_{ai} = 1$.

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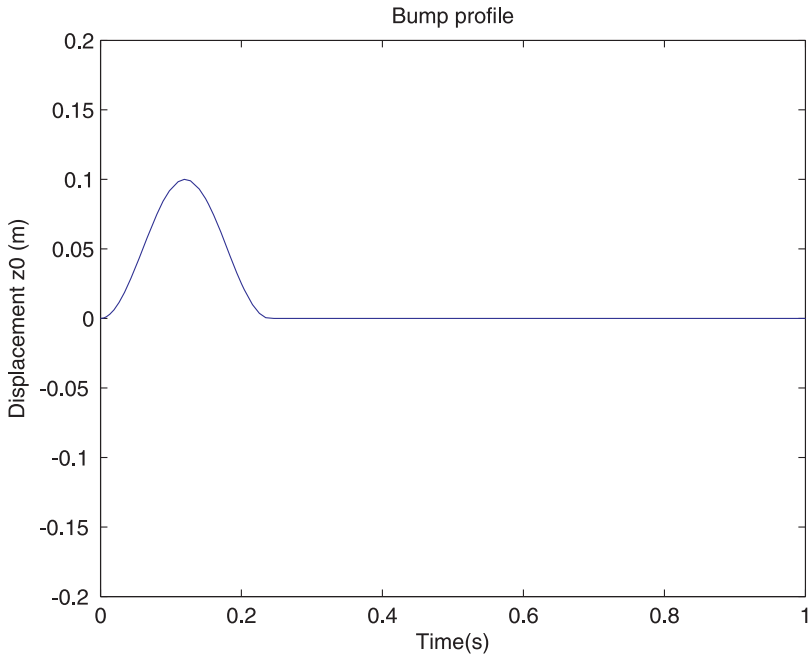


Figure 2. Bump input from the ground.

First, we give a conventional controller K_c with an H_∞ performance $\gamma = 10.3848$ without considering the load-dependent and fault-tolerant method, which is designed by employing the approach in Theorem 1 with the M_{a0}, J_a terms and λ being dropped out. The controller gain matrix is given by

$$K_c = \begin{bmatrix} 90 & 274 & -10518 & 53 & -4840 & 763 & -9324 \\ 541 & -14894 & 1859 & -39004 & -416 \end{bmatrix}.$$

In the context of seat suspension performance, road disturbances can be generally assumed to be shocks. Shocks are discrete events of relatively short duration and high intensity, caused by, for example, a pronounced bump or pothole on an otherwise smooth road surface. In this work, this case of a road profile is considered to reveal the transient response characteristic, which is given by

$$z_0(t) = \begin{cases} \frac{a}{2}(1 - \cos(\frac{2\pi V_0}{l}t)), & 0 \le t \le \frac{l}{V_0}, \\ 0, & t > \frac{l}{V_0}, \end{cases} \tag{26}$$

and illustrated in Figure 2, where a is the height of the bump and l is the length of the bump. Here we choose $a = 0.1$ m, $l = 2$ m, and the vehicle forward velocity $V_0 = 30$ km/h.

Figure 3 shows a plot of the bump responses of the open-loop system and the closed-loop system with controller K_c. It is obvious that the closed-loop system obtains a much better

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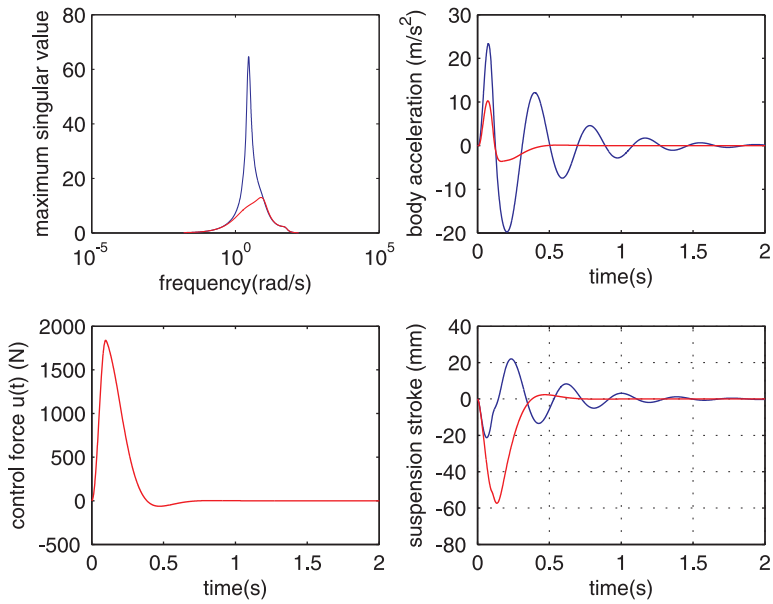


Figure 3. Bump responses of the open-loop system and the closed-loop system with controller K_c (open-loop system: blue solid curve, closed-loop system: red solid curve).

performance in both frequency and time domains than the passive mode, and the suspension stroke and maximum control force constraints are guaranteed simultaneously.

Next, we employ a load-dependent controller K_l with an H_∞ performance $\gamma = 11.0346$, which is designed by employing the approach in Theorem 1 with the M_{a0}, J_a terms dropped out.

Figure 4 depicts the bump responses of the closed-loop system with the conventional controller K_c versus the one with the load-dependent controller K_l. From Figure 4 we can see that the load-dependent approach has reduced the conservatism and further improved the closed-loop performance.

Finally, we use the controller K_{lf} designed by considering both the load-dependent and fault-tolerant control methods by solving the convex optimization problem 24 in the MATLAB environment.

In Figure 5, the $\left\|T_{zo1w}\right\|_\infty$ of closed-loop systems with the load-dependent controller K_l and the fault-tolerant controller K_{lf} versus the uncertain parameter λ is shown with 0%, 30%, 60%, and 90% actuator thrust loss. It can be seen that, when the failure extent is slight, the standard load-dependent controller K_l yields smaller closed-loop H_∞ norm than the fault-tolerant controller K_{lf}. However, if the failure extent keeps increasing, the fault-tolerant controller K_{lf} is capable of providing a far better closed-loop H_∞ performance than the standard load-dependent controller K_l.

In Figures 6–9, the bump responses of closed-loop systems with standard load-dependent controller K_l and fault-tolerant controller K_{lf} with 0%, 30%, 60%, and 90% actuator thrust loss are illustrated, respectively. Similar results to Figure 5, revealing the advantage of the fault-tolerant controller K_{lf} over the load-dependent controller K_l in both frequency and

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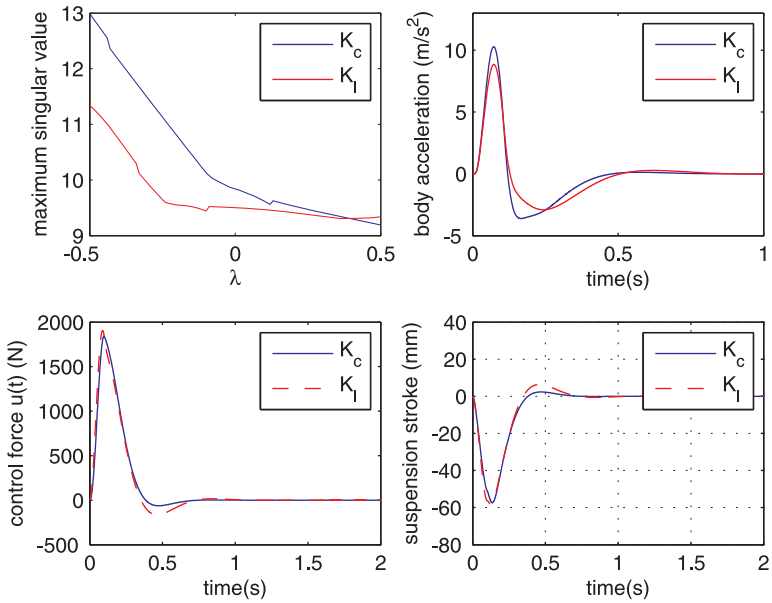


Figure 4. Bump responses of the closed-loop systems with conventional controller K_c and load-dependent controller K_l.

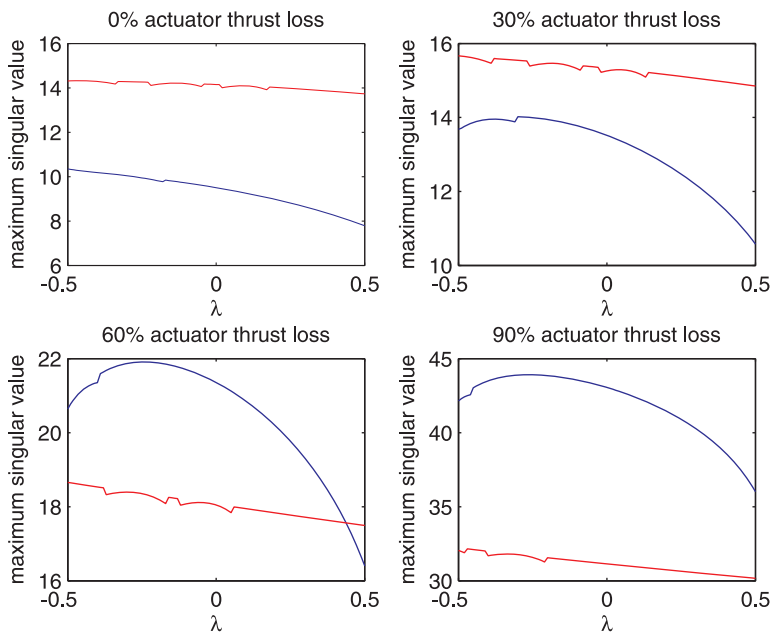


Figure 5. $\left\|T_{z_{o1}w}\right\|_{\infty}$ of the closed-loop systems with load-dependent controller K_l and fault-tolerant controller K_{lf} versus the uncertain parameter λ (K_l: blue solid curve, K_{lf}: red solid curve).

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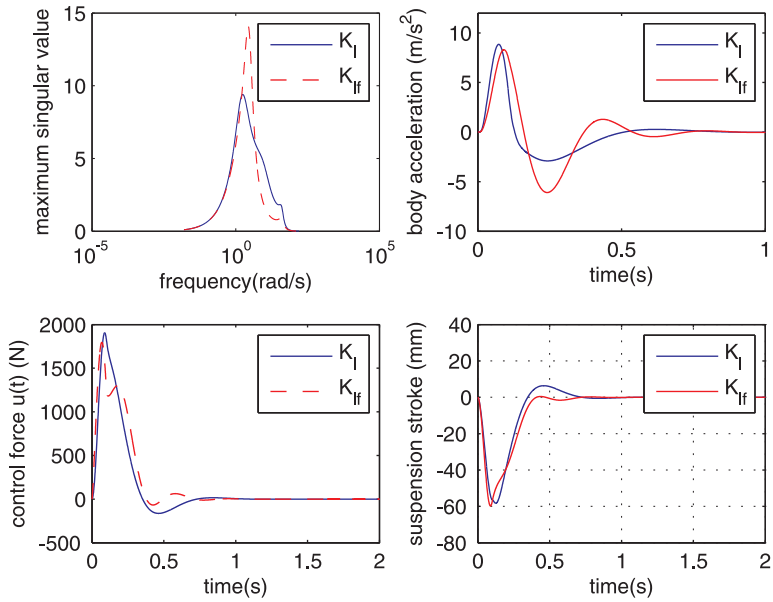


Figure 6. Bump responses of the closed-loop systems with load-dependent controller K_l and fault-tolerant controller K_{lf} with 0% actuator thrust loss.

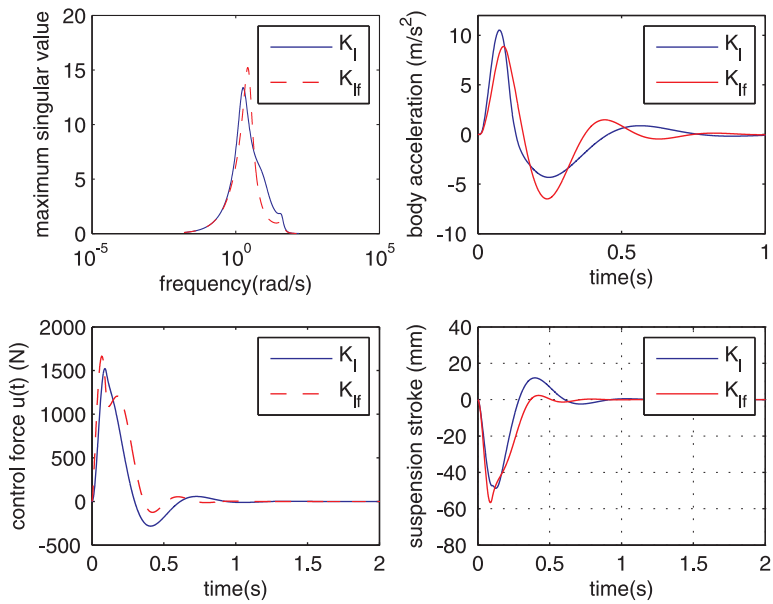


Figure 7. Bump responses of the closed-loop systems with load-dependent controller K_l and fault-tolerant controller K_{lf} with 30% actuator thrust loss.

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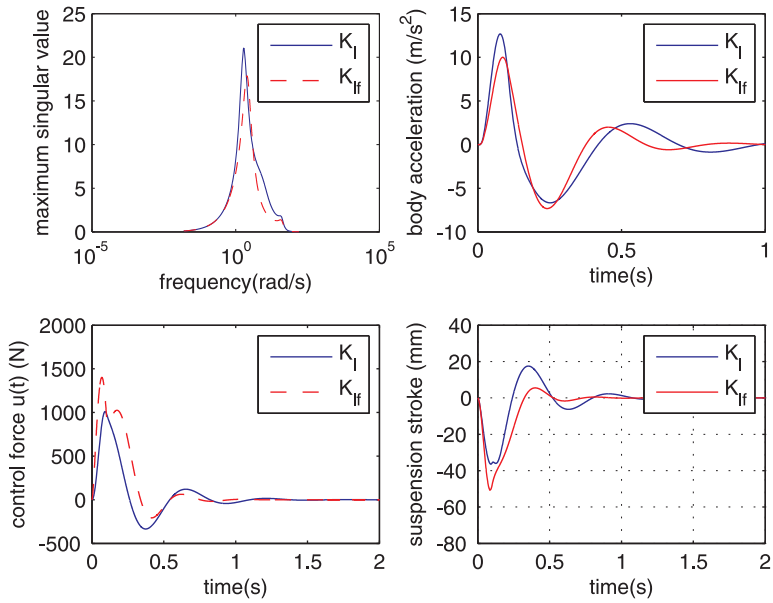


Figure 8. Bump responses of the closed-loop systems with load-dependent controller K_l and fault-tolerant controller K_{lf} with 60% actuator thrust loss.

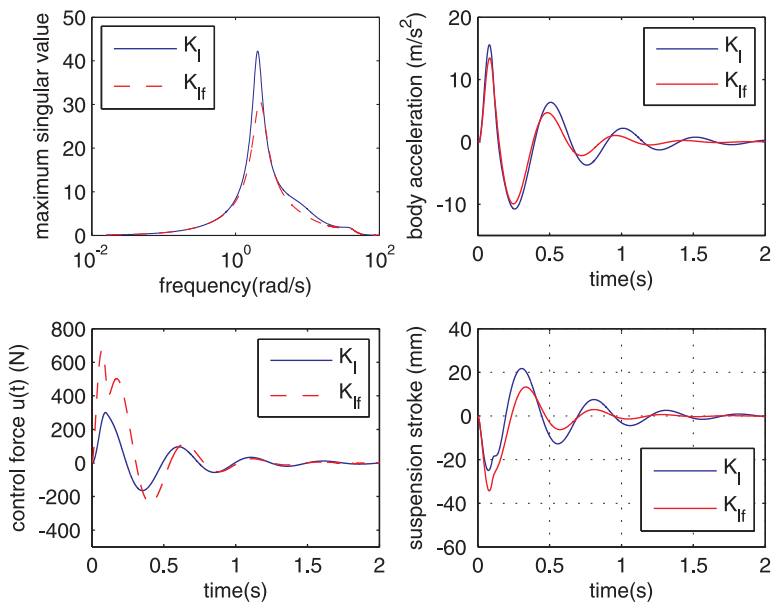


Figure 9. Bump responses of the closed-loop systems with load-dependent controller K_l and fault-tolerant controller K_{lf} with 90% actuator thrust loss.

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time domains, can be inferred from these figures. Moreover, we can see that the reliable controller K_{lf} is capable of providing a much steadier control force in the fault condition than the conventional controller K_l.

It is worth mentioning that most of the figures only illustrate the nominal case for clarity. However, the change tendencies can be readily indicated from the figures containing the uncertain parameter λ.

From the simulation results it can be seen that our controller design approach provides robust and reliable asymptotical stability. The reliable controller K_{lf} can maintain a satisfactory performance in spite of parameter uncertainties and the partial fault in the actuator, which makes it more practical with more application potentials than the regular controller design approaches.

5. CONCLUSIONS

This paper has dealt with the problem of multi-objective robust H_∞ reliable control for a class of semi-active seat suspension systems with a human body model by proposing a load-dependent state-feedback controller. A mathematical human body model has been employed for a better insight into the vibration control performance of the system, and polytopic parameter uncertainties have been introduced to characterize the real uncertain situation. The H_∞ performance has been established via a parameter-dependent Lyapunov approach and the controller design has been cast into a convex optimization problem with LMI constraints via some algebraic manipulations. The desired controller can then be achieved by solving the corresponding LMIs. Finally, a design example has been given to demonstrate the effectiveness and advantages of the proposed controller design approach.

APPENDIX. PROOF OF THEOREM 1

To begin with, we define a Lyapunov–Krasovskii functional candidate for system 14 as

$$V(t, \lambda) = x^T(t)P(\lambda)x(t), \tag{A1}$$

where $P(\lambda) > 0$ is to be determined.

Then, the derivative of $V(t, \lambda)$ along the solution of system 14 is given by

$$\begin{aligned}\dot{V}(t, \lambda) &= \dot{x}^T(t)P(\lambda)x(t) + x^T(t)P(\lambda)\dot{x}(t)\\ &= x^T(t)\Pi_{11}(\lambda)x(t) + x^T(t)P(\lambda)B_w(\lambda)w(t) + w^T(t)B_w^T(\lambda)P(\lambda)x(t),\\ &= \zeta^T(t)\Pi(\lambda)\zeta(t),\end{aligned} \tag{A.2}$$

where $\zeta^T(t) = \begin{bmatrix} x^T(t) & w^T(t) \end{bmatrix}$ and

$$\Pi(\lambda) = \begin{bmatrix} \Pi_{11}(\lambda) & P(\lambda)B_w(\lambda) \\ * & 0 \end{bmatrix}, \tag{A.3}$$

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with $\Pi_{11}(\lambda) = \text{sym}(P(\lambda)A(\lambda) + P(\lambda)B(\lambda)M_{a0}K_a(\lambda) + P(\lambda)B(\lambda)M_{a0}L_aK_a(\lambda))$.

Now, we establish the asymptotic stability of the system in equation 14 with $w(t) = 0$, that is,

$$\dot{x}(t) = (A(\lambda) + B(\lambda)K(\lambda))x(t).$$

For the above system, $\dot{V}(t, \lambda)$ in equation A.2 reduces to

$$\dot{V}(t, \lambda) = x^T(t)\Pi_{11}(\lambda)x(t). \tag{A.4}$$

By the Schur complement, equation 18 in Theorem 1 guarantees $\Pi_{11}(\lambda) < 0$, which further leads to $\dot{V}(t, \lambda) < 0$ for any $x(t) \neq 0$. Therefore, we conclude that the system in equation 14 with $w(t) = 0$, parameter uncertainty equation 15, and actuator fault equation 13 is robust asymptotically stable.

Next, we will investigate the H_∞ performance of the seat suspension system under zero initial condition. Consider the following index:

$$J \triangleq \int_0^\infty \left[z_{o1}^T(t)z_{o1}(t) - \gamma^2 w^T(t)w(t)\right] dt. \tag{A.5}$$

Then we have

$$J \leq \int_0^\infty \left[z_{o1}^T(t)z_{o1}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, \lambda)\right] dt. \tag{A.6}$$

Via some algebraic manipulations and the Schur complement, it is not difficult to obtain

$$z_{o1}^T(t)z_{o1}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, \lambda) = \zeta^T(t, \alpha)\hat{\Pi}(\lambda)\zeta(t, \alpha), \tag{A.7}$$

where

$$\hat{\Pi}(\lambda) = \begin{bmatrix} \Pi_{11}(\lambda) + C_1^T(\lambda)C_1(\lambda) & P(\lambda)B_w(\lambda) \\ * & -\gamma^2 \end{bmatrix}. \tag{A.8}$$

After performing a congruence transformation to equation A.8 with $\left\{P^{-1}(\lambda), I\right\}$ and defining $\bar{P}(\lambda) = P^{-1}(\lambda)$, we have an equivalent matrix denoted as

$$\bar{\Pi}(\lambda) = \begin{bmatrix} \bar{\Pi}_{11}(\lambda) + \bar{P}(\lambda)C_1^T(\lambda)C_1(\lambda)\bar{P}(\lambda) & B_w(\lambda) \\ * & -\gamma^2 \end{bmatrix}, \tag{A.9}$$

where

$$\bar{\Pi}_{11}(\lambda) = \text{sym}(A(\lambda)\bar{P}(\lambda) + B(\lambda)M_{a0}K_a(\lambda)\bar{P}(\lambda) + B(\lambda)M_{a0}L_aK_a(\lambda)\bar{P}(\lambda)). \tag{A.10}$$

Given the structure of M_{a0} and L_a, it can be easily obtained that $M_{a0}L_a = L_aM_{a0}$. Thus, by defining $\bar{K}_a(\lambda) = K_a(\lambda)\bar{P}(\lambda)$, we have

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$$\bar{\Pi}_{11}(\lambda) = \mathrm{sym}(A(\lambda)\bar{P}(\lambda) + B(\lambda)M_{a0}\bar{K}_a(\lambda) + B(\lambda)L_aM_{a0}\bar{K}_a(\lambda)). \tag{A.11}$$

By Lemma 1, for scalar $\varepsilon_{a3} > 0$, it holds that

$$\begin{aligned} B(\lambda)L_aM_{a0}\bar{K}_a(\lambda) + \bar{K}_a^T(\lambda)L_a^TM_{a0}^TB^T(\lambda) \;\le\; & \varepsilon_{a3}B(\lambda)J_aB^T(\lambda) \\ & + \varepsilon_{a3}^{-1}\bar{K}_a^T(\lambda)M_{a0}^TJ_aM_{a0}\bar{K}_a(\lambda). \end{aligned} \tag{A.12}$$

Therefore,

$$\bar{\Pi}(\lambda) \le \begin{bmatrix} \Phi(\lambda) + \bar{P}(\lambda)C_1^T(\lambda)C_1(\lambda)\bar{P}(\lambda) & B_w(\lambda) \\ * & -\gamma^2 \end{bmatrix}, \tag{A.13}$$

where

$$\Phi(\lambda) = \mathrm{sym}(A(\lambda)\bar{P}(\lambda)+B(\lambda)M_{a0}\bar{K}_a(\lambda))+\varepsilon_{a3}B(\lambda)J_aB^T(\lambda)+\varepsilon_{a3}^{-1}\bar{K}_a^T(\lambda)M_{a0}^TJ_aM_{a0}\bar{K}_a(\lambda).$$

Notice that the right-hand side of equation A.13 is equivalent to equation 18 in Theorem 1 according to the Schur complement. Thus, if equation 18 holds, i.e. $\bar{\Pi}(\lambda) < 0$, we have

$$z_{o1}^T(t)z_{o1}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t,\lambda) < 0 \tag{A.14}$$

for any non-zero $w \in L_2[0,\infty)$, which indicates that $J < 0$. Hence $\|z_{o1}\|_2 < \gamma\,\|w\|_2$ is guaranteed for any non-zero $w \in L_2[0,\infty)$.

Finally, we give the sufficient conditions that guarantee the suspension stroke and maximum control force constraints. Inequality A.14 indicates that $\dot{V}(t) - \gamma^2 w^T(t)w(t) < 0$. Integrating both sides of the above inequality from zero to any $t > 0$, we obtain

$$V(t) - V(0) < \gamma^2 \int_0^t w^T(t)w(t)dt < \gamma^2\,\|w\|\,. \tag{A.15}$$

Then it is true that $x^T(t)Px(t) < \rho$, where $\rho = \gamma^2 w_{\max}$. Consider

$$\begin{aligned} \max_{t>0}|z_{o2}(t)|^2 &= \max_{t>0}\left\|x^T(t)C_2^T(\lambda)C_2(\lambda)x(t)\right\|_2 \\ &= \max_{t>0}\left\|x^T(t)P^{\frac{1}{2}}(\lambda)P^{-\frac{1}{2}}(\lambda)C_2^T(\lambda)C_2(\lambda)P^{-\frac{1}{2}}(\lambda)P^{\frac{1}{2}}(\lambda)x(t)\right\|_2 \\ &< \rho\cdot\theta_{\max}(P^{-\frac{1}{2}}(\lambda)C_2^T(\lambda)C_2(\lambda)P^{-\frac{1}{2}}(\lambda)), \\ \max_{t>0}|u(t)|^2 &= \max_{t>0}\left\|x^T(t)K^T(\lambda)K(\lambda)x(t)\right\|_2 < \rho\cdot\theta_{\max}(P^{-\frac{1}{2}}(\lambda)K^T(\lambda)K(\lambda)P^{-\frac{1}{2}}(\lambda)), \end{aligned}$$

where $\theta_{\max}(\cdot)$ represents the maximal eigenvalue. From the above inequalities, we know that the constraints 8 and 9 are guaranteed if

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$$\rho \cdot P^{-\frac{1}{2}}(\lambda)C_2^T(\lambda)C_2(\lambda)P^{-\frac{1}{2}}(\lambda) < z_{\max}^2 I,$$
$$\rho \cdot P^{-\frac{1}{2}}(\lambda)K^T(\lambda)K(\lambda)P^{-\frac{1}{2}}(\lambda) < u_{\max}^2 I. \quad \text{(A.16)}$$

By the Schur complement and Lemma 1, equation A.16 is equivalent to equation 19 in Theorem 1, and the proof is complete.

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