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Vibration suppression using two-terminal flywheel. Part II: application to vehicle passive suspension

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Abstract

The two-terminal flywheel introduced in Part I is suggested to be attached in parallel to the suspension strut whose vibration suppression performance improvement is accordingly investigated in this paper. The passenger comfort, tire grip and suspension deflection are respectively taken as the vibration suppression indices to design the optimal parameters of the proposed and conventional passive suspensions. The optimal single-objective performances of both suspensions are compared to each other in actual dynamic parameter ranges. Due to the three conflicting and non-commensurable performance objectives, the Chebyshev goal programming approach is subsequently applied to find a multi-objective compromise for a design case. The non-ideal factors of the inverse screw transmission mechanism, which is modeled in Part I, are also taken into consideration to discuss the influence on the vibration suppression performance. The results show that, owing to the presence of the two-terminal flywheel, the proposed suspension employing this novel component significantly outperforms the conventional one in terms of both passenger comfort and tire grip, with practically identical suspension deflection performance.

Keywords

Flywheel, passive suspension, two-terminal manipulation, vehicle, vibration suppression

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1. Introduction

This paper investigates the vehicle passive suspension application of the two-terminal flywheel reported in Part I (Li et al., 2011b) of the study. In a vehicle, a suspension system is employed between the body and the wheel to isolate the disturbance resulting from the road roughness and vibration. Although active or semi-active suspensions may lead to better performance in terms of vibration suppression, passive suspensions still have the largest market share due to their acceptable performance and simple strut structure (Yim et al., 2010).

The mechanical model of a conventional vehicle passive suspension strut can be regarded as the parallel combination of a damper and a spring. Although the mass is also one of three basic vibration components, it has only one genuine manipulation terminal (Willems, 2010), and thus difficult to be directly integrated into a passive suspension where two terminals are necessary to connect to the body and the wheel respectively.

To further improve the performance of the passive suspension by employing inertial force, a dynamic vibration absorber consisting of a mass, a damper and a spring was introduced to reduce the wheel resonance without penalty on the ride comfort (Liu and Coppola, 2010). Due to its larger attached mass, however, the application of such a design has been quite limited with probably only one exception in the French subcompact Citroen 2 CV (Hrovt, 1997).

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In Part I of this work, we modeled and characterized a two-terminal flywheel with an inverse screw transmission. The results show that the device can be regarded as an ideal two-terminal mass in much of the frequency band for practical applications. The proposed approach can release the virtual grounded terminal for the flywheel, which has more extensive applications in the vibration suppression. Compared to classical mass components, the two-terminal flywheel has two main advantages: it provides two genuine terminals and has the ability to amplify inertial mass effect. The proposed device makes it possible for the mass to be integrated into mechanical systems with the topology like springs and dampers, whereas the amplified inertial mass effect makes it feasible to use smaller mass.

Based on the modeling and characterization of the proposed component in Part I, this paper aims at applying the proposed two-terminal flywheel to vehicle vibration suppression. The optimal parametric design is presented to examine the performance of the proposed suspension in terms of multiple single-objective and multi-objective.Accordingly the paper is structured as follows. Section 2 provides the background information of suspension vibration suppression performance measures. Section 3 derives the optimal parameter combination and performance in terms of three separate criteria, i.e., passenger comfort, tire grip and suspension deflection for suspensions with and without the two-terminal flywheel. Section 4 presents a Chebyshev goal programming approach for multi-objective optimal design which is applied to a passive suspension design example. The non-ideal factors of the two-terminal flywheel are also taken into account to investigate their influence on the optimal performance drift. The conclusions are drawn in Section 5.

2. Vibration suppression performance measures

Before presenting the performance measures, the passive suspension model is introduced in the following subsection.

2.1. Passive suspension model

Figure 1(a) shows the elementary two degrees-of-freedom quarter-car model where m_1, m_2, k_1, k_2, and c stand for the unsprung mass, the sprung mass, the tire stiffness, the strut stiffness and the strut damping, respectively. $x_0(t)$ is the disturbance generated by the road force $F_0(t)$, $x_2(t)$ is the absolute displacement of the vehicle body, and $x_{12}(t)$ is absolute displacement of the tire. Let $x_1(t) = x_2(t) - x_{12}(t)$ represent the relative displacement between the vehicle body and the tire. The governing equations in the Laplace domain for the suspension model (namely, S_1) shown in Figure 1(a) are given by

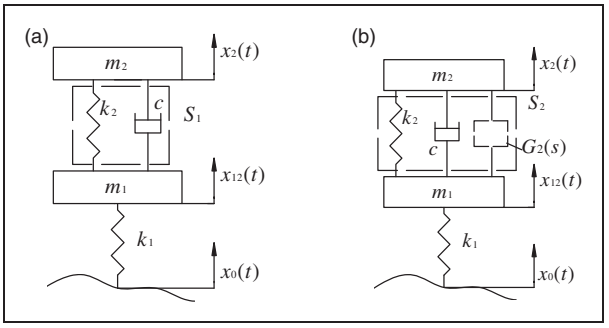


Figure 1. Two degrees-of-freedom quarter-car models. (a) is a conventional passive suspension (S_1) whose strut consists of a spring k_2 and a damper c. (b) is the proposed suspension (S_2) with an ideal two-terminal flywheel $G_2(s)$ in parallel.

$$\left.\begin{aligned} &m_2 s^2 \hat{x}_2 + cs\hat{x}_1 + k_2\hat{x}_1 = 0 \\ &m_1 s^2(\hat{x}_2 - \hat{x}_1) - cs\hat{x}_1 - k_2\hat{x}_1 + k_1(\hat{x}_2 - \hat{x}_1 - \hat{x}_0) = 0 \end{aligned}\right\} \tag{3}$$

where, '$\wedge$' represents the Laplace transform of the corresponding variables. The road disturbance is a white noise displacement $w(t)$ whose spectrum densities are expressed as (Hazra and Ghosh, 2009)

$$G_q(f) = \frac{1}{v} G_q(n_0)\left(\frac{n}{0.1}\right)^{-2} \tag{4}$$

where $f = \frac{\omega}{2\pi}$ is the motion frequency; v the velocity of the vehicle; $G_q(n_0)$ the road roughness parameter; and $n = f/v$ the wave number.

In order to take advantage of the inertial force for the conventional passive suspension, an inverse screw transmission-based two-terminal flywheel is suggested to attach to the suspension strut in parallel. The ideal model of the proposed suspension (namely, S_2) is shown in Figure 1(b). The governing equations are given by

$$\left.\begin{aligned} &m_2 s^2 \hat{x}_2 + cs\hat{x}_1 + k_2\hat{x}_1 + G_2(s)\hat{x}_1 = 0 \\ &m_1 s^2(\hat{x}_2 - \hat{x}_1) - cs\hat{x}_1 - k_2\hat{x}_1 - G_2(s)\hat{x}_1 \\ &\quad + k_1(\hat{x}_2 - \hat{x}_1 - \hat{x}_0) = 0 \end{aligned}\right\} \tag{5}$$

where, $G_2(s) = m_t s^2$ (m_t is the ideal inertial mass of the two-terminal flywheel) is the ideal governing equation for the inverse screw transmission-based two-terminal flywheel shown in equation (21) in Part I.

With the introduction of following variables: tire natural frequency ω_1, body natural frequency ω_2,

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damping ratio ζ, tire body mass ratio α and inertial mass ratio β, i.e.

$$\left.\begin{array}{l}\omega_1=\sqrt{\frac{k_1}{m_1}}\\ \omega_2=\sqrt{\frac{k_2}{m_2}}\\ \zeta=\frac{c}{2\sqrt{k_2 m_2}}\\ \alpha=\frac{m_1}{m_2}\\ \beta=\frac{m_t}{m_2}\end{array}\right\} \tag{6}$$

equation (3) can be rewritten as

$$\left.\begin{array}{l}s^2\hat{x}_2+2\zeta\omega_2 s\hat{x}_1+\omega_2^2\hat{x}_1=0\\ s^2(\hat{x}_2-\hat{x}_1)-\frac{2\zeta\omega_2}{\alpha}s\hat{x}_1-\frac{\omega_2^2}{\alpha}\hat{x}_1+\omega_1^2(\hat{x}_2-\hat{x}_1-\hat{x}_0)=0\end{array}\right\} \tag{7}$$

Similarly, for the proposed suspension S_2, equation (5) can be recast as

$$\left.\begin{array}{l}s^2\hat{x}_2+2\zeta\omega_2 s\hat{x}_1+\omega_2^2\hat{x}_1+\beta s^2\hat{x}_1=0\\ s^2(\hat{x}_2-\hat{x}_1)-\frac{2\zeta\omega_2}{\alpha}s\hat{x}_1-\frac{\omega_2^2}{\alpha}\hat{x}_1-\frac{\beta}{\alpha}s^2\hat{x}_1\\ \quad+\omega_1^2(\hat{x}_2-\hat{x}_1-\hat{x}_0)=0\end{array}\right\} \tag{8}$$

For an actual vehicle, the tire natural frequency ω_1, body natural frequency ω_2, tire body mass ratio α, damping ratio ζ and inertial mass ratio β are generally selected in the following ranges: $0.3<\omega_1<100$, $0.3<\omega_2<100$ (GB/T 4783-1984), $0.01<\alpha<0.2$, $0.01\le\zeta\le10$ and $0\le\beta\le10$. Usually former three parameters are fixed to optimize the other parameters in accordance with the vibration suppression performance criteria.

2.2. Performance criteria

The vibration suppression performance of a suspension can be embodied by several criteria or objectives among which are three most commonly used ones, namely, passenger comfort, tire grip and suspension deflection. Since these indices are the basis for the multi-objective design of actual suspensions, the benefit analyses of the proposed suspension are firstly carried out in terms of the three single-objective criteria, respectively. The multi-objective parametric optimization of the two-terminal flywheel will be put forward in Section 4 to validate the actual design steps.

The passenger comfort related to the body vertical acceleration is defined as (Fleury and Mistrop, 2006)

$$\begin{aligned}M_{1i}&=\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{s^2\hat{x}_2}{\hat{x}_0}\right)^2G_q(f)\mathrm{d}\omega\right)^{\frac{1}{2}}\\&=0.2\pi\sqrt{vG_q(n_0)}\left\|H_{1i}(s)\right\|_2\end{aligned} \tag{9}$$

where, $i=1$, 2 for S_1, S_2 respectively, $H_{1i}(s)=s\hat{x}_2/\hat{x}_0$, and $\left\|H_{1i}(s)\right\|_2$ is the 2-norm of $H_{1i}(s)$. For a given road, $0.2\pi\sqrt{vG_q(n_0)}$ is constant, which means that the passenger comfort is determined mainly by transfer function$H_{1i}(s)$.

The tire grip is another important criterion to characterize the ride safety of the vehicle, which is specified by

$$M_{2i}=0.2\pi\sqrt{vG_q(n_0)}\left\|H_{2i}(s)\right\|_2 \tag{10}$$

where,

$$H_{2i}(s)=(\hat{x}_2-\hat{x}_1-\hat{x}_0)/(s\hat{x}_0)$$

The suspension deflection is related to the vehicle center of gravity and the suspension life. Proceeding in a similar way, when $H_{3i}(s)=\hat{x}_1/(s\hat{x}_0)$, the suspension deflection is given by

$$M_{3i}=0.2\pi\sqrt{vG_q(n_0)}\left\|H_{3i}(s)\right\|_2 \tag{11}$$

It should be noted that, the lower the three objective values, the better the suspension is. Hence the performance optimization issue is to determine concerning parameters such that M_1, M_2 and M_3 can be minimized. However, it is unlikely to achieve the best solutions for all the three objectives simultaneously. In this study, the Chebyshev goal programming method will be applied to minimize the worst deviation from any single objective-optimum (Xu and Liang, 2005), which will be discussed later in Section 4.

3. Application optimizations and performance comparisons

In following subsections, we will discuss the optimal vibration suppression performance in terms of the passenger comfort, tire grip and suspension deflection respectively. In each subsection, the counterparts of the conventional suspension are also derived so as to compare the contribution of the proposed two-terminal flywheel.

3.1. Passenger comfort benefit with the proposed component

For the conventional passive suspension S_1 depicted by equation (7), we have

$$H_{11}(s)=\frac{s\omega_1^2\left(2\xi\omega_2 s+\omega_2^2\right)}{\left(2\xi\omega_2 s+\omega_2^2\right)\left(s^2+\omega_1^2\right)+s^2\left(s^2+\frac{2\xi\omega_2}{\alpha}s+\frac{\omega_2^2}{\alpha}+\omega_1^2\right)} \tag{12}$$

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Let $s = \mathrm{j}\omega$ and rewrite equation (12) as

$$H_{11}(\mathrm{j}\omega) = \frac{-\mathrm{j}\omega^3 B_3 - \omega^3 B_2 + \mathrm{j}\omega B_1 + B_0}{\omega^4 A_4 - \mathrm{j}\omega^3 A_3 - \omega^3 A_2 + \mathrm{j}\omega A_1 + A_0} \tag{13}$$

According to the literature (Crandall and Mark, 1963), the integral of the foregoing equation is

$$\begin{aligned}\int_{-\infty}^{+\infty} (H_{11}(\mathrm{j}\omega))^2 \mathrm{d}\omega = \frac{\pi}{D}\Bigg(&\frac{(A_2A_3 - A_1A_4)B_0^2}{A_0} \\ &+ A_3\left(B_1^2 - 2B_0B_2\right) + A_1\left(B_2^2 - 2B_1B_3\right) \\ &+ \frac{(A_1A_2 - A_0A_3)B_3^2}{A_4}\Bigg)\end{aligned} \tag{14}$$

where $D = A_1(A_2A_3 - A_1A_4) - A_0A_3^2$. Combining equations (12) to (14) gives

$$\left\|H_{11}(s)\right\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} (H_{11}(\mathrm{j}\omega))^2 \mathrm{d}\omega\right)^{1/2} = \sqrt{\frac{a_{11}}{\xi} + a_{12}\xi} \tag{15}$$

where, $a_{11} = \frac{(\alpha+1)\omega_2^3}{4}$, $a_{12} = \alpha\omega_1^2\omega_2$. Owing to the positive real number ξ, α, ω_1 and ω_2, there are $a_{11} > 0$ and $a_{12} > 0$ which leads to

$$\frac{\mathrm{d}^2\left(\frac{a_{11}}{\xi} + a_{12}\xi\right)}{\mathrm{d}\xi^2} = \frac{2a_{11}}{\xi^3} \geq 0 \leftrightarrow \frac{\mathrm{d}^2\left(\left\|H_{11}(s)\right\|_2\right)}{\mathrm{d}\xi^2} \geq 0 \tag{16}$$

Hence $\left\|H_{11}(s)\right\|_2$ is a concave function with respective to ξ in the positive real domain. In the other words, there is a unique optimal ξ related to the minimal $\left\|H_{11}(s)\right\|_2$. This optimal damping ratio, named $\xi_{11\mathrm{opt}}$ (Subscript 'opt' means optimal parameter. The same below.), is the real, positive solution of following equation

$$\frac{\mathrm{d}\left\|H_{11}(s)\right\|_2}{\mathrm{d}\xi} = 0 \tag{17}$$

Solving above equation yields

$$\xi_{11\mathrm{opt}} = \frac{\omega_2}{2\omega_1}\sqrt{\frac{\alpha+1}{\alpha}} \tag{18}$$

Substituting equations (15) and (18) into equation (9), the optimal passenger comfort for the conventional passive suspension is obtained as

$$M_{11\mathrm{opt}} = 0.2\pi\sqrt{vG_q(n_0)}\sqrt{\frac{\omega_1\omega_2^2(1+\alpha)}{\sqrt{1+1/\alpha}}} \tag{19}$$

In contrast to the conventional passive suspension, for the proposed suspension S_2 described by equation (8), the following is obtained:

$$H_{12}(s) = \frac{s\omega_1^2\left(\beta s^2 + 2\xi\omega_2 s + \omega_2^2\right)}{\left(\begin{array}{l}\left(\beta s^2 + 2\xi\omega_2 s + \omega_2^2\right) \times \left(s^2 + \omega_1^2\right) \\ + s^2\left(s^2 + \frac{\beta}{\alpha}s^2 + \frac{2\xi\omega_2}{\alpha}s + \frac{\omega_2^2}{\alpha} + \omega_1^2\right)\end{array}\right)} \tag{20}$$

Proceeding in a similar way leads to

$$\begin{aligned}\left\|H_{12}(s)\right\|_2 &= \left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} (H_{12}(\mathrm{j}\omega))^2 \mathrm{d}\omega\right)^{1/2} \\ &= \sqrt{a_{21}\xi + \frac{a_{22}\beta^3 + a_{23}\beta^2 + a_{24}\beta + a_{25}}{(a_{26}\beta + a_{27})\xi}}\end{aligned} \tag{21}$$

where,

$$\begin{aligned}a_{21} &= \alpha\omega_1^2\omega_2 \\ a_{22} &= \omega_1^4\alpha^2 \\ a_{23} &= \alpha^2\omega_1^4 - 2\alpha^2\omega_1^2\omega_2^2 - 2\alpha\omega_1^2\omega_2^2 \\ a_{24} &= \alpha^2\omega_2^4 - 2\alpha^2\omega_1^2\omega_2^2 + 2\alpha\omega_2^4 + \omega_2^4 \\ a_{25} &= \alpha^2\omega_2^4 + \alpha\omega_2^4 \\ a_{26} &= 4\alpha\omega_2 + 4\omega_2\end{aligned}$$

and

$$a_{27} = 4\alpha\omega_2.$$

It is easy to prove that $\left\|H_{12}(s)\right\|_2$ is also a concave function with relative to ξ.

Hence there is a optimal damping ratio $\xi_{12\mathrm{opt}}$ which can be obtained from $\frac{\mathrm{d}\left\|H_{12}(s)\right\|_2}{\mathrm{d}\xi} = 0$, that is

$$\xi_{12\mathrm{opt}} = \sqrt{\frac{\alpha_{22}\beta^3 + \alpha_{23}\beta^2 + \alpha_{24}\beta + \alpha_{25}}{\alpha_{21}\alpha_{27} + \alpha_{21}\alpha_{26}\beta}} \tag{22}$$

Due to the inclusion of the two-terminal mass, we need to also find the optimal inertial mass ratio β. Setting $\frac{\mathrm{d}\left\|H_{12}(s)\right\|_2}{\mathrm{d}\beta} = 0$ yields

$$\begin{aligned}\frac{\mathrm{d}\left\|H_{12}(s)\right\|_2}{\mathrm{d}\beta} &= \frac{3\alpha_{22}\beta^2 + 2\alpha_{23}\beta + \alpha_{24}}{\xi(\alpha_{27} + \alpha_{26}\beta)} \\ &\quad - \frac{\alpha_{26}\left(\alpha_{22}\beta^3 + \alpha_{23}\beta^2 + \alpha_{24}\beta + \alpha_{25}\right)}{\xi(\alpha_{27} + \alpha_{26}\beta)^2} = 0\end{aligned} \tag{23}$$

There are three solutions for this equation. Only one of them is real and positive, which is selected as the optimal inertial mass ratio $\beta_{12\mathrm{opt}}$. Substituting equations (21), (22) and (23) into equation (9), the optimal passenger comfort for suspension S_2 can be obtained. Due to

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space limit, the analytical solution of $M_{12\text{opt}}$ is omitted here.

Figure 2 illustrates the comparison between suspension models with and without the two-terminal flywheel (i.e. S_1 and S_2) for tire body mass ratios $\alpha = (0.01,\ 0.03,\ 0.06,\ 0.1)$, under possible vehicle parameter ranges. From Figures 2(a) to (h) we can see that the suspension with the two-terminal flywheel has much smaller M_{1i}(in the order of 10^2 to 10^3) for all α values under consideration, which means that the optimal passenger comfort level of S_2 is much higher than that of S_1. The performance contribution of the proposed two-terminal flywheel is more clearly illustrated by $M_{12\text{opt}}/\ M_{11\text{opt}}$ in Figure 2(i) for $\alpha = 0.01$. It is also noticed that the magnitude of improvement increases with the tire natural frequency ω_1 and body natural frequency ω_2. Figures 2(j) to (k) show the optimal damping ratios of two suspensions for $\alpha = 0.01$. Owing to the presence of the proposed two-terminal flywheel, a smaller damping ratio ξ_{opt} is necessary for the suggested suspension, which means that smaller energy dissipation component can also lead to better performance. Our result (Figure 2(l)) also indicates that the increase of ω_2/ω_1leads to reduced inertial mass ratio to achieve the optimal passenger comfort level.

3.2. Tire grip benefit

For the conventional passive suspension S_1 without the two-terminal flywheel, we have

$$H_{21}(s) = \frac{\omega_1^2\left(s^2 + 2\xi\omega_2 s + \omega_2^2\right)}{s\left(2\xi\omega_2 s + \omega_2^2\right)\left(s^2 + \omega_1^2\right) + s^3\left(s^2 + \frac{2\xi\omega_2}{\alpha}s + \frac{\omega_2^2}{\alpha} + \omega_1^2\right)} - 1 \tag{24}$$

The 2-norm of $H_{21}(s)$ can be solved in accordance with equation (14), i.e.

$$\left\|H_{21}(s)\right\|_2 = \sqrt{b_{11}\xi + \frac{b_{12}}{\xi}} \tag{25}$$

with

$$b_{11} = \frac{(1+\alpha)^2\omega_2}{\alpha\omega_1^2}$$

and

$$b_{12} = \frac{\alpha^3\omega_1^4 - 2\alpha^3\omega_1^2\omega_2^2 - 2\alpha^2\omega_1^2\omega_2^2 + \alpha^3\omega_2^4 + 3\alpha^2\omega_2^4 + 3\alpha\omega_2^4 + \omega_2^4}{4\alpha^2\omega_1^4\omega_2}$$

The optimal damping ratio is then given by

$$\xi_{21\text{opt}} = \sqrt{\frac{b_{12}}{b_{11}}} \tag{26}$$

leading to the optimal tire grip

$$M_{21\text{opt}} = 0.2\pi\sqrt{vG_q(n_0)}\sqrt{2\sqrt{b_{11}b_{12}}} \tag{27}$$

On the other hand, for the proposed suspension S_2 having a two-terminal flywheel included, combining equations (8) and (10) gives

$$H_{22}(s) = \frac{\omega_1^2\left(s^2 + \beta s^2 + 2\xi\omega_2 s + \omega_2^2\right)}{\begin{pmatrix} s\left(\beta s^2 + 2\xi\omega_2 s + \omega_2^2\right)\left(s^2 + \omega_1^2\right) \\ +s^3\left(s^2 + \frac{\beta}{\alpha}s^2 + \frac{2\xi\omega_2}{\alpha}s + \frac{\omega_2^2}{\alpha} + \omega_1^2\right) \end{pmatrix}} - 1 \tag{28}$$

Following the approach similar to that for the conventional passive suspension, one has

$$\left\|H_{22}(s)\right\|_2 == \sqrt{b_{21}\xi + \frac{b_{22}\beta^2 + b_{23}\beta + b_{24}}{\xi}} \tag{29}$$

where

$$b_{21} = \frac{\alpha(2\omega_2 + 2\omega_2/\alpha)^2}{4\omega_2\omega_1^2}$$

$$b_{22} = \frac{1+\alpha}{4\omega_2}$$

$$b_{23} = \frac{2\omega_1^2\alpha + \omega_1^2 - 2\omega_2^2\alpha - 4\omega_2^2 - 2\omega_2^2/\alpha}{4\omega_2\omega_1^2}$$

and

$$b_{24} = (\omega_1^4\alpha - 2\omega_1^2\omega_2^4\alpha - 2\omega_1^2\omega_2^2 + \omega_2^4\alpha + 3\omega_2^4 + 3\omega_2^4/\alpha + \omega_2^4/\alpha^2)/(4\omega_2\omega_1^4)$$

For any fixed ω_1, ω_2 and α, the optimal inertial mass ratio is

$$\beta_{22\text{opt}} = \begin{cases} \frac{-b_{23}}{2b_{22}}, & b_{23} > 0 \\ 0, & b_{23} \le 0 \end{cases} \tag{30}$$

and the optimal damping ratio can be calculated as

$$\xi_{22\text{opt}} = \sqrt{\frac{b_{22}\beta^2 + b_{23}\beta + b_{24}}{b_{21}}} \tag{31}$$

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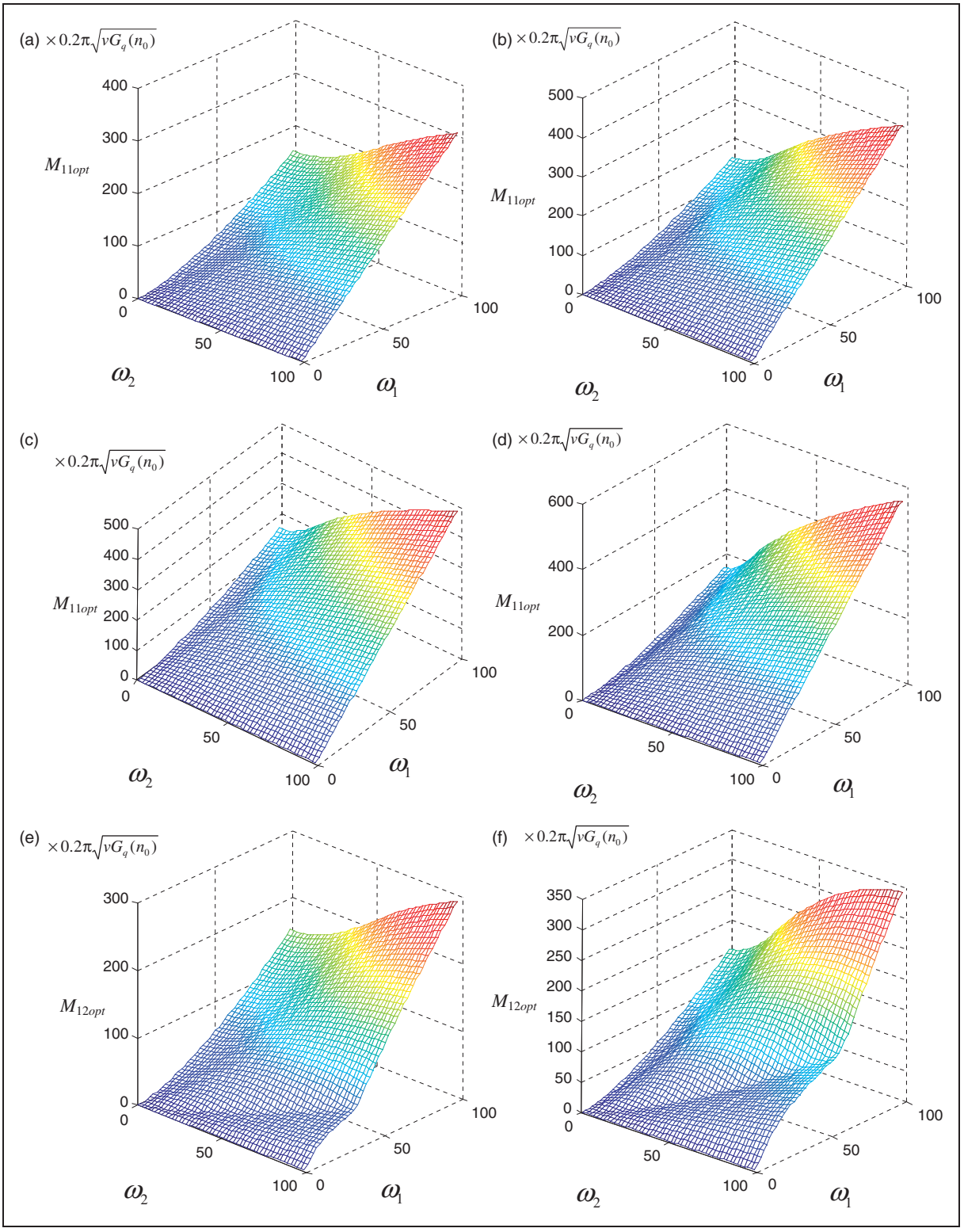


Figure 2. Comparison of optimal passenger comfort performance between S_1 and S_2. (a), (b), (c) and (d) are the optimal passenger comfort indexes of S_1 with $\alpha = (0.01, 0.03, 0.06, 0.1)$ respectively. (e), (f), (g) and (h) are those of S_2 with same α respectively. (i) illustrates the optimal passenger comfort performance improvements of the proposed suspension S_2 compared to the conventional suspension S_1. (j) shows the optimal damping ratio for S_1 when $\alpha = 0.01$. (k) and (l) respectively shows the optimal damping ratio and the optimal inertial mass ratio for S_2 with $\alpha = 0.1$.

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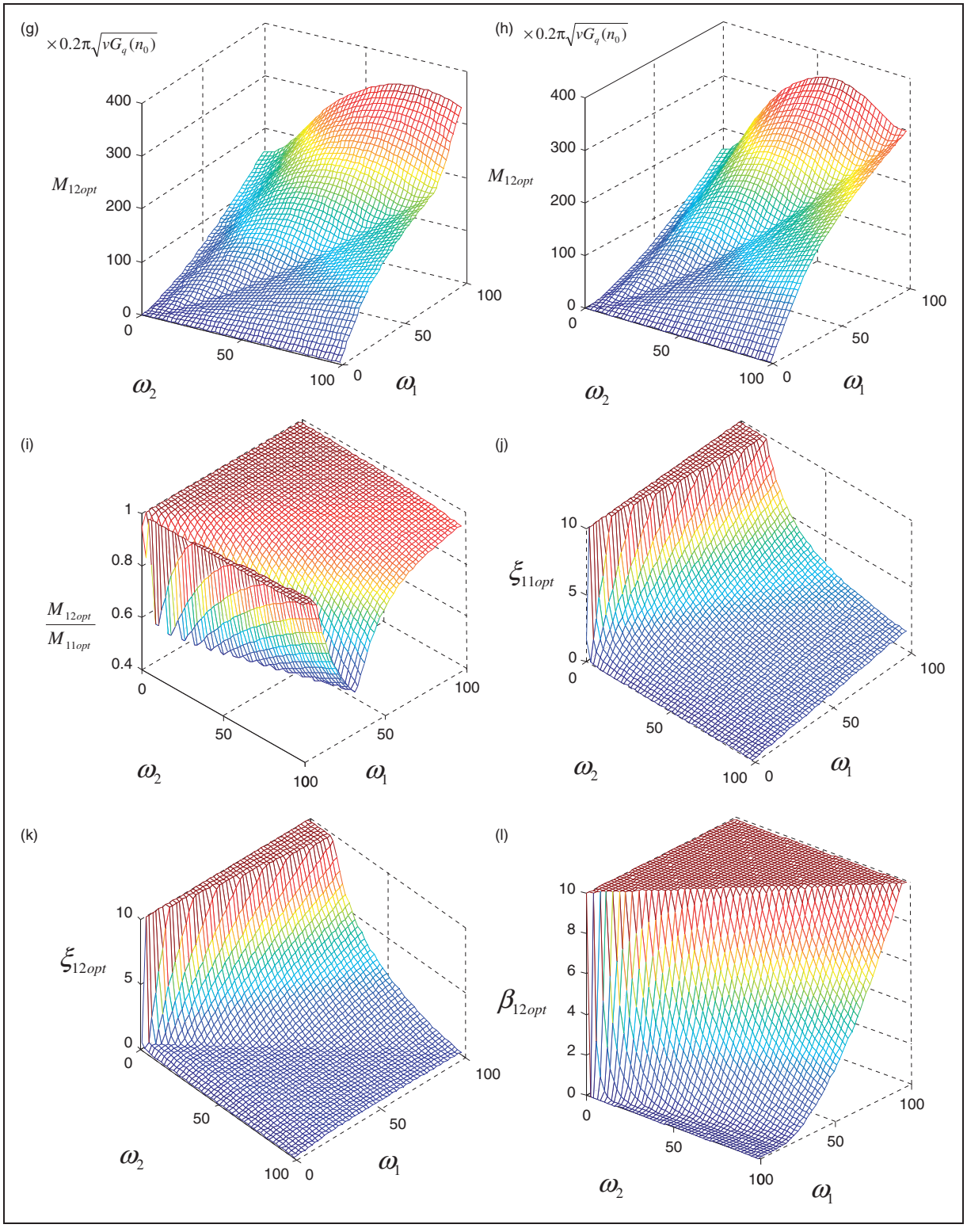


Figure 2. Continued.

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leading to the optimal tire grip performance

$$M_{22\text{opt}} = \begin{cases} 0.2\pi\sqrt{vG_q(n_0)}\sqrt[4]{\frac{4b_{21}b_{22}b_{24}-b_{21}b_{23}^2}{b_{22}}}, & b_{23} > 0 \\ 0.2\pi\sqrt{vG_q(n_0)}\sqrt[4]{4b_{21}b_{24}}, & b_{23} \le 0 \end{cases} \tag{32}$$

Figure 3 illustrates the comparison of the optimal tire grip performance between S_1 and S_2 (with and without two-terminal flywheel) in possible vehicle parameter ranges. Figures 3(a) to (d) describe the variety of $M_{21\text{opt}}$ values when $\alpha = (0.01, 0.03, 0.06, 0.1)$ respectively. Figures 3(e) to (h) depict the counterparts of $M_{22\text{opt}}$ under the same conditions. Figures 3(i) illustrates the performance improvement $M_{22\text{opt}}/M_{21\text{opt}}$ when $\alpha = 0.1$. It is clear that, thanks to the presence of a two-terminal flywheel, the proposed suspension yields better tire grip performance in these cases.

3.3. Comparison of suspension deflection performances

For suspensions S_1 and S_2, the transfer functions related to deflection are respectively

$$H_{31}(s) = \frac{-s\omega_1^2}{\left(2\xi\omega_2 s + \omega_2^2\right)\left(s^2 + \omega_1^2\right) + s^2\left(s^2 + \frac{2\xi\omega_2}{\alpha}s + \frac{\omega_2^2}{\alpha} + \omega_1^2\right)} \tag{33}$$

and

$$H_{32}(s) = \frac{-s\omega_1^2}{\left(\begin{array}{l}\left(\beta s^2 + 2\xi\omega_2 s + \omega_2^2\right)\left(s^2 + \omega_1^2\right) \\ \quad + s^2\left(s^2 + \frac{\beta}{\alpha}s^2 + \frac{2\xi\omega_2}{\alpha}s + \frac{\omega_2^2}{\alpha} + \omega_1^2\right)\end{array}\right)} \tag{34}$$

Considering equations (33) and (34), equation (11) yields

$$\left\|H_{31}(s)\right\|_2 = \left\|H_{32}(s)\right\|_2 = \sqrt{\frac{\alpha\omega_1^2}{4\xi\omega_2}} \tag{35}$$

and the optimal suspension dynamic displacement performance

$$M_{31\text{opt}} = M_{32\text{opt}} = 0.2\pi\sqrt{vG_q(n_0)}\sqrt{\frac{\alpha\omega_1^2}{4\xi\omega_2}} \tag{36}$$

The above results suggest that both suspensions have same optimal suspension deflection performance. In the other words, paralleling a two-terminal flywheel with the strut can not improve the deflection performance of the suspension.

4. Multi-objective design application case study

4.1. Chebyshev goal programming

It should be pointed out that the three design criteria described in the previous sections are non-commensurable and an optimal design in terms of one criterion may be inferior in terms of the other two. To achieve a "balanced" optimal design with respect to all the three objectives, the Chebyshev goal programming is adopted in this paper.

Besides the best single-objective performances $M_{1i\text{opt}}$, $M_{2i\text{opt}}$ and $M_{3i\text{opt}}$ ($i = 1,2$ for S_1, S_2 respectively) resulting from the optimal parameters $\xi_{1i\text{opt}}$, $\xi_{2i\text{opt}}$, $\xi_{3i\text{opt}}$ (and $\beta_{12\text{opt}}$, $\beta_{22\text{opt}}$, $\beta_{32\text{opt}}$ for suspension S_2), there are also the worst performances $M_{1i\text{wor}}$, $M_{2i\text{wor}}$ and $M_{3i\text{wor}}$ caused by $\xi_{1i\text{wor}}$, $\xi_{2i\text{wor}}$, $\xi_{3i\text{wor}}$ (and $\beta_{12\text{wor}}$, $\beta_{22\text{wor}}$, $\beta_{32\text{wor}}$ for suspension S_2, subscript 'wor' means the worst one) respectively for both suspensions in given ranges. Suppose that there is a multi-objective optimal parameter combination $\xi_{i\text{opt}}$ (and $\beta_{2\text{opt}}$) which is related to the multi-objective optima $M_{1i\text{mop}}$, $M_{2i\text{mop}}$ and $M_{3i\text{mop}}$ (subscript 'mop' denotes multi-objective optimum). Then the Chebyshev goal programming model can be formulated as follows

$$\min\ \delta_i \tag{37}$$

subject to

$$\left.\begin{array}{l} \delta_i \ge \frac{M_{1i\text{mop}} - M_{1i\text{opt}}}{M_{1i\text{wor}} - M_{1i\text{opt}}} \\ \delta_i \ge \frac{M_{2i\text{mop}} - M_{2i\text{opt}}}{M_{2i\text{wor}} - M_{2i\text{opt}}} \\ \delta_i \ge \frac{M_{3i\text{mop}} - M_{3i\text{opt}}}{M_{3i\text{wor}} - M_{3i\text{opt}}} \end{array}\right\}$$

and all other constraints in single objective design (e.g. all parameters are in the positive, real domain). Solving the foregoing model yields an overall optimal solution in terms of all the three design criteria.

4.2. Application case study

Here we present an application case to illustrate the Chebyshev goal programming design process, during which three single-objective design processes are also included.Consider a suspension with following given parameters: $m_1 = 30$ kg, $m_2 = 320$ kg, $k_1 = 100$ kN/m and $k_2 = 80$ kN/m. Suppose that the feasible ζ and β ranges are $0.1 \le \zeta \le 1$ and $0 \le \beta \le 1$. The optimum c_{opt} should, on one hand, be found with respect to the best

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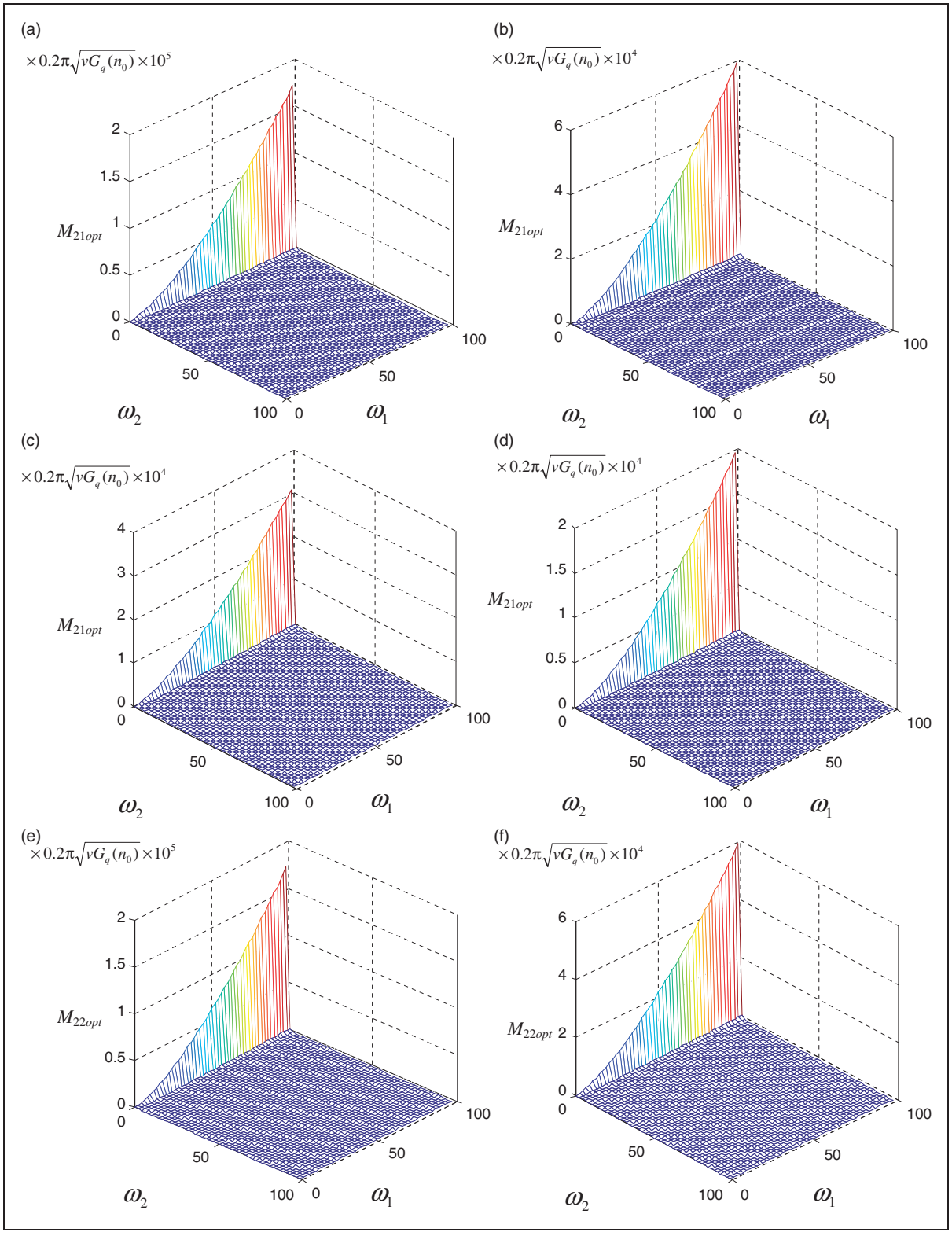


Figure 3. Comparison of optimal tire grip performance between S_1 and S_2. (a), (b), (c) and (d) are the optimal tire grip indexes of S_1 with $\alpha = (0.01, 0.03, 0.06, 0.1)$ respectively. (e), (f), (g) and (h) are those of S_2 with the same conditions. (i) shows the optimal tire grip performance comparison M_{22opt}/M_{21opt} between the proposed suspension S_2 and the conventional suspension S_1.

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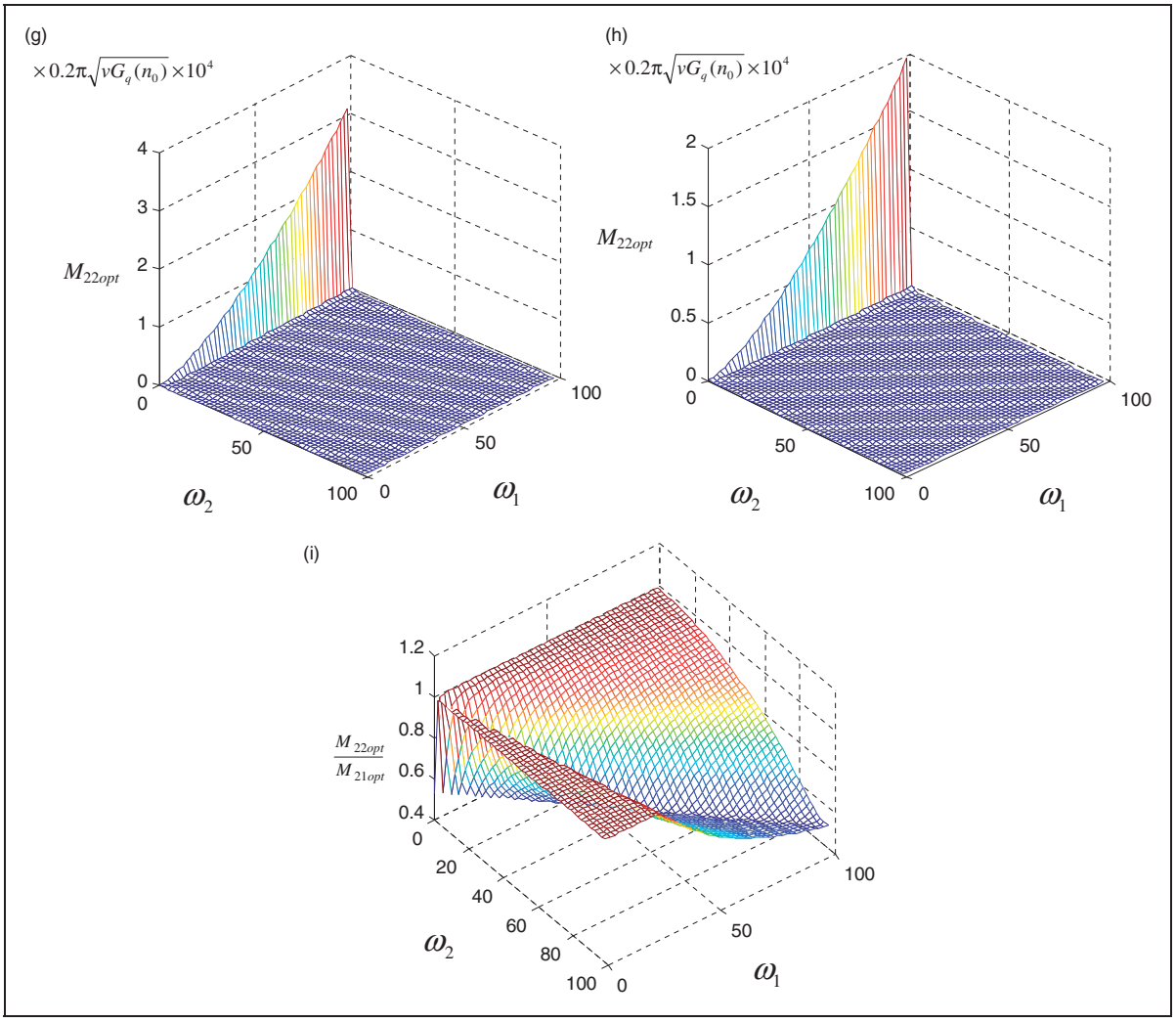


Figure 3. Continued.

performance of the conventional passive suspension. On the other hand, optimize both the unknown c_{opt} and $m_{t\text{opt}}$ for the suggested suspension. Both single- and multi-objectives should be taken into consideration for this design application.

Substituting the given values into equation (6) obtains $\omega_1 = 57.7350$, $\omega_2 = 15.8114$, $\alpha = 0.0938$. The first step is to calculate the single-objective parameters (both the single-objective optimal and the worst ones) in accordance with equations (12) to (36). Table 1 shows the optimal and the worst performances in terms of the three single objectives. The single-objective parametric design results further validate that the proposed two-terminal flywheel yields better optimal passenger comfort and tire grip performance, and the same optimal suspension deflection, comparing to the conventional one.

Solving the Chebyshev goal programming model expressed by equations (37) with parameters in Table 1 leads to the overall optimal designs for both the conventional and proposed approaches as summarized in Table 2. The multi-objective optimal design results show that the proposed suspension yields better passenger comfort and tire grip performance, with practically identical suspension deflect performance as compared with that of the conventional approach. Figures 4(a) and 4(b) show the numerical optimization process of the multi-objective design for both conventional and proposed suspensions.

4.3. Influence of non-ideal factors on performance

The ideal dynamics model relative to governing equation $G_2(s)$ of the inverse screw transmission-based two-terminal flywheel is suggested to be incorporated into the conventional passive suspension model, which is shown in Figure 1(b). The performance improvement brought about by the proposed component is

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Table 1. Comparison of single-objective design results between conventional and proposed approaches

Design Objective	Passenger comfort	Tire grip	Suspension deflection	
S_1	$M_{opt}(\times 0.2\pi\sqrt{vG_q(n_0)})$	67.9848	0.2328	2.2228
	c_{opt}(kN.s/m)	4.7329	4.5298	10.119
	$M_{wor}(\times 0.2\pi\sqrt{vG_q(n_0)})$	222.5279	0.3568	7.0293
	c_{wor}(kN.s/m)	10.119	1.0119	1.0119
S_2	$M_{opt}(\times 0.2\pi\sqrt{vG_q(n_0)})$	61.8897	0.2230	2.2228
	c_{opt}(kN.s/m)	3.9223	4.1583	10.119
	m_{topt}(kg)	137.7481	106.2857	0-320
	$M_{wor}(\times 0.2\pi\sqrt{vG_q(n_0)})$	112.2629	0.4306	7.0293
	c_{wor}(kN.s/m)	1.0119	1.0119	1.0119
	m_{twor}(kg)	320	320	0–320

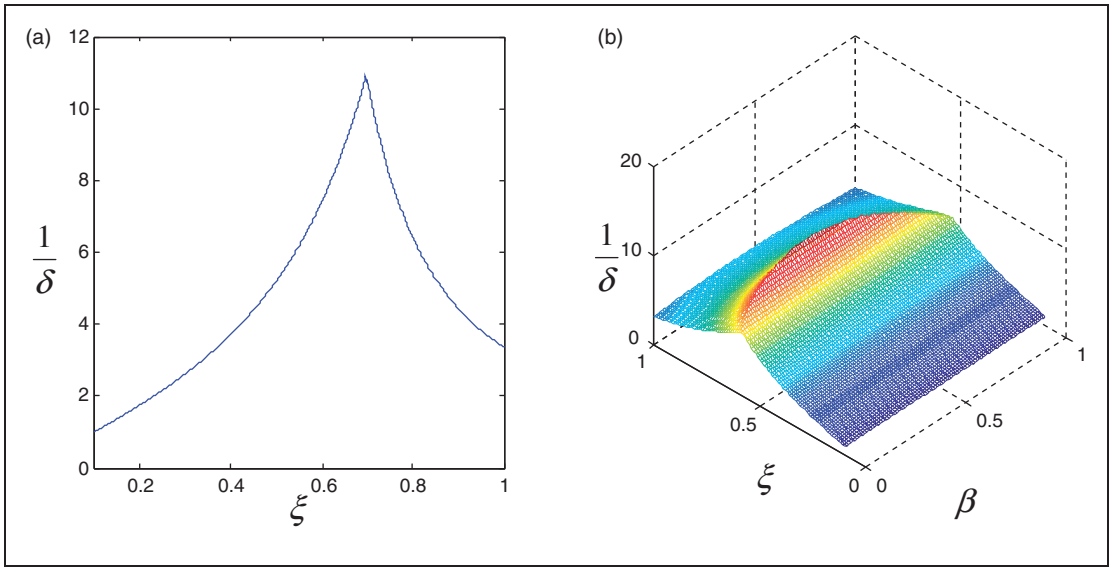


Figure 4. Numerical optimization of the multi-objective design. (a) shows the optimal damping ratio with respective to the maximal $1/\delta$ (i.e. min δ) for the conventional suspension S_1. (b) shows the variety of the Chebyshev goal δ (represented by $1/\delta$ in the figure) with ξ and β for multi-objective optimal design of the proposed suspension S_2.

Table 2. Multi-objective optimal design results

Suspension type	S_1	S_2
c_{mop}(kN.s/m)	7.0531	6.8912
m_{tmop}(kg)	/	137.6
δ	0.0915	0.0983
$M_{1mop}(\times 0.2\pi\sqrt{vG_q(n_0)})$	70.6728	66.7451
$M_{2mop}(\times 0.2\pi\sqrt{vG_q(n_0)})$	0.2441	0.2376
$M_{3mop}(\times 0.2\pi\sqrt{vG_q(n_0)})$	2.6625	2.6936

demonstrated using an example described in Subsection 4.2. However, as depicted in Part I, there are still some non-ideal factors included in an actual device, which is governed by $G_1(s)$.In this subsection, we will discuss the performance drift resulting from the non-ideal factors of the prototype device presented in Part I of this research.

Since the ideal governing equation $G_2(s)$ is replaced by the actual governing equation $G_1(s)$ of the device, equation (5) governing the aforementioned application case is recast as

$$\left.\begin{array}{l} m_2 s^2\hat{x}_2 + cs\hat{x}_1 + k_2\hat{x}_1 + G_1(s)\hat{x}_1 = 0 \\ m_1 s^2(\hat{x}_2 - \hat{x}_1) - cs\hat{x}_1 - k_2\hat{x}_1 - G_1(s)\hat{x}_1 \\ \quad + k_1(\hat{x}_2 - \hat{x}_1 - \hat{x}_0) = 0 \end{array}\right\} \tag{38}$$

where, $k_1 = 100 \times 10^3$, $k_2 = 80 \times 10^3$, $c = 6.8912 \times 10^3$, $G_1(s)$ can be obtained using equation (20) in Part I of this study where $m_t = 137.6$, $c_s = 1200$, $k_s = 2.1 \times 10^6$, and $c_f = 31.4$. The structure mass $m_r = 1.515$ of the prototype device is divided into two identical parts,

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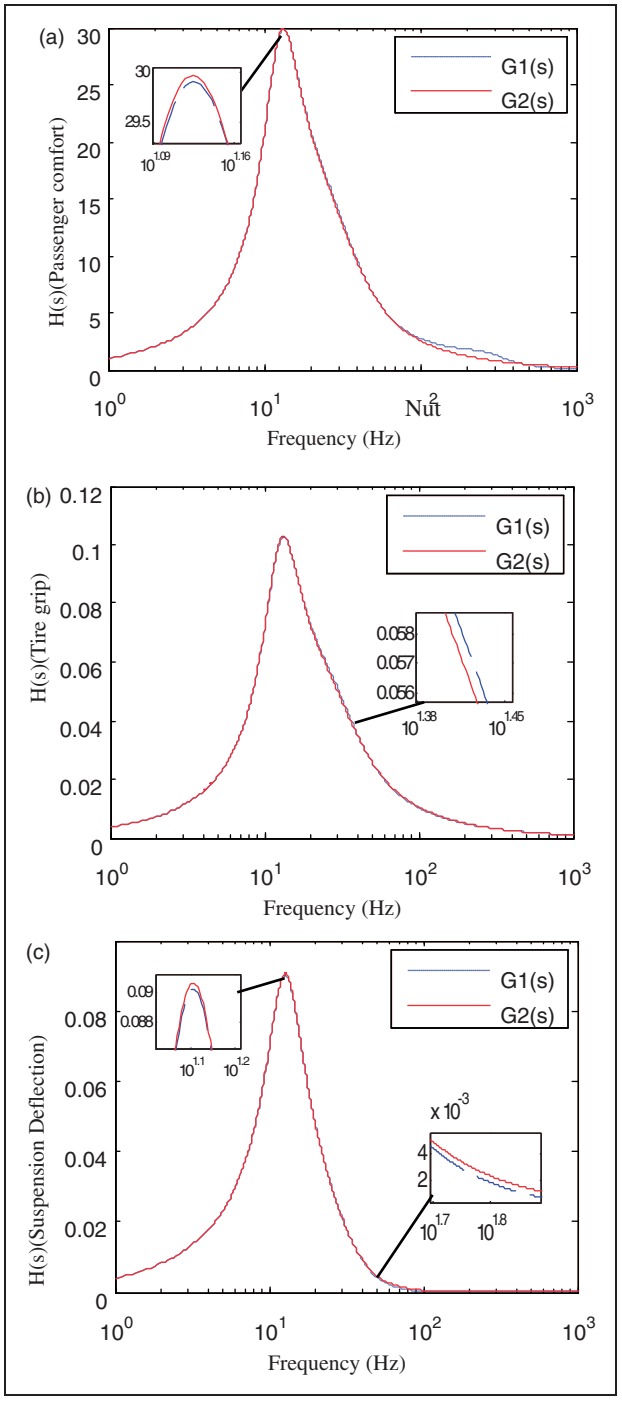


Figure 5. Transfer functions associated to suspension vibration suppression performances: (a) comparison between ideal and actual dynamics governing equations in terms of passenger comfort; (b) comparison in terms of tire grip; and (c) comparison in terms of suspension deflection.

one of which is added to $m_1 = 30.7575$ and the other for $m_2 = 320.7575$. The initial value of F is negligible.

As introduced in Section 2, the vibration suppression performance of the proposed vehicle passive suspension can be measured in terms of M_{12}, M_{22} and M_{32} which are directly related to three transfer functions, i.e. $H_{12}(s)$, $H_{22}(s)$ and $H_{32}(s)$. The three transfer functions incorporating the non-ideal factors are denoted $H'_{12}(s)$, $H'_{22}(s)$ and $H'_{32}(s)$, as expressed below

$$\left.\begin{aligned} H'_{12}(s) &= \frac{sx_2}{x_0} = \frac{k_1 s}{P} \\ H'_{22}(s) &= \frac{x_2 - x_1 - x_0}{sx_0} = \frac{k_1\left(1+\frac{m_2 s^2}{G_1(s)+cs+k_2}\right)}{sP} - \frac{1}{s} \\ H'_{32}(s) &= \frac{x_1}{sx_0} = \frac{-m_2 s k_1}{P(G_1(s)+cs+k_2)} \end{aligned}\right\} \quad (39)$$

where

$$P = \frac{m_2 s^2\left(m_1 s^2 + G_1(s) + cs + k_1 + k_2\right)}{G_1(s) + cs + k_2} + m_1 s^2 + k_1$$

Let $s = \mathrm{j}\omega$, $\omega \in [\omega_1, \omega_2]$. In this example, $\omega_1 = 1$ and $\omega_2 = 1000$ (Li et al., 2010, 2011) are selected to examine the performance drift in vibration response due to the difference between the ideal and the actual dynamic characteristics. Figures 5 (a), (b), and (c) show the comparisons between $|H_{12}(s)|$ and $|H'_{12}(s)|$, $|H_{22}(s)|$ and $|H'_{22}(s)|$, $|H_{32}(s)|$ and $|H'_{32}(s)|$ respectively.

As shown in Figure 5, the performance drifts due to the non-ideal factors are insignificant in terms of all three performance measures. Some small discrepancies are observed around 15 Hz and between 90 and 450 Hz for passenger comfort and suspension deflection (Figures 5(a) and (c)), and between 30 and 80 Hz for tire grip (Figure 5(b)). Therefore, the optimal design using the ideal model of the two-terminal flywheel can be applied to obtain acceptable results with much simplified computation.

5. Conclusions

Following the modeling and characterization of the two-terminal flywheel in Part I of this research, we propose that this component be applied to suppress vibration for a vehicle passive suspension. The proposed design has been analyzed in terms of three important design criteria, i.e., passenger comfort, tire grip and suspension deflection.

The tire natural frequency, the suspension natural frequency and the tire body mass ratio are assumed to be known to optimize the damping ratio and the inertial mass ratio to achieve the desirable performance. Compared to the conventional passive suspension, the proposed two-terminal flywheel has demonstrated superior passenger comfort and tire grip, with equal suspension deflection performance. To achieve a "balanced" optimal design in terms of the all the three criteria, we adopted the Chebyshev goal programming approach. The results of a case study demonstrate that the

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proposed design outperforms the conventional one in terms of passenger comfort and tire grip, with practically identical suspension deflection performance. It is also illustrated that the effects of non-ideal factors on the three performance measures are negligible and thus the ideal dynamics model can be directly applied.

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