International Journal of Information Technology & Decision Making
Vol. 12, No. 4 (2013) 729–755
© World Scientific Publishing Company
DOI: 10.1142/S0219622013500272

World Scientific
www.worldscientific.com

A FUZZY MULTI-CRITERIA DECISION ANALYSIS APPROACH FOR RETAIL LOCATION SELECTION

A. CAGRI TOLGA*
Department of Industrial Engineering
Galatasaray University
34357, Ortakoy, Istanbul, Turkey
ctolga@gsu.edu.tr

FATIH TUYSUZ
Department of Industrial Engineering
Istanbul University
34320, Avcilar, Istanbul, Turkey
fatih.tuysuz@istanbul.edu.tr

CENGIZ KAHRAMAN
Department of Industrial Engineering
Istanbul Technical University
34367, Macka, Istanbul, Turkey
kahramanc@itu.edu.tr

This paper proposes fuzzy multi-criteria decision-making approach integrated with fuzzy real option value theory. The applicability of the proposed method was shown on a real-world supermarket location selection problem. Based on the interviews with the experts, the evaluation criteria for retail location selection were identified. Then the network for fuzzy analytic network process (ANP) method was constructed. The fuzzy real option value for each alternative was calculated and used in the proposed approach as the representative of the financial dimension. Finally, the preference ranking of alternatives and the relative importance of the criteria were obtained. The significant contribution of the proposed approach is that it integrates the financial dimension (FROV) of the location problem methodologically with the multi-criteria characteristic (FANP) of the problem. Another importance of this study is the first usage of real options valuation in the area of location selection science.

Keywords: Location selection; multi-criteria; fuzzy; ANP; real options valuation, trinomial lattice approach.

1. Introduction

Facility location decision is a key element in any organization's strategic plan and often has a great effect on overall success of the firm. Location selection decisions are of high priority since such decisions involve long-term commitment of resources and generally represent a substantial investment which may affect the long-term

*Corresponding author.

profitability and sustainability of the firm. Another important property of the location selection decisions is that they are usually irreversible. Location analysis refers to the modeling, formulation and solution of class problems that can be defined as placing facilities in some given space. Location selection of public and private sector facilities such as plants, warehouses, retail outlets, terminals, storage yards, distribution centers, hospitals, police stations, etc. are examples of facility location problems.

The irreversibility of location selection decisions needs to be evaluated by a more complex method. Current engineering economy techniques (net present worth, decision tree method, etc.) usually discuss the returnable investment projects. Option based real options valuation overcomes these inadequacies. Real options give a right but not an obligation to make or not to make an investment for a certain period. For instance, investing in the expansion of a firm's factory gives the company the right to produce more but not the obligation. In addition, any investment can be postponed by utilizing real options.

Although the facility location decisions usually depend on the type of the business, there are many different approaches and techniques used in location selection literature. The use of each model or technique depends on the type of the problem on hand. For example, the factory location problems may involve a different set of criteria or objectives when compared to the retail location problems. Facility location problems are solved to minimize the total cost of serving all customers whereas retail location problems focus on how to identify a location that can maximize the number of the customers. The classification of the techniques and models used in location analysis can be done in a variety of ways. For different classification of techniques and models and for more details, see Refs. 1–4. The formulation and solution of each technique and model vary widely in terms of fundamental assumptions, mathematical complexity and computational performance. In general we can classify the location analysis approaches into two main categories: mathematical programming models (network, continuous, discrete models) and multi-criteria methods (simple additive weighting, analytic hierarchy process (AHP), analytic network process (ANP), TOPSIS), and sales forecasting models (time series methods, econometric methods, etc.).

Mathematical models use linear programming, integer programming, nonlinear programming, goal programming and dynamic programming to find the optimal location. The aim of these models is to maximize the utilization of the sources and minimize the overall cost and/or maximize the profit. Mathematical models can be either static-and-deterministic models or dynamic-and-stochastic models. Multi-criteria methods deal with the ranking and selection of one or more location sites from the alternatives. The appropriateness or suitability of a location mostly depends on the factors that are selected and evaluated together with their effects on organization's objectives and operations. The mathematical models and their extensions usually take into consideration a few factors and one or two objectives. Since the mathematical models seek optimal solutions as the number of the alternatives and

objectives increase, the complexity of the problem increases which results in a long solution procedure difficult to solve and may sometimes give no feasible solution. Due to the reason that location selection problem encompasses several independent criteria and each criterion has numerous detailed sub-criteria, location selection is a typical multiple criteria decision-making problem which requires both quantitative and qualitative factors to be evaluated.

The main problem for qualitative criteria is that, the values for such criteria are often imprecisely defined for the decision makers. In the evaluation of the qualitative criteria, the desired value and the weight of importance for the criteria are usually defined in linguistic terms. In crisp or conventional methods, it is difficult to express the character and significance of criteria exactly or clearly.[5] The conventional approaches tend to be less effective in dealing with the imprecision or vagueness nature of the linguistic assessment.[6] Due to this reason, the use of the fuzzy set theory and multi-criteria approach for evaluating location selection factors seems more convenient by allowing decision makers to express their ideas more adequately.

In this study, we present a fuzzy multi-criteria decision-making (MCDM) approach for location selection which we think this fuzzy model is more effective than the traditional ones. We propose a new model that integrates the vagueness of investment analysis and qualitative criteria of location selection. Vagueness and irreversibility of location selection are investigated by fuzzy real options analysis. We choose fuzzy ANP because this method takes both qualitative and quantitative criteria into account and considers dependency among criteria. Fuzzy real options valuation (FROV) by trinomial lattice approach is integrated to fuzzy ANP as a quantitative criterion. The main contribution of this study is the first use of FROV together with the fuzzy ANP method. Although fuzzy ANP and FROV methods have been widely applied in many real life problems separately, they have never been used together in such an integrated manner. We also think that this proposed approach makes a significant contribution to the location selection literature since the proposed method takes into account the two important characteristics of location selection problems which are financial evaluation (FROV) and multi-criteria evaluation (FANP).

The organization of the paper is as follows. In Sec. 2, a literature review on fuzzy multi-criteria methods for location selection is given. In Sec. 3, the fuzzy analytic network process (FANP), and in Sec. 4, FROV are presented. In Sec. 5, the proposed model for location selection is presented. In Sec. 6, an application of the proposed model for a retail location selection is given. Finally the conclusions are presented.

2. Literature Review: Fuzzy Multi-Criteria Approaches to Location Selection

Humans are more successful and efficient in making qualitative forecasting in comparison to making quantitative predictions. Since fuzzy linguistic models allow the

transformation of verbal expressions into numerical ones, multi-attribute methods based on fuzzy sets are used to deal quantitatively with the imprecision in the expression of the importance of each criterion.[6] Applications of the fuzzy set theory in MCDM are usually the extensions or fuzzification of the crisp ones.

In Ref. 7, they propose a decision support system for location selection of a convenience store which integrates the fuzzy set theory with the AHP. Reference 8 proposes a multiple criteria decision-making method for locating distribution centers under fuzzy environment. In this method, the weight of each criterion and the rating of alternatives described as triangular fuzzy numbers linguistically. After calculating the difference of final evaluation value among each pair of distribution center locations, a fuzzy preference relation matrix is constructed to represent the intensity of the preferences of one location over another. Finally a stepwise ranking procedure is used to find the ranking order of all candidate locations. One can deduce from Ref. 9 that examination of the usefulness of fuzzy AHP in the evaluation of site selection for convenient stores and its influence on the prediction accuracy of neuro-based location selection system are offered. The authors conclude that the fuzzy AHP is really constructive and useful for weighting and sieving decision factors. Reference 10 develops a decision support system for locating convenience store which consists of hierarchical structure development for fuzzy AHP, weight determination, data collection and decision making. Reference 11 applies fuzzy TOPSIS approach for selecting plant location, where the ratings of various alternative locations under various criteria and the weights of various criteria are assessed in linguistic terms represented by fuzzy numbers.

In Ref. 6, the authors present four different fuzzy multi-attribute decision making approaches which are Ref. 12's fuzzy relations method, fuzzy synthetic evaluation, Ref. 13's weighted goal method and fuzzy AHP proposed by Ref. 14. All these approaches are used for selecting the best facility location alternative by taking into account both quantitative and qualitative criteria. The authors also present a short comparative analysis among the approaches and a numeric example to each approach is given. Reference 15 presents a fuzzy multiple criteria decision-making method for selecting the most suitable location of reverse logistics reprocessing center. Their approach consists of two stages. In the first stage, they evaluate a certain alternative with fuzzy evaluation method. In the second stage, they evaluate other alternatives and choose the best suitable alternative by using fuzzy AHP. Reference 16 proposes a new multiple criteria decision-making method in order to solve the location of base station problem under fuzzy environment. The proposed method allows decision makers to determine the ranking of alternatives and also to indicate the degree of preference of each pair of alternatives. They also claim that their method is more suitable and effective in dealing with subjective judgments in an imprecise environment. Reference 17 proposes a new fuzzy TOPSIS method for location selection. In this approach, to avoid complicated fuzzy arithmetic operations, the linguistic variables, which are represented by triangular fuzzy numbers, are transformed into crisp numbers based on graded

mean representation. The canonical representation of multiplication operations on triangular fuzzy numbers is used to obtain the positive ideal solution and the negative ideal solution. The closeness efficient is defined to determine the ranking order of all alternatives.

In Ref. 5, the authors propose a new fuzzy multiple attribute decision-making method which integrates the fuzzy set theory, the factor rating system and simple additive weighting to evaluate facility location alternatives. Their approach produces an overall desirability score for each alternative and also does not insist on consensus but rather synthesizes a representative outcome from decision makers judgments. Reference 18 applies a combined AHP and fuzzy TOPSIS method for transshipment site selection. Fuzzy TOPSIS is used to select a solid waste transshipment site for ranking the alternative sites and the AHP is applied to calculate the criteria weights. Reference 19 applies fuzzy AHP based on Ref. 14's extent analysis method for location selection. They give a numerical example and a sensitivity analysis for their example. Reference 20 presents a comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection. They apply both methods for location selection of a textile company in Turkey. They also present the similarities and differences of both methods clearly.

In Ref. 21, the authors propose a fuzzy TOPSIS method for distribution center location selection. Their fuzzy TOPSIS method is based on α-level sets. They claim that their proposed method is more efficient and reasonable than the other methods. Reference 22 applies fuzzy ANP method which is based on Ref. 14's extent analysis method for shipyard location selection. Reference 23 also uses fuzzy ANP method for selecting the location of a regional hospital in Taiwan. In their approach, instead of the classical eigenvector prioritization method which is employed in the prioritization stage of the ANP, they apply a fuzzy preference programming method, which obtains crisp priorities from inconsistent interval and fuzzy judgments. Reference 24 proposes a combined MCDM method for shopping site selection which includes the usage of fuzzy AHP for assigning the weights of criteria and fuzzy TOPSIS for the determination of the most suitable alternative. Reference 25 uses generalized Choquet integral as a multi-criteria method for warehouse location selection. The important property of Choquet integral is that it includes interactions between criteria as ANP does.

3. Fuzzy Analytic Network Process (FANP)

Both AHP and ANP were developed by Saaty in Refs. 26 and 27 which can be used for examining hierarchy and network model representations. AHP and ANP mainly focus on how to solve decision problems with uncertainty and with multiple criteria characteristics by decomposing a complex MCDM problem into a hierarchy. They both incorporate the evaluations of all decision makers into a final decision, without having to elicit their utility functions on subjective and objective criteria, by pairwise comparisons of alternatives.

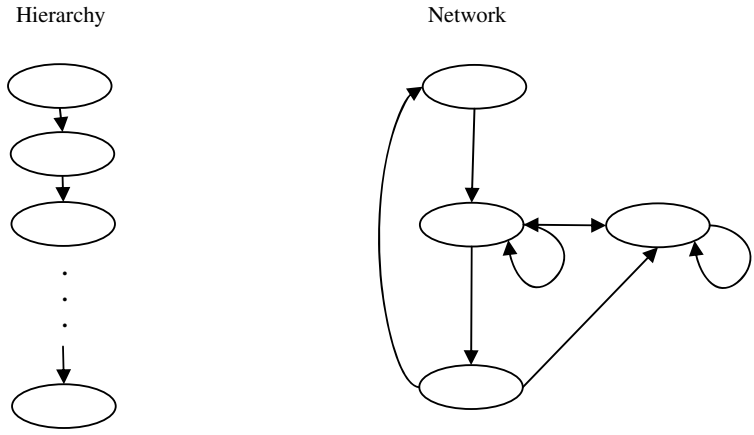


Fig. 1. Hierarchy and network illustration.

The ANP, the general form of AHP, is a comprehensive decision-making technique that captures the outcome of dependence and feedback within and between clusters of elements. AHP represents a framework based on a unidirectional hierarchical relationship whereas ANP permits more complex interrelationships among decision levels and attributes. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of alternatives may influence the importance of the criteria.[27] The main advantage of ANP over AHP is that ANP does not require strictly hierarchical structure and therefore can treat problems having complex interrelationships among criteria so that it can handle the complexities of real-world problems, see Refs. 28 and 29, for more details. Figure 1 illustrates a hierarchy and a network with interrelationships among criteria and feedback between factors of a complex structure.

The algorithmic procedure or steps of the ANP method is similar to the AHP. ANP has four basic steps which are deconstructing a problem into a complete set of network models, generating pairwise comparisons to estimate priorities at each level, building a supermatrix to represent the influence priority of elements and making decisions based on the supermatrix. The main difference is that, in ANP method, the supermatrix concept which resembles the Markov chain process is used.[27] This supermatrix handles the interdependence characteristics among elements and components. To obtain global priorities in a system with interdependent influences, the local priority vectors are added to the appropriate columns of a matrix, known as a supermatrix. A supermatrix is actually a segmented matrix, in which each matrix partition denotes a relationship between the two nodes in a system.

Conventional ANP method cannot consider the uncertainty in human judgments. In addition to this, human assessment on qualitative criteria is subjective and thus imprecise. In order to overcome this, the fuzzy set theory is the most convenient and mostly applied approach. Fuzzy ANP is an extension of crisp ANP

in which fuzzy sets are incorporated with the pairwise comparisons to model the uncertainty in human judgment and preference. There are some fuzzy ANP methods proposed by various authors. These methods are systematic approaches to the alternative selection and justification problem by using the concepts of the fuzzy set theory and hierarchical structure analysis. The methods differ from each other mainly in deriving priorities from fuzzy pairwise comparison matrices. One of the fuzzy ANP approaches is the one that is based on Ref. 14's extent analysis method. In this approach, a simple arithmetic mean algorithm is used for deriving fuzzy priorities from comparison matrices whose elements are defined as triangular fuzzy numbers. Although the computational procedure or the steps of this approach is much easier than the other fuzzy ANP approaches, the drawback of this method is that it is possible to obtain the value of zero for initial weights or local priorities to some elements of the decision structure. Such a computed zero local priority may cause some paths of interactions not to be considered in the supermatrix calculations.[30]

In Refs. 31 and 32, the authors propose a fuzzy extension of ANP which obtains crisp priorities from crisp, interval and fuzzy judgments using a technique called fuzzy preference programming (FPP). Reference 33 states that this approach does not require the construction of fuzzy comparison matrices and can derive priorities from an incomplete set of fuzzy judgments. The proposed approach is also invariant to the specific form of the fuzzy sets used to represent the judgments, and can be applied when some of the judgments are represented as intervals or crisp values. By using α-cuts, the initial fuzzy judgments are transformed into a series of interval judgments. The fuzzy preference programming method is employed to derive crisp priorities from the interval judgments, corresponding to each α-cut level, thus eliminating the need for an additional fuzzy ranking procedure. A simple aggregation is then used to obtain crisp overall values of the priorities. Finally, a nonlinear modification of the FPP method is proposed for a direct assessment of priorities without decomposing the fuzzy judgments by α-cut. The main drawback of this approach is that each comparison matrix must be constructed as an individual FPP model and thus the complexity of solving a network structure problem by the FPP technique is proportional to the number of the comparison matrices.[34]

To overcome this disadvantage, Ref. 34 proposes a multiple objective programming approach for the ANP to obtain all local priorities for crisp or interval judgments at one time. The main advantages of this approach are that the priorities of all comparison matrices are obtained simultaneously and the limiting supermatrix is obtained with less power.

The studies related to fuzzy ANP are limited in comparison to fuzzy AHP because of the computational complexity of the fuzzy ANP methods. Reference 35 presents a fuzzy optimization model for quality function deployment (QFD) planning using analytic network approach. They apply an integrated fuzzy ANP approach which is based on Ref. 14's extent analysis method to formulate and solve quality function

deployment problems. Reference 36 uses fuzzy ANP method to model the dynamic nature of manufacturing performance and its relationship to the environmental practices. Reference 37 proposes a fuzzy ANP approach to evaluate the cooperation risk of virtual logistics enterprise. They use Ref. 14's extent analysis method and with a numerical example show the appropriateness and efficiency of the fuzzy ANP in cooperation risk evaluation.

In Ref. 38, the authors develop a fuzzy ANP model based on the approach of Refs. 31 and 32 for selecting the best dispatching rule with respect to multiple criteria and system conditions. They also apply the model in a company to prove the applicability of their model. Reference 30 presents a fuzzy ANP approach for selecting third party logistics service suppliers and apply their approach to an experimental example. Reference 39 uses fuzzy ANP for selecting best competitive priority for a manufacturing firm to build up its most critical competitive priority through cleaner production implementation. They claim that their proposed approach serves as a guideline to managers in focusing on manufacturing decisions. Reference 40 presents a fuzzy ANP-based approach to transportation-mode selection between Turkey and Germany. In their study, a decision-making group from different management levels and from different functional areas related to transportation activities are brought together to evaluate criteria and alternative nodes. They apply their model to a firm and compare the results of the fuzzy ANP results and the current preferences of the company. Reference 41 uses fuzzy ANP-based model to identify and determine the factors that cause faulty behavior risk in work system safety. They apply their model to two different work systems and compare the results. Reference 42 applies fuzzy ANP method in production line selection evaluation indices system in ERP. They use fuzzy ANP for calculating the weights of factors and sub-factors of the model. Reference 43 applies a fuzzy ANP approach based on Ref. 14's extent analysis method to identify the main determinants of organizational readiness of a firm to implement an ERP project.

In Ref. 44, the authors present an intelligent approach for machine tool selection by using fuzzy ANP. They apply preference ratio analysis to the results of the fuzzy ANP to reach a final solution. Reference 45 presents a fuzzy approach for selecting material handling equipment. Their approach includes both fuzzy ANP and fuzzy TOPSIS methods. They use fuzzy ANP for determining the weights of the criteria, and use fuzzy TOPSIS for selecting the best alternative.

In this study we use Ref. 46's method for deriving the priorities from pairwise comparisons. Arithmetic procedure for the fuzzy ANP method based on Ref. 46's approach is given in the following steps:

Step 1. *Model construction and problem structuring.* The fuzzy ANP decision problem consists of (1) alternatives, $M_i (i = 1, 2, \ldots, m)$, (2) a set of evaluation criteria, $C_j (j = 1, 2, \ldots, n)$, (3) a linguistic judgment a_{ij} representing the relative importance of each criteria, (4) a weighting vector, $w = (w_1, w_2, \ldots, w_n)$. In this first

Table 1. Fuzzy comparison scale.

Numerical rating	Linguistic term	Fuzzy number
$\tilde{1}$	Equally important	(1,1,2)
$\tilde{2}$	Equally to moderately more imp.	(1,2,3)
$\tilde{3}$	Moderately more important	(2,3,4)
$\tilde{4}$	Moderately to strongly more imp.	(3,4,5)
$\tilde{5}$	Strongly more important	(4,5,6)
$\tilde{6}$	Strongly to very strongly more imp.	(5,6,7)
$\tilde{7}$	Very strongly more important	(6,7,8)
$\tilde{8}$	Very strongly to absolutely more imp.	(7,8,9)
$\tilde{9}$	Absolutely more important	(8,9,9)

step, the network consisting of all the criteria, the alternatives, and the relations are formed. This step is crucial since the selected criteria and defined relations can affect the result.

Step 2. *Construction and evaluation of the fuzzy pairwise comparison matrix.* Each pairwise comparison is performed by using linguistic terms. Each linguistic term is expressed by a triangular fuzzy number denoted as $\tilde{N} = (n_1, n_2, n_3)$ and shown in Table 1.

In Ref. 46's approach, the negative judgment is treated as the reversed order or the inverse of the fuzzy number of the positive judgment.

Step 3. *Calculating the fuzzy priorities and defuzzification.* The fuzzy priority of each attribute or criterion is calculated.

Since the fuzzy priority values are represented as fuzzy numbers, it is necessary to transform these fuzzy values into crisp values. Various defuzzification methods are available and the center of area method is applied for defuzzification which can be given as below:

$$Z(\tilde{N}_i) = \frac{\int_{n_1}^{n_3} z \cdot \mu_{\tilde{N}}(z)\,dz}{\int_{n_1}^{n_3} \mu_{\tilde{N}}(z)\,dz} = \frac{\int_{n_1}^{n_2} \left(z \cdot \frac{z-n_1}{n_2-n_1}\right) dz + \int_{n_2}^{n_3} \left(z \cdot \frac{n_3-z}{n_3-n_2}\right) dz}{\int_{n_1}^{n_2} \left(\frac{z-n_1}{n_2-n_1}\right) dz + \int_{n_2}^{n_3} \left(\frac{n_3-z}{n_3-n_2}\right) dz}$$

$$= \frac{(n_1 + n_2 + n_3)}{3}. \tag{3.1}$$

where $Z(\tilde{N}_i)$ represents the value of the fuzzy mean of the TFN, $\tilde{N}_i$.

Step 4. *Supermatrix formation and evaluation.* After the defuzzification of the fuzzy values of the local priorities which are generated from pairwise comparisons, these crisp values are placed into the appropriate positions in the supermatrix. In the supermatrix, each element is represented at one row and one respective column. Let a decision system be C_k, $k = 1, 2, \ldots, n$ and each component k has m_k elements, denoted by $e_{k1}, e_{k2}, \ldots, e_{km_k}$. The influence of set of elements in one component on another is represented by a priority vector obtained from pairwise comparisons.

A general form of a supermatrix can be formulated as follows.[27]

$$W = \begin{array}{c} \\ \\ C_1 \\ \\ \\ \\ C_2 \\ \\ \\ \vdots \\ \\ C_n \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccccccccc} & C_1 & & & & C_2 & & & \cdots & & C_n & \\ e_{11} & e_{12} & \cdots & e_{1m_1} & e_{21} & e_{22} & \cdots & e_{2m_2} & \cdots & e_{n1} & e_{n2} & \cdots & e_{nm_1} \end{array} \\ \begin{bmatrix} & & & & & & & & & & & \\ & W_{11} & & & & W_{12} & & & \cdots & & W_{1n} & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & W_{21} & & & & W_{22} & & & \cdots & & W_{2n} & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & \vdots & & & & \vdots & & & \vdots & & \vdots & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & W_{n1} & & & & W_{n2} & & & \ldots & & W_{nn} & \\ & & & & & & & & & & & \end{bmatrix} \end{array} . \tag{3.2}$$

This supermatrix is a partitioned matrix and its each part represents a relationship between two components or clusters in the system. Since this matrix must be stochastic, column normalization is applied and the weighted supermatrix is obtained. The weighted supermatrix is then raised to a significantly large power in order to have the converged or limiting values. The values of this limit matrix are the desired priorities or the final weights of the elements of the decision network. The limit supermatrix has the same form as the weighted supermatrix with the only difference that all its columns are the same. Finally, the alternatives are ranked according to their priority weights with respect to the goal and the one with the highest priority weight is selected.

4. Fuzzy Real Options Valuation (FROV)

An option is defined as the right, but not the obligation, to buy (if a call) or sell (if a put) a nominated asset by paying a preset price on or before a specified date. In financial options there are six basic managerial options; option to defer, option to abandon, option to switch, option to expand, option to grow, option to stage. Real options are based on financial options. In this study, option to grow-based real option calculation is used. However, the nature of real options involves permanent, fixed or immovable assets. In contrast to financial options, real options are not tradable — e.g., the factory owner cannot sell the right to extend his factory to another party, he can only make this decision. The valuation of real options necessitates a real option

analysis (ROA). The key advantage and value of ROA is to integrate managerial flexibility into the valuation process and thereby assist in making the best decisions.[47]

There are three valuation methods for real options: (i) Partial differential equations (the most famous one is Black−Scholes formula[48]), (ii) Simulations (Monte Carlo), (iii) Lattices (binomial, trinomial, etc.). Partial differential equation methods necessitate the solution of a partial differential equation with specified boundary conditions (i.e., option values at specified periods, type of option, etc.). Reference 48 offers the valuation method which is based on partial differential equations. Reference 49 proposes Monte Carlo simulation approach for financial options built on the insight that whatever the distribution of stock value would be at the time the option expired. The distribution is determined by operations driven by the activities of the asset value between now and the expiration date. Reference 50 develops a simplified option pricing model (the binomial options pricing model) based on a discrete-time approach. Later trinomial tree method is developed for more complex situations.

In daily life, real circumstances are very often uncertain and vague in several ways. When there is a lack of information or no voluntary financial data out the company, a system might not be known completely. Reference 51 suggests a strict mathematical outline named fuzzy set theory that overcomes these inadequacies. Many engineering and decision problems implemented classes of groupings of data with boundaries, which are not explained exactly, can be simplified by the fuzzy set theory. The fuzzy approach to ROV is first studied in Ref. 52. This work is based on Black−Scholes' ROV, but under fuzziness. Then, Ref. 53 offers fuzzy compound options for R&D project selection based on the Black−Scholes real option valuation. Reference 54 offers a model that integrates this fuzzy Black−Scholes real option valuation and fuzzy AHP for selection of R&D projects. Another fuzzy real option valuation based on Ref. 50's method is investigated by Ref. 55. They modeled pricing of grid/distributed computing resources as a problem of real option pricing.

Making a decision of investing on a location in retailer sector is similar to purchasing of an option on a future investment with the difference that it is a real investment. The nature of the investment in location selection decisions is discrete and you have to decide whether to carry on the option or exercise it every year. Since there is a lack of data or vagueness fuzzy real option analysis is needed. In real option analysis lattice models for evaluating the alternatives are fit like a glove for this type of investment.

In lattice models binomial tree method is developed in Ref. 50 for pricing the options simply. Later trinomial tree method is developed for more complex situations. Trinomial tree method provides much better approximation to the continuous time process than the binomial lattice for the same number of steps. The trinomial tree is easier to work with because of its more regular grid and it is more flexible, allowing relatively easy extension to time-varying drift and volatility parameters.[56]

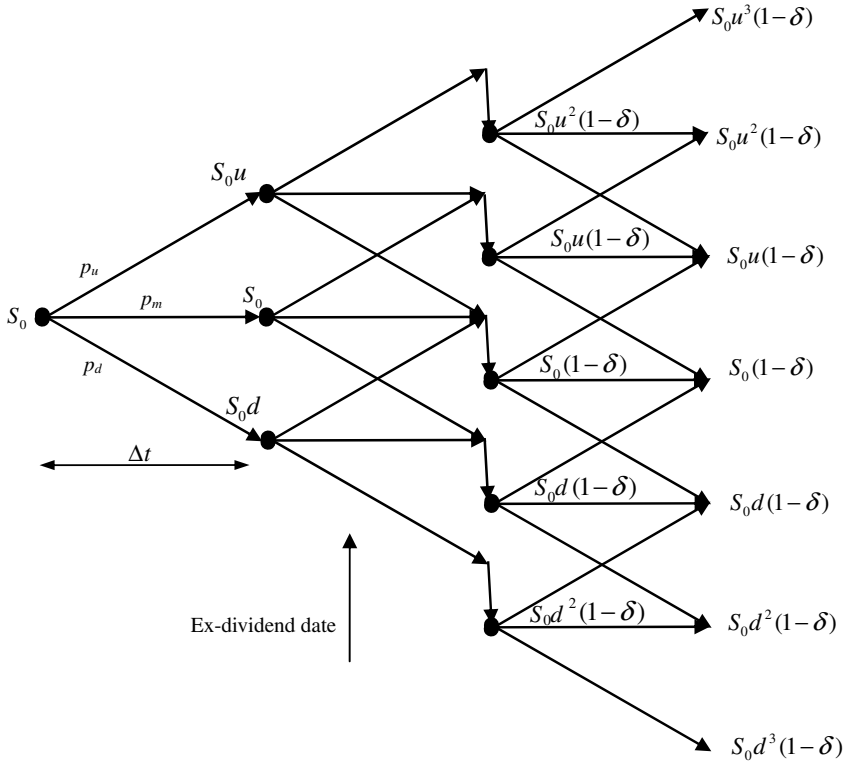


Fig. 2. Trinomial tree model with dividend yield.

In this section, we first present general form of the trinomial tree model as illustrated in Ref. 56. Then, the fuzzy forms of these representations will be developed for the case of incomplete data or vagueness. The trinomial tree approach is equivalent to the explicit finite difference method introduced in Ref. 57 and it is illustrated in Fig. 2. Suppose that p_u, p_m, and p_d are the probabilities of up, middle, and down movements at each node and Δt is the length of the time step. Dividend paying stocks' real option valuation techniques for binomial lattices are evaluated in Ref. 58. For a dividend paying stock in trinomial lattice, parameter values that match the mean and standard deviation of price changes are as follows[56]:

$$v = r - \delta - \frac{1}{2}\sigma^2, \tag{4.1}$$

$$\Delta x = \sigma\sqrt{3\Delta t}, \tag{4.2}$$

$$u = \mathrm{e}^{\Delta x}, \tag{4.3}$$

$$d = \frac{1}{u}, \tag{4.4}$$

$$p_u = \frac{1}{2}\left(\frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2} + \frac{v\Delta t}{\Delta x}\right), \tag{4.5}$$

$$p_m = 1 - \frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2}, \tag{4.6}$$

$$p_d = \frac{1}{2}\left(\frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2} - \frac{v\Delta t}{\Delta x}\right), \tag{4.7}$$

where σ points out the uncertainty of expected cash flows, r quantifies the annualized continuously compounded rate on a safe asset, δ illustrates the value lost over the duration of the option, and u depicts the proportional up movement, vice versa d depicts the proportional down movement of cash flow in a trinomial model.

Computations for a trinomial tree lattice are similar to those for a binomial tree. At time zero, the present value of expected cash flows, S_0 is known. At time Δt, there are three possible S_0 values, S_0u, S_0, and S_0d; at time $2\Delta t$, there are five possible S_0 values, S_0u^2, S_0u, S_0, S_0d, and S_0d^2; and so on. In general, at time $i\Delta t$, $2i+1$ S_0 values are considered. These are:

$$S_{i,j} = S_0(1-\delta)u^j, \quad j = -i, \ldots, i. \tag{4.8}$$

Notice that the $u = 1 \div d$ relationship is used in computing the S_0 value at each node of the tree in Fig. 2. For example, $S_0u^{-2} = S_0d^2$. Real option valuations by lattice methods are evaluated by starting with the end of the tree (time T) and working backward. The value of the real option is known at time T. Let us express the approach algebraically. Because the value of the real option at its expiration date is max $(0, S_{i,j} - X)$, we know that

$$c_{T,j} = \max(0, S_{T,j} - X), \quad j = -T, \ldots, 0, \ldots, T. \tag{4.9}$$

After computing the $c_{T,j}$ values (no early exercise), $c_{i,j}$ values at each node could be calculated by the formula below:

$$c_{i,j} = \mathrm{e}^{-r\Delta t}[p_u c_{i+1,j+1} + p_m c_{i+1,j} + p_d c_{i+1,j-1}], \tag{4.10}$$

for $0 \le i \le T-1$ and $-i \le j \le i$. When early exercise is considered, this value for $c_{i,j}$ must be compared with the option's intrinsic value, and we obtain:

$$c_{i,j} = \max\{\mathrm{e}^{-r\Delta t}[p_u c_{i+1,j+1} + p_m c_{i+1,j} + p_d c_{i+1,j-1}], S_0u^j - X\}. \tag{4.11}$$

In case of incomplete data or vagueness, all the calculations are same, but S_0 (the present value of expected cash flows), and X (the nominal value of stationary costs) will be integrated to these formulas in a fuzzy form (S_0, X). In this way, Eq. (4.8) turns into Eq. (4.12):

$$\tilde{S}_{i,j} = \tilde{S}_0(1-\delta)u^j, \quad j = -i, \ldots, i. \tag{4.12}$$

Then, Eq. (4.9) turns into Eq. (4.13):

$$\tilde{c}_{T,j} = \max(0, \tilde{S}_{T,j} - \tilde{X}), \quad j = -T, \ldots, 0, \ldots, T. \tag{4.13}$$

To select the maximum term in Eq. (4.13), a comparison between 0 and the fuzzy term $(\tilde{S}_{T,j} - \tilde{X})$ has to be made. In this study, an alternative ranking approach offered in Ref. 59 is used for this fuzzy comparison. For detailed information the readers could glance at quidam paper.

Equations (4.10) and (4.11) turn into Eqs. (4.14) and (4.15) consecutively:

$$\tilde{c}_{i,j} = \mathrm{e}^{-r\Delta t}[p_u \tilde{c}_{i+1,j+1} + p_m \tilde{c}_{i+1,j} + p_d \tilde{c}_{i+1,j-1}] \tag{4.14}$$

for $0 \leq i \leq T-1$ and $-i \leq j \leq i$. And in early exercise:

$$\tilde{c}_{i,j} = \max\{\mathrm{e}^{-r\Delta t}[p_u \tilde{c}_{i+1,j+1} + p_m \tilde{c}_{i+1,j} + p_d \tilde{c}_{i+1,j-1}], \tilde{S}_0 w^j - \tilde{X}\}. \tag{4.15}$$

5. The Proposed Approach

In the literature, for location selection problems multi-criteria methods and economical evaluation methods are used separately. In this proposed approach, location selection problem is handled by ANP which takes into account both qualitative and quantitative criteria. The real options valuation method is applied for evaluating the irreversible economical factors and is used in the decision network as a criteria cluster. The fuzzy forms of ANP and real options methods are integrated in the proposed approach. The main contribution of this model is that it integrates both economical and the multi-criteria evaluation for location selection problem.

5.1. *Notation*

$\tilde{a}_{ij}$ = fuzzy comparison value of criterion i to criterion j
$\tilde{c}_{0,0}$ = fuzzy real option value at time zero means fuzzy real option value of the alternative
$\tilde{c}_{i,j}$ = fuzzy real option value at time i, and level j
$c_{i,j}$ = real option value at time i, and level j
C_j = the jth evaluation criteria
d = the proportional down movement in a trinomial model
δ = value lost over the duration of the option
i = time step
j = the level of the cash flow or real option
$\tilde{N}$ = triangular fuzzy number
p_u = probability of up movement
p_m = probability of staying right
p_d = probability of down movement
r = risk-free interest rate

$\tilde{r}_i$ = the geometric mean of fuzzy comparison value of criterion i to each criterion j
S_0 = the present value of expected cash flows
$S_{i,j}$ = cash flow value at time i, and level j
$\tilde{S}_0$ = the present value of expected cash flows in fuzzy form
$\tilde{S}_{i,j}$ = cash flow value at time i, and level j in fuzzy form
σ = the uncertainty of expected cash flows
t = time
u = the proportional up movement in a trinomial model
v = parameter related to uncertainty, risk free interest rate and lost value during the option life
$\tilde{w}_i$ = fuzzy priority or the weight of the ith criterion
x = parameter related to uncertainty and time
X = the nominal value of stationary costs
$\tilde{X}$ = the nominal value of stationary costs in fuzzy form
$Z(\tilde{N}_i)$ = value of the fuzzy mean of the triangular fuzzy number $\tilde{N}_i$

5.2. *Algorithm of the model*

The algorithmic procedure for the proposed approach is as follows:

Step 1. Construct the pairwise comparison matrix containing fuzzy numbers obtained from questionnaires as follows:

$$\tilde{A} = [\tilde{a}_{ij}] = \begin{matrix} C_1 \\ C_2 \\ \cdot \\ C_n \end{matrix} \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix} = \begin{matrix} C_1 \\ C_2 \\ \cdot \\ C_n \end{matrix} \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \dots & 1 \end{bmatrix}, \tag{5.1}$$

where $\tilde{a}_{ij} = 1$ if $i = j$; and $\tilde{a}_{ij} = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ or $\tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}$, if $i \neq j$.
Step 2. For economical criteria cluster; apply FROV as follows:

Step 2.1. Calculate all $\tilde{S}_{i,j}$ values by Eq. (4.12) for fuzzy real options value.
Step 2.2. Calculate $\tilde{c}_{T,j}$ values by Eq. (4.13), compare them with 0 by using Eq. (5.2):

$$P_{KT}(\tilde{N}, \tilde{U}) = \begin{cases} \tilde{N} \succ \tilde{U} & \text{if } I(w) \in (0.5, 1] \\ \tilde{N} = \tilde{U} & \text{if } I(w) = 0.5 \\ \tilde{U} \succ \tilde{N} & \text{if } I(w) \in [0, 0.5) \end{cases}. \tag{5.2}$$

Calculation of index $I(w)$ is the key factor in this method and the method can be applied to both triangular and trapezoidal fuzzy numbers. Using Eq. (5.3) in

triangular fuzzy numbers is as easy as winking.

$$I(\omega) = \begin{cases} 0, & u_1 \geq n_4 \\ \dfrac{\frac{(n_4-u_1)^2}{(u_2-u_1-n_3+n_4)}}{(n_4+n_3-n_2-n_1)+(u_4+u_3-u_2-u_1)}, & u_2 \geq n_3, u_1 < n_4 \\ \dfrac{n_4+n_3-u_2-u_1}{(n_4+n_3-n_2-n_1)+(u_4+u_3-u_2-u_1)}, & u_3 \geq n_2, u_2 < n_3\,. \\ \dfrac{(n_4+n_3-u_2-u_1)-\frac{(n_2-u_3)^2}{(u_4-u_3+n_2-n_1)}}{(n_4+n_3-n_2-n_1)+(u_4+u_3-u_2-u_1)}, & u_3 < n_2, u_4 > n_1 \\ 1, & u_4 \leq n_1 \end{cases} \tag{5.3}$$

Step 2.3. Compute all $\tilde{c}_{i,j}$ values by Eq. (4.14), till finding $\tilde{c}_{0,0}$ value. This value is FROV of one alternative.

Step 2.4. Repeat Steps 2.1–2.3 for each alternative and find all alternatives' FROVs.

Step 2.5. Defuzzify the values in Step 2.4 by Eq. (3.1) and normalize these values for integrating fuzzy ANP as weight.

Step 3. Apply Eqs. (5.4) and (5.5) to obtain the fuzzy local priorities of criteria.

$$\tilde{r}_i = \left[\prod_{j=1}^{n} \tilde{a}_{ij}\right]^{\frac{1}{n}} = [\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \cdots \otimes \tilde{a}_{in}]^{\frac{1}{n}}, \quad \forall\, i = 1, 2, \ldots, n, \tag{5.4}$$

$$\tilde{w}_i = \tilde{r}_i \otimes \left[\sum_{j=1}^{n} \tilde{r}_j\right]^{-1} = \frac{\tilde{r}_i}{\tilde{r}_1 \oplus \cdots \oplus \tilde{r}_n}, \tag{5.5}$$

where $\tilde{a}_{ij}$ is the fuzzy comparison value of criterion i to criterion j, $\tilde{r}_i$ is the geometric mean of fuzzy comparison value of criterion i to each criterion j, and $\tilde{w}_i$ is the fuzzy priority or the weight of the ith criterion.

Step 4. Transform the fuzzy values into crisp values by using Eq. (3.1).

Step 5. Construct the supermatrix as in Eq. (3.2).

Step 6. Normalize each column of the supermatrix to obtain the weighted supermatrix.

Step 7. Raise this weighted supermatrix to a large power until all its columns are the same.

Step 8. Rank the alternatives in descending order according to their final priority weights obtained in Step 7.

6. An Application of the Proposed Approach to a Retail Location Selection Problem

The main problem is selecting the best supermarket location among three alternatives named Beşiktaş (Ortaköy), Çekmeköy, and Halkalı. Each of these

Table 2. Criteria and relation matrix.

		Alternatives			Competition			Environmental						FROV	Site/Store						Transport./Acces.		
		Beşiktaş	Çekmeköy	Halkalı	C12. Location of competitors	C13. Competition density	C14. Competitive Advantage Opportunity	C7. Population density	C8. Level of urbanization	C9. Amount of saving account per person	C10. Retail expenditure (food+drink)	C11. Demographic profile	FROV	C1. Sales area	C2. Storage area	C3. Parking area	C4. Physical condition	C5. Visibility	C6. Ability to expand or modify the facility	C15. Availability of public transp.	C16. Closeness to the target market	C17. Closeness to the major roads and highways
Alternati	Beşiktaş	■			*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	Çekmeköy		■		*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	Halkalı			■	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Competit	C12. Location of competitors	*	*	*	■	*	*						*									
	C13. Competition density	*	*	*	*	■	*						*									
	C14. Competitive Advantage Opportunit	*	*	*			■															
Environmental	C7. Population density	*	*	*	*	*		■			*		*							*		
	C8. Level of urbanization	*	*	*					■		*		*							*		*
	C9. Amount of saving account per person	*	*	*						■	*		*									
	C10. Retail expenditure (food+drink)	*	*	*		*					■		*	*								
	C11. Demographic profile	*	*	*	*						*	■	*									
FROV		*	*	*									■									
Site/Store	C1. Sales area	*	*	*			*						*	■	*	*						
	C2. Storage area	*	*	*											■							
	C3. Parking area	*	*	*			*									■			*			
	C4. Physical condition	*	*	*			*										■	*	*			
	C5. Visibility	*	*	*			*											■			*	
	C6. Ability to expand or modify the facility	*	*	*			*												■			
Transport. /Acces.	C15. Availability of public transp.	*	*	*																■	*	
	C16. Closeness to the target market	*	*	*									*								■	
	C17. Closeness to the major roads and highways	*	*	*			*														*	■

locations is a candidate for a supermarket opening in which more than 10,000 different types of products are to be sold, and each of which will have a sales area over 1000 m^2.

The evaluation criteria for location selection problems are classified into various categories in the literature as presented before. Considering these works and by applying Delphi method to the five experts from three different supermarket retailers who have at least 12 years of experience in supermarket retailing, we produce six clusters named alternatives, competition, environmental, FROV, site/store, transportation/accessibility and 17 criteria. Table 2 presents the criteria and the relations. Based on the experience and knowledge, the decision maker makes the necessary pairwise comparisons by using the fuzzy comparison scale given in Table 1.

The clusters and the included criteria reflect the relationships between the spatial attributes and the overall output or performance. Especially environmental and the competition clusters are aimed at revealing the sales performance like customer number, sales, and revenue.

Network considering all clusters and criteria with three alternatives is illustrated in Fig. 3. This network is developed in Super Decisions software.

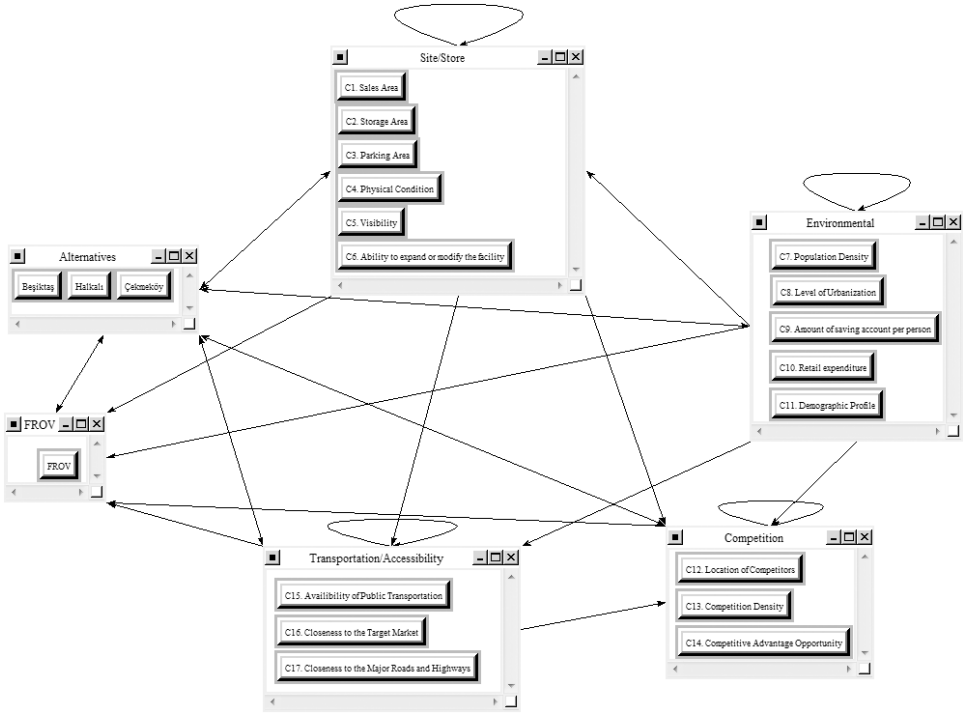


Fig. 3. Network of location selection decision.

The first step of the proposed method is constructed with the rehearsals above. Then the second step of the algorithm can be applied as follows:

The financial costs for each location alternatives gathered from the firm are listed in Table 3. And the expected incomes for the first year after the opening of the store are listed in Table 4.

One can easily compute the following $\tilde{S}_0$ and $\tilde{X}$ values for each alternative with a tax rate 25% as illustrated in Table 5.

The investment period T is five years for both alternatives. For the annualized continuously compounded rate on a safe asset; the company will make a portfolio that contains five year Treasury bond interest rate belonging Central Bank of the Republic of Turkey, and the other instruments (bonds, stocks, etc.). This portfolio's expected rate of return is 10.29%.

For the value lost over the duration of the option, the estimated δ value will be used. The value lost over the duration of the option means the entrance of the competitors to the market, so at this time in Turkey, there are many competitors in supermarket sector. That is why we take the value in this manner, however in the latest days, there may be bankruptcies or new foundations. In this study the future big competitors' entrance to the chosen location area will be considered. The existing competitors' decreasing profit is included in expected revenues. The experts guessed

Table 3. Costs for each alternative.

Initial costs (TL)	Beşiktaş (1100 m^2) (b_1, b_2, b_3)	Çekmeköy (1200 m^2) (c_1, c_2, c_3)	Halkalı (1100 m^2) (h_1, h_2, h_3)
1. Refurbishment cost	(27,500, 440,000, 605,000)	(300,000, 480,000, 660,000)	(27,500, 440,000, 605,000)
2. Technological requirement cost	(88,000, 110,000, 132,000)	(96,000, 120,000, 144,000)	(88,000, 110,000, 132,000)
TOTAL	(363,000, 550,000, 737,000)	(396,000, 600,000, 804,000)	(363,000, 550,000, 737,000)
Regular costs per annum (TL)			
1. Rental cost	(126,000, 234,000, 414,000)	(108,000, 180,000, 306,000)	(99,000, 153,000, 270,000)
2. Fixed cost	(300,000, 324,000, 372,000)	(240,000, 276,000, 312,000)	(210,000, 228,000, 276,000)
3. Transportation/Delivery cost	(61,875, 74,250, 86,625)	(45,375, 54,450, 63,525)	(44,550, 59,400, 74,250)
TOTAL	(487,875, 632,250, 872,625)	(393,375, 510,450, 681,525)	(353,550, 440,400, 620,250)

Table 4. Incomes after the opening of the supermarket.

(TL)	Beşiktaş (1100 m^2) (b_1, b_2, b_3)	Çekmeköy (1200 m^2) (c_1, c_2, c_3)	Halkalı (1100 m^2) (h_1, h_2, h_3)
Expected incomes after the opening of the retailer	(420,000, 780,000, 1380,000)	(360,000, 600,000, 1020,000)	(330,000, 510,000, 900,000)

Table 5. Expected revenues and initial costs of alternatives after tax payment.

(TL)	Beşiktaş (1100 m^2) (b_1, b_2, b_3)	Çekmeköy (1200 m^2) (c_1, c_2, c_3)	Halkalı (1100 m^2) (h_1, h_2, h_3)
$\tilde{S}_0$	(−50906.25, 110812.5, 380531.3)	(−25031.25, 67162.5, 253856.3)	(−17662.5, 52,200, 209812.5)
$\tilde{X}$	(363,000, 550,000, 737,000)	(396,000, 600,000, 804,000)	(363,000, 550,000, 737,000)

that in five year period there may be one entrance to the market. Estimation of the waiting cost is very difficult; but as a result of consultancy with the experts in the firm, 1761.75 TL value lost per five year period for Beşiktaş alternative is reached. The delta is calculated by $\delta =$ the cost of waiting $\div E(\tilde{S}_0)$. So one can easily calculate the delta for Beşiktaş as: $\delta = (5) \times 1761.75 \div 146812.5 = 6\%$. $E(\tilde{S}_0)$ can be computed by Eq. (3.1).

For FROV by trinomial lattice method, at first step, v, Δx, u, and d values have to be calculated from Eqs. (4.1)−(4.4); and those could be found −0.191, 1.184, 3.268, 0.306, respectively. For detailed information the readers could glance at the paper in Ref. 60.

Then, the probabilities of up, middle, and down movements for each node are calculated by Eqs. (4.5)−(4.7) at the second step:

$$p_u = 0.099, \quad p_m = 0.641, \quad p_d = 0.260.$$

According to the proposed approach at Step 2.1, the $\tilde{S}_{i,j}$ values are computed by Eq. (4.12). For example, $\tilde{S}_{5,5}$ and $\tilde{S}_{5,-5}$ could be found as below:

$$\tilde{S}_{5,5} = (-17835955.46, 38825229.01, 133326230.60)\text{TL};$$
$$\tilde{S}_{5,-5} = (-128.38, 279.46, 959.67)\text{TL}.$$

After finding these 11 ends of the period values, Eq. (4.13) have to be computed to find the values of the fuzzy real option values at its expiration date. As illustrated in Step 2.2 in the proposed model, $I(w)$ value has to be calculated to compare the $\tilde{S}_{T,j} - \tilde{X}$ values with 0. This operation has to be repeated for both 11 ends of period values. For example for Beşiktaş alternative for $\tilde{c}_{5,5}$, $I(w) = 0.960$ means $\tilde{S}_{T,j} - \tilde{X} > 0$.

At the next step, all $\tilde{c}_{i,j}$ values at each node are calculated by Eq. (4.14) with backward induction. Finally, $\tilde{c}_{0,0}$ value for Beşiktaş alternative — shown below — will be carried out which means FROV:

$$\tilde{c}_{0,0} = (-28859.06, 19677.68, 98127.50)\text{TL}.$$

That process has to be applied for other alternatives and after finding the other values as below, according to Step 2.5 the normalization of these values has to be put into practice as seen in Table 6.

In the third step, the fuzzy local priorities of criteria are to be calculated. 31 different pairwise comparisons have to be performed and Eqs. (5.4) and (5.5) have to

Table 6. FROV and weights of alternatives.

Alternative	$\tilde{c}_{0,0}$ (TL)	Defuzzified (TL)	Weights
Beşiktaş	(−28859.06, 19677.68, 98127.57)	29648.73	0.5175
Çekmeköy	(−22913.55, 6742.49, 62645.24)	15491.39	0.2704
Halkalı	(−19219.11, 3959.68, 51702.61)	12147.72	0.2120

Table 7. Pairwise comparisons of the criteria w.r.t. "Beşiktaş" node in "Competition" cluster.

	Location of competitors (C12)	Competition density (C13)	Competitive advantage opportunity (C14)
Location of competitors (C12)	(1, 1, 2)	(1, 2, 3)	(5, 6, 7)
Competition density (C13)		(1, 1, 2)	(2, 3, 4)
Competitive advantage opportunity (C14)			(1, 1, 2)

Table 8. Fuzzy priorities of criteria C12, C13, and C14 w.r.t. "Beşiktaş" node in "Competition" cluster.

	Fuzzy priorities
Location of competitors (C12)	(0.282, 0.600, 1.193)
Competition density (C13)	(0.144, 0.300, 0.687)
Competitive advantage opportunity (C14)	(0.054, 0.100, 0.201)

Table 9. The importance of criteria and preference of alternatives.

Cluster	Criteria	Limiting priority or importance (%)	Normalized by cluster (%)
Competition	A12. Location of competitors	6.36	41.91
	A13. Competition density	5.63	37.10
	A14. Competitive advantage opportunity	3.18	20.99
Environmental	A7. Population density	0.51	7.47
	A8. Level of urbanization	1.03	15.30
	A9. Amount of saving account per person	1.13	16.71
	A10. Retail expenditure	3.61	53.43
	A11. Demographic profile	0.48	7.09
FROV	FROV1	34.15	100.00
Site/Store	A1. Sales area	0.80	29.26
	A2. Storage area	0.18	6.52
	A3. Parking area	0.61	22.15
	A4. Physical condition	0.37	13.57
	A5. Visibility	0.57	20.75
	A6. Ability to expand or modify the facility	0.21	7.75
Transportation/Accessibility	A15. Availability of public transportation	0.27	22.74
	A16. Closeness to the target market	0.82	68.13
	A17. Closeness to the major roads and highways	0.11	9.13
Alternatives	Beşiktaş	0.2099	**52.49**
	Halkalı	0.1015	**25.38**
	Çekmeköy	0.0885	**22.13**

be applied for each of these 31 pairwise matrices. In Table 7, one of these pairwise comparison matrices is given as an example.

By applying Eqs. (5.2) and (5.3) successively, the fuzzy priority values given in Table 8 are obtained.

Having performed the same for all the rest of pairwise comparison matrices and having defuzzified the fuzzy values, the unweighted supermatrix can now be constructed. After applying column normalization to the unweighted supermatrix, the weighted supermatrix is evaluated. Table 9 gives the limiting priorities or importance criteria, and also the preference of alternatives. These values are taken from the limit supermatrix which is obtained by raising the weighted supermatrix to its powers (see Appendix A for supermatrix).

According to the limiting priorities in Table 9, FROV, competition and environmental clusters are the main determinants of the supermarket location selection problem. Especially, financial analysis (FROV, 34.15%), location of competitors (6.36%), competition density (5.63%) and retail expenditure (3.61%) are more important than the other evaluation criteria. The preferences for the alternatives are 52.49% for Beşiktaş, 25.38% for Halkalı and 22.13% for Çekmeköy. Thus, Beşiktaş of which preference is more than the total of other alternatives' preferences is the best supermarket location among all the alternatives.

7. Conclusion

This paper presents a new approach called the fuzzy real options value integrated fuzzy ANP method for retail location selection problems. The retail sector specific location selection criteria which were derived from literature are determined as a result of the interviews with the experts. Due to the dependency between criteria and vagueness, fuzzy ANP is used as the multi-criteria method. The significant contribution of the proposed approach is that it integrates the financial dimension (FROV) of the location problem methodologically with the multi-criteria characteristic (FANP) of the problem. Another importance of this study is the first usage of real options valuation in the area of location selection science. Although mathematical modeling and forecasting methods mainly focus on obtaining results based on one of the mostly used objectives like maximizing the sales revenue, profitability, or customer number, the proposed approach gives results not only based on one or two sales performance related objectives but also includes other dimensions in a holistic manner.

The presented approach is applied successively to a real-world problem of selecting the best supermarket location among three alternative locations. Since this approach makes it possible to determine the weights or importance of the evaluation criteria, this can help the retail sector experts better understand the nature of their business. One of the practical implications of this study is that the presented network framework can be used with some modifications in other location selection decisions (either public or private sector) rather than retail sector. Another one can

be applying the offered methodology to other multi-criteria selection problems that require the evaluation of economic and risky aspect.

For future research, usage of the fuzzy real option value with other MCDM methods and comparing them with the results of this study can be offered. In addition, for theoretical studies, the test of consistency for pairwise comparison matrices of ANP method under fuzzy environment may be a promising area. Although there are some recent important studies related to identify the inconsistent elements and to improve the consistency ratio in crisp AHP/ANP methods, for detailed information the readers could glance at the papers in Refs. 61 and 62, there is not enough study for fuzzy forms of these methods.

Appendix A

		Alternatives			Competition			Environmental					FROV	Site/Store						Transportation/Accessibility		
		Beşiktaş	Halkalı	Çekmeköy	A12. Location of competitors	A13. Competition Density	A14. Competitive Advantage Opportunity	A7. Population Density	A8. Level of Urbanization	A9. Amount of saving account per person	A10. Retail expenditure	A11. Demographic Profile	FROV1	A1. Sales Area	A2. Storage Area	A3. Parking Area	A4. Physical Condition	A5. Visibility	A6. Ability to expand or modify the facility	A15. Availibility of Public Transportation	A16. Closeness to the Target Market	A17. Closeness to the Major Roads and Highways
Alternatives	Beşiktaş	0.0000	0.0000	0.0000	0.1704	0.3333	0.6674	0.6846	0.6102	0.6846	0.6390	0.6717	0.5175	0.2645	0.2956	0.2955	0.3411	0.6282	0.1464	0.7255	0.6717	0.6282
	Halkalı	0.0000	0.0000	0.0000	0.5775	0.3333	0.1040	0.0941	0.1464	0.0941	0.0853	0.1257	0.2704	0.1072	0.1063	0.1765	0.1040	0.1072	0.2434	0.0836	0.1257	0.1072
	Çekmeköy	0.0000	0.0000	0.0000	0.2521	0.3333	0.2287	0.2213	0.2434	0.2213	0.2757	0.2027	0.2120	0.6282	0.5981	0.5279	0.5549	0.2645	0.6102	0.1909	0.2027	0.2645
Competition	A12. Location of competitors	0.5828	0.5904	0.5763	0.0000	0.4770	0.0000	0.7963	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A13. Competition Density	0.3175	0.3131	0.3402	0.8435	0.0000	0.0000	0.2037	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A14. Competitive Advantage Opportunity	0.0997	0.0965	0.0834	0.1565	0.5230	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000
Environmental	A7. Population Density	0.1185	0.0479	0.0539	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A8. Level of Urbanization	0.0639	0.2073	0.4090	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A9. Amount of saving account per person	0.2649	0.1341	0.0900	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A10. Retail expenditure	0.5013	0.5319	0.2894	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A11. Demographic Profile	0.0514	0.0787	0.1576	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FROV	FROV1	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000	0.0000
Site/Store	A1. Sales Area	0.2776	0.2627	0.1670	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A2. Storage Area	0.0488	0.0948	0.0361	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7963	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A3. Parking Area	0.3082	0.1913	0.2007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A4. Physical Condition	0.1708	0.1765	0.1110	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A5. Visibility	0.1618	0.2189	0.4112	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5613	0.0000	0.0000	0.0000	0.0000	0.0000
	A6. Ability to expand or modify the facility	0.0329	0.0558	0.0741	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.4387	0.0000	0.0000	0.0000	0.0000	0.0000
Transportation/ Accessibility	A15. Availibility of Public Transportation	0.2040	0.2374	0.2144	0.0000	0.0000	0.0000	1.0000	0.8313	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	A16. Closeness to the Target Market	0.6952	0.6452	0.7230	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000
	A17. Closeness to the Major Roads and Highways	0.1008	0.1174	0.0626	0.0000	0.0000	0.0000	0.0000	0.1687	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Fig. A.1. Supermatrix.

References

1. S. S. Owen and M. S. Daskin, Strategic facility location, *European Journal of Operational Research* **111** (1998) 423–447.
2. T. S. Hale and C. R. Moberg, Location science research: A review, *Annals of Operations Research* **123** (2003) 21–35.
3. A. Klose and A. Drexl, Facility location models for distribution system design, *European Journal of Operational Research* **162** (2005) 4–29.
4. C. S. ReVelle, H. A. Eiselt and M. S. Daskin, A bibliography for some fundamental problem categories in discrete location science, *European Journal of Operational Research* **184** (2008) 817–848.
5. S. Y. Chou, Y. H. Chang and C. Y. Shen, A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes, *European Journal of Operational Research* **189** (2008) 132–145.
6. C. Kahraman, D. Ruan and I. Doğan, Fuzzy group decision-making for facility location selection, *Information Sciences* **157** (2003) 135–153.
7. R. J. Kuo, S. C. Chi and S. S. Kao, A decision support system for locating convenience store through fuzzy AHP, *Computers & Industrial Engineering* **37** (1999) 323–326.
8. C. T. Chen, A fuzzy approach to select the location of the distribution center, *Fuzzy Sets and Systems* **118** (2001) 65–73.
9. S. C. Chi and R. J. Kuo, Examination of the influence of fuzzy analytic hierarchy process in the development of an intelligent location selection support system of convenience store, in *Proc. IFSA WC and 20th NAFIPS Int. Conf.*, Vol. 3 (2001), pp. 1312–1316.
10. R. J. Kuo, S. C. Chi and S. S. Kao, A decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network, *Computers in Industry* **47** (2002) 199–214.
11. T.-C. Chu, Facility location selection using fuzzy TOPSIS under group decisions, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **10**(6) (2002) 687–701.
12. J. M. Blin, Fuzzy relations in group decision theory, *Journal of Cybernetics* **4** (1974) 17–22.
13. R. R. Yager, Fuzzy decision-making including unequal objectives, *Fuzzy Sets and Systems* **1** (1978) 87–95.
14. D. Y. Chang, Applications of the extent analysis method on fuzzy AHP, *European Journal of Operational Research* **95** (1996) 649–655.
15. X. Y. Chang and Z. G. Chen, A fuzzy-based location selection method for reverse logistics reprocessing center, in *Proc. Int. Conf. Management Science and Engineering, ICMSE'06* (2006), pp. 561–565.
16. Z. Gacovski and I. Cvetanoski, Fuzzy decision-making for selection of mobile base station location, in *Proc. 28th Int. Conf. Information Technology Interfaces* (2006), pp. 401–406.
17. D. Yong, Plant location selection based on fuzzy TOPSIS, *International Journal of Advanced Manufacturing Technology* **28** (2006) 839–844.
18. S. Onüt and S. Soner, Transshipment site selection using the AHP and TOPSIS approaches under fuzzy environment, *Waste Management* **28** (2008) 1552–1559.
19. M. Tabari, A. Kaboli, M. B. Aryanezhad, K. Shahanaghi and A. Siadat, A new method for location selection: A hybrid analysis, *Applied Mathematics and Computation* **206**(2) (2008) 598–606.
20. İ. Ertuğrul and N. Karakaşoğlu, Comparison of fuzzy AHP and TOPSIS methods for facility location selection, *International Journal of Advanced Manufacturing Technology* **39** (2008) 783–795.

21. Y. Hu, S. Wu and L. Cai, Fuzzy multi-criteria decision-making TOPSIS for distribution center location selection, in *Proc. Int. Conf. Networks Security, Wireless Comm. and Trusted Comp.*, China (2009), pp. 707−710.
22. A. F. Guneri, M. Cengiz and S. Seker, A fuzzy ANP approach to shipyard location selection, *Expert Systems with Applications* **36** (2009) 7992−7999.
23. C. R. Wu, C. T. Lin and C. H. Chen, Integrated environmental assessment of the location selection with fuzzy analytical network process, *Quality and Quantity* **43** (2007) 351−380.
24. S. Önüt, T. Efendigil and S. S. Kara, A combined fuzzy MCDM approach for selecting shopping center site: An example from Istanbul, Turkey, *Expert Systems with Applications* **37** (2010) 1973−1980.
25. T. Demirel, N. Ç. Demirel and C. Kahraman, Multi-criteria warehouse location selection using Choquet integral, *Expert Systems with Applications* **37** (2010) 3943−3952.
26. T. L. Saaty, *The Analytic Hierarchy Process* (McGraw Hill, New York, 1980).
27. T. L. Saaty, *Decision Making with Dependence and Feedback: The Analytic Network Process* (RWS Publications, Pittsburgh, 1996).
28. R. W. Saaty, *Decision Making in Complex Environments* (Creative Decisions Foundation, Pittsburgh, 2003).
29. S. Burnaz and Y. I. Topcu, A multiple-criteria decision-making approach for the evaluation of retail location, *Journal of Multi-Criteria Decision Analysis* **14** (2006) 67−76.
30. C. Li, Y. Sun and Y. Du, Selection of 3PL service suppliers using a fuzzy analytic network process, in *Proc. Control and Decision Conf.*, China (2008), pp. 2174−2179.
31. L. Mikhailov and M. G. Singh, Comparison analysis of methods for deriving priorities in AHP, in *Proc. IEEE Conf. Systems, Man, and Cybernetics*, Tokyo, Japan (1999), pp. 1037−1042.
32. L. Mikhailov and M. G. Singh, Fuzzy analytic network process and its application to the development of decision support systems, *IEEE Transactions on Systems* **33** (2003) 33−41.
33. L. Mikhailov, Deriving priorities from fuzzy pair-wise comparison judgments, *Fuzzy Sets and Systems* **134** (2003) 365−385.
34. J. R. Yu and S. J. Cheng, An integrated approach for deriving priorities in analytic network process, *European Journal of Operational Research* **180** (2007) 1427−1432.
35. C. Kahraman, T. Ertay and G. Büyüközkan, A fuzzy optimization model for QFD planning process using analytic network approach, *European Journal of Operational Research* **171** (2006) 390−411.
36. M. L. Tseng, Y. H. Lin, J. H. Chiang and J. H. Guo, Environmental practices for manufacturing performance: FANP approaches, in *Proc. IEEE Int. Conf. Industrial Eng. and Eng. Manag.*, Singapore (2007), pp. 99−103.
37. X. Cui, F. Wang, X. Li, Y. Zhang and S. Yu, A fuzzy-ANP approach to cooperation risk evaluation of virtual logistics enterprise, in *Proc. IEEE Int. Conf. Automation and Logistics*, China (2007), pp. 1452−1457.
38. S. Sadi-nezhad, H. Didekhani and S. M. Seyedhosseini, Developing a fuzzy ANP model for selecting the suitable dispatching rule for scheduling a FMS, in *Proc. IEEE Int. Conf. Industrial Eng. and Eng. Manag.*, Singapore (2008), pp. 405−409.
39. M. L. Tseng, Y. H. Lin, A. S. F. Chiu and J. C. H. Liao, Using FANP approach on selection of competitive priorities based on cleaner production implementation: A case study in PCB manufacturer, Taiwan, *Clean Technologies and Environmental Policy* **10** (2008) 17−29.
40. U. R. Tuzkaya and S. Önüt, A fuzzy analytic network process based approach to transportation-mode selection between Turkey and Germany: A case study, *Information Sciences* **178** (2008) 3133−3146.

41. M. Dağdeviren, İ. Yüksel and M. Kurt, A fuzzy analytic network process (ANP) model to identify faulty behavior risk (FBR) in work system, *Safety Science* **46** (2008) 771−783.
42. R. Bi and J. Wei, Application of fuzzy ANP in production line selection evaluation indices system in ERP, in *Proc. IEEE Int. Conf. Automation and Logistics*, China (2008), pp. 1604−1608.
43. J. Razmi, R. Ghodsi and M. S. Sangari, A fuzzy ANP model to assess the state of organizational readiness for ERP implementation, in *Proc. 4th Int. Conf. Information and Automation for Sustainability*, Colombo (2008), pp. 481−488.
44. Z. Ayağ and R. G. Özdemir, An intelligent approach to machine tool selection through fuzzy analytic network process, *Journal of Intelligent Manufacturing* **45**(10) (2009) 2169−2194.
45. S. Onut, S. S. Kara and S. Mert, Selecting the suitable material handling equipment in the presence of vagueness, *International Journal of Advanced Manufacturing Technology* **44** (2009) 818−828.
46. J. J. Buckley, Fuzzy hierarchical analysis, *Fuzzy Sets and Systems* **17**(1) (1985) 233−247.
47. M. A. Brach, *Real Options in Practice* (John Wiley & Sons, Inc, Hoboken, New Jersey, 2003).
48. F. Black and M. Scholes, The pricing of options and corporate liabilities, *Journal of Political Economy* **81** (1973) 637−654.
49. P. P. Boyle, Options: A Monte Carlo approach, *Journal of Financial Economics* **4** (1977) 323−338.
50. J. C. Cox, S. A. Ross and M. Rubinstein, Option pricing: A simplified approach, *Journal of Financial Economics* **7** (1979) 229−263.
51. L. Zadeh, Fuzzy sets, *Information Control* **8** (1965) 338−353.
52. C. Carlsson and R. Fullér, A fuzzy approach to real option valuation, *Fuzzy Sets and Systems* **139** (2003) 297−312.
53. J. Wang and W. L. Hwang, A fuzzy set approach for R&D portfolio selection using a real options valuation model, *Omega* **35** (2007) 247−257.
54. A. C. Tolga and C. Kahraman, Fuzzy multiattribute evaluation of R&D projects using a real options valuation model, *International Journal of Intelligent Systems* **23**(11) (2008) 1153−1176.
55. D. Allenotor and R. K. Thulasiram, A grid resources valuation model using fuzzy real option, *Parallel and Distributed Processing and Applications, Proceedings*, Lecture Notes in Comput. Sci., Vol. 4742 (2007), pp. 622−632.
56. L. Clewlow and C. Strickland, *Implementing Derivatives Models* (John Wiley & Sons, Inc, Chichester, 1998).
57. E. S. Schwartz, The valuation of warrants: Implementing a new approach, *Journal of Financial Economics* **4** (1977) 79−94.
58. L. E. Brandao and J. S. Dyer, Decision analysis and real options: A discrete time approach to real option valuation, *Annals of Operations Research* **135** (2005) 21−39.
59. C. Kahraman and A. Ç. Tolga, An alternative ranking approach and its usage in multi-criteria decision-making, *International Journal of Computational Intelligent Systems* **3**(2) (2009) 219−235.
60. A. C. Tolga, C. Kahraman and M. L. Demircan, A comparative fuzzy real options valuation model using trinomial lattice and Black−Scholes approaches: A call center application, *Journal of Multiple-Valued Logic and Soft Computing* **16**(1−2) (2010) 135−154.
61. D. Ergu, G. Kou, Y. Peng and Y. Shi, A simple method to improve the consistency ratio of the pair-wise comparison matrix in ANP, *European Journal of Operational Research* **213** (2011) 246−259, doi:10.1016/j.ejor.2011.03.014.
62. D. Ergu, G. Kou, Y. Shi and Y. Shi, Analytic network process in risk assessment and decision analysis, *Computers & Operations Research* (2011) doi:10.1016/j.cor.2011.03.005.