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# A Study on Sixth Grade Students' Misconceptions and Errors in Spatial Measurement: Length, Area, and Volume

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**Abstract** The purpose of the present study was to portray students' misconceptions and errors while solving conceptually and procedurally oriented tasks involving length, area, and volume measurement. The data were collected from 445 sixth grade students attending public primary schools in Ankara, Türkiye via a test composed of 16 constructed-response format tasks. The findings revealed a wide range of misconceptions and errors such as "believing that all rulers are 30 cm long," "confusing area formula with perimeter formula," "believing a box has more than one surface area," "using the volume formula for surface area," "believing that ruler must be longer than the object measured," etc. These misconceptions and errors could be considered as the evidences indicating the sixth graders' lack of comprehension of the fundamental concepts of spatial measurement and their relationships and the procedures and formulas used for measuring length, area, and volume. The possible causes of such misconceptions and overcoming ways were also discussed.

**Keywords** Area · Conceptual knowledge · Errors · Length · Mathematical misconceptions · Procedural knowledge · Spatial measurement · Volume measurement

## Introduction

Several theories of learning and instruction assigned the fundamental role for "knowledge" in one's cognitive development and learning process (De Jong &

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Ferguson-Hessler, 1996). Schneider & Stern (2006) claimed that at least two types of knowledge are shaping one's understanding and actions: the first one is knowledge of concepts and the second is knowledge of procedures in a certain domain. A bulk of research and theory in cognitive science supports the notion that deep understanding depends on how well a learner represents and connects bits of knowledge (Kilpatrick, Swafford & Findell, 2001). Understanding is more than acquiring concepts or skills at a time (Carpenter & Lehrer, 1999), it is a complex and multifaceted mental activity that is emerged and deepened in a continuous and generative process. Success in mathematics relies mostly on how learners internalize the meaning related to a procedure they are learning or a concept that is being taught and connections made between them. Thus, understanding of students' conceptions, ideas, and skills of mathematical strands is central to good teaching and learning of mathematics (Even & Tirosh, 2008; Fennema, Carpenter, Frankie, Levi, Jacobs & Empson, 1996; National Council of Teachers of Mathematics [NCTM], 2000). Among mathematical strands, "measurement" has a unique place in almost all mathematics curricula. Considering its vital role in mathematics, science, and our life, students should fully understand not only "how to measure" but also "what it means to measure." However, several research studies on teaching and learning "measurement" have indicated that students have poor and superficial understanding of length, area, and volume measurement (Bragg & Outhred, 2004; Chappell & Thompson, 1999; Curry, Mitchelmore & Outhred, 2006; Grant & Kline, 2003; Kamii & Clark, 1997; Martin & Strutchens, 2000). In this respect, portraying students' errors and misconceptions in length, area, and volume measurement might shed light to their conceptions, ways of thinking, and learning difficulties.

## Review of Literature

### Conceptual and Procedural Knowledge

The essential aspect of mathematical understanding is to be able to make links not only among bits of knowledge but also among other areas (Putnam, 1987). Considering the place of richly connected links in the learner's mathematical understanding, the types of knowledge have continued to receive a great deal of attention over the last decades (Even & Tirosh, 2008). Especially after the Skemp's influential work (1978), various definitions of knowledge of concepts and procedures have been characterized under different labels (e.g. Declarative knowledge vs. Procedural knowledge by Anderson, 1983; Proceptual thinking vs. Procedural thinking by Gray & Tall, 1993), yet the distinctions between types of mathematical knowledge are generally overlapping and based on emphasis rather than kind (Hiebert, 1986). After the publication of Hiebert's edited book (1986), the terms used for mathematical knowledge have been recognized mostly in the form of conceptual and procedural knowledge (Putnam, 1987). As defined by Hiebert (1986), conceptual knowledge is "connected web of knowledge" and "rich in relationships" (p. 3); procedural knowledge, on the other hand, is composed of both the knowledge of mathematical symbols and the knowledge of algorithms or procedures that are "step-by-step instructions that prescribe how to complete tasks" (p. 6).

In the last 30 years, several research studies conducted on conceptual and procedural knowledge have mainly tried to shed light on the following issues: (a) which type of

knowledge is more essential, (b) whether the balance between types of knowledge is needed, (c) what is the relationship between them, (d) which type of knowledge develops first, and (e) what is the interaction between them (Gelman & Williams, 1998;-Haapasalo & Kadijevich, 2000; Rittle-Johnson, Siegler & Alibali, 2001; Sophian, 1997; Star, 2005). Although drawing a clear picture about conceptual and procedural knowledge still remains an unsolved problem in the field, the bulk of theoretical and research-based arguments clearly indicate that building links between types of knowledge is the most effective way of learning mathematics with understanding which is one of the ultimate goals of almost all mathematics curricula (Hiebert & Carpenter, 1992; Ohlsson & Rees, 1991; Star, 2000, Van de Walle, 2007). The main points related to conceptual and procedural knowledge in the mathematics education literature are summarized in Appendix A.

## Misconceptions and Errors in Length, Area, and Volume Measurement

Students' errors and misconceptions while learning mathematics are considered as rich vehicles for uncovering their conceptions, ways of thinking, and learning difficulties (Ashlock, 1990; Greeno, Collins & Resnick, 1996). An error is "erroneous responses to a question" (Hadjidemetriou & Williams, 2002, p. 69) and "caused by insufficient mastery of basic facts, concepts, and skills" (Legutko, 2008, p. 143). A misconception is a "student conception that produces a systematic pattern of error" (Smith, diSessa & Rochelle, 1993, p. 119); may be due to "the misapplication of a rule, an over- or under-generalization, or an alternative 'conception' of the situation" (Drews, 2005, p. 15) and "part of a faulty cognitive structure that causes, lies behind, explains, or justifies the error." (Hadjidemetriou & Williams, 2002, p. 69). Indeed, errors and misconceptions have potential to open a window onto the way that how students internalize concepts and skills (Radatz, 1980; Ryan & Williams, 2007). Thus, considering students' misconceptions and errors as the aspects of their informal knowledge that have the same basic structure with experts, yet have been used incorrectly is more productive and functional for learning than labeling them as wrong or failure (Smith et al., 1993). For several decades, researchers have studied the errors and misconceptions in mathematics education and attempted to shed light on three focal points as follows: (a) types of students' errors and misconceptions, (b) the causes of errors and misconceptions, and (c) the ways to overcome and remediate these errors and misconceptions (e.g. Borasi, 1996; Gagatsis & Kyriakides, 2000; Radatz, 1979; Watson, 1980). Furthermore, several research studies on teaching and learning "measurement" have indicated that students have difficulties in learning measurement especially in the domains of length, area, and volume (Chappell & Thompson, 1999; Martin & Strutchens, 2000; Robinson, Mahaffey & Nelson, 1975). The common point emerged from the past research is that the reason behind students' poor understanding of measurement is putting more emphasis on how to measure rather than what to measure (Grant & Kline, 2003; Kamii & Clark, 1997). Among the domains of measurement, *linear measurement* has a unique place in grasping the main ideas of measurement. The concepts and skills involved in linear measurement are particularly essential for students' understanding of area and volume measurement as well as understanding of more advanced topics taught in secondary school (Nührenbörger, 2001; Outhred, Mitchelmore, McPhail & Gould, 2003; Outhred & Mitchelmore, 2000). The foundational concepts of length

measurement are conservation, unit iteration, additivity, transitivity, partitioning, zero-point, accumulation of distance, and relation between number and measurement (Kamii & Clark, 1997; Lehrer, 2003; Stephan & Clements, 2003). Nevertheless, the lines of several research reports have revealed elementary and middle school students' difficulties while learning linear measurement are as follows: (a) starting from 1, (b) ignoring the idea of unit iteration, (c) incorrect alignment with a ruler, (d) counting hash marks or numbers on a ruler, and (e) focusing on end point while measuring with a ruler (Barrett, Clements, Klanderman, Pennisi & Polaki, 2006; Battista, 2006; Bragg & Outhred, 2001, 2004; Nunes, Light & Mason, 1993; Schwartz, 1995; Wilson & Rowland, 1993). Moreover, *measuring of area* not only expands students' understanding of spatial measurement but also provides foundations for the development of students' understanding of multiplication, fractions, algebraic multiplication, and enlargement (Sarama & Clements, 2009; Hirstein, Lamb & Osborne, 1978; Outhred & Mitchelmore, 2000). Stephan & Clements (2003) have stated that partitioning, unit iteration, conservation, and structuring an array are the foundational ideas of area measurement. What research has found about the students' difficulties are as follow: (a) realizing how length units produce area units, (b) grasping the conservation of area, (c) understanding array and grid structure, (d) comprehending two-dimensional structure of area, and (e) understanding the difference between not only the concept of area and perimeter (i.e. a special linear dimension for a closed two-dimensional figure, Larsen, 2006, p. 41) but also the formulas for these concepts (Battista, 2003; Chappell & Thompson, 1999; Furinghetti & Paola, 1999 ;Nunes et al., 1993). Furthermore, *volume measurement* is also a significant topic from elementary to high school levels as it provides rich context for extension of students' knowledge about arithmetic, geometric reasoning, and spatial structuring (Battista, 2003; Lehrer, 2003). In order to gain understanding of volume measurement, conservation, spatial visualization, and meaningful enumeration of arrays of cubes are the foundational ideas (Battista, 1999; Ben-Haim, Lappan & Houang, 1985; Lehrer, 2003). Studies have shown, however, that students have tough times to make sense of volume measurement. The following are the mostly reported students' difficulties in volume measurement: (a) treating three-dimensional figures as two-dimensional ones, (b) counting visible faces/unit cubes, (c) enumerating the cubes in 3D arrays incorrectly, and (d) confusing the concept of volume with the surface area and their formulas (Battista, 1999, 2003; Ben-Haim et al. 1985; Owens & Outhred, 2006; Voulgaris & Evangelidou, 2004). The theoretical foundations and research-based claims about length, area, and volume measurement are summarized in Appendix B.

In the light of abovementioned issues, it is obvious that devaluing measurement strand can be seriously debilitating for children's mathematics learning since the concepts and skills involved in measurement provide a route into mathematics as well as into daily life, science and technology practices (Ryan & Williams, 2007). Thus, doing a research study on students' errors and misconceptions in length, area, and volume measurement while solving conceptually and procedurally orientated tasks is important from several perspectives. First of all, many of the studies have explored either the students' performance or the development of students' concept growth in a single domain of measurement. In addition, researchers in mathematics education concerning

students' misconceptions and errors have been mostly focused on addition, subtraction, multiplication, division, and fractions. Therefore, there is a paucity of research on students' errors and misconceptions in length, area, and volume measurement *altogether* through conceptually and procedurally oriented tasks (Borasi, 1996; Eames, Miller, Barrett, Cullen, Kara, Clements, Sarama et al., 2014; Lewis & Schad, 2006; Radatz, 1979). Furthermore, constructivist way of teaching and learning mathematics considers students' misconceptions as the reflections of their conceptual structure generated through the interactions between previous and new (Smith et al., 1993). In this sense, it is obvious that students' previously learnt concepts and skills, and the relations between them play a crucial role in making sense of mathematics. For instance, Bragg & Outhred (2000) noted that students' understanding of length measurement is crucial for understanding of rulers, perimeter, area, and volume measurement. Battista (2003) underlined the importance of area measurement for understanding of volume measurement. In order to promote mathematical understanding, recognition of students' (*mis*)conceptions and then restructuring instruction accordingly that allow a student to refine his/her existing naïve ideas and to lead toward conceptual mastery of a particular topic are of the utmost importance (Smith et al., 1993). Furthermore, it is also believed that the results of this study may contribute to the previous research by giving an example from the Turkish educational context where the constructivist way of learning and student-centered approach are taken as basis and reflected in the content, delivery, and assessment aspects of the Turkish Mathematics curriculum.

In this respect, analyzing specifically students' misconceptions and errors while solving conceptually and procedurally orientated tasks involving length, area, and volume measurement will be valuable both for teachers and curriculum developers who aim to develop and promote students' mathematical understanding by making use of misconceptions and errors as catalysts for learning.

### The Purpose of the Study

The purpose of the present study was to investigate students' errors and misconceptions while engaging in the tasks on length, area, and volume measurement. The study was intended to address the following research questions:

1. What are the students' errors and misconceptions while solving conceptually orientated tasks involving length, area, and volume measurement?
2. What are the students' errors and misconceptions while solving procedurally orientated tasks involving length, area, and volume measurement?

## Method

### Participants

The study was carried out with 445 sixth grade students. The reasons behind focusing on sixth grade as a target level are mainly due to experience with the

content, and the learning objectives of length, area, and volume measurement in the Turkish National Mathematics Curriculum from first to sixth grades which included the fundamental concepts and skills. The participants were selected among the public primary schools in Ankara, Türkiye through the use of purposive sampling method (Fraenkel & Wallen, 2006). The public schools' average mathematics score in the Selection Examination for Secondary Education Institutions (OKS) was used as the main criteria. One of the reasons is that the OKS was a highly-competitive nationwide examination administered at the end of eighth grade. It was also mandatory for those who would like to enroll in one of the well-resourced, qualified, and prestigious high schools. Moreover, the performances of the schools on the OKS exam were perceived by parents as a strong indicator of the quality of education provided in primary schools (Sahin, 2004), since the exam covered the content of primary education curriculum (first to eighth grades) subjects including mathematics as well as other subject areas as Turkish language, science, etc. In total, there were 100 multiple-choice items, 25 of which were on mathematics. According to the official records of the Ministry of National Education, the total number of public primary schools in Ankara that participated in the OKS that year was 685. Based on the range, the minimum, and the maximum scores obtained from the OKS, public primary schools in Ankara were classified as low, medium, and high-achieving schools and two schools from each achievement level were selected for the study. In total, 445 sixth grade students (203 male and 242 female) attending the selected public schools voluntarily participated in the study. Their ages ranged between 11 and 14 and most of them were 12 years old.

**Data Collection and Analysis**

In the present study, the data was collected via a test (see Appendix C and D) composed of 16 constructed-response format tasks prepared by considering previous research on students' understanding of the measurement, the theoretical bases of measurement, and students' errors and misconceptions about measurement as well as the content and the learning objectives of length, area, and volume measurement topics in the sixth grade mathematics curriculum. Although most of the mathematics educators have underlined the notion that it was difficult to design procedure-free items to assess conceptual knowledge in mathematics and vice versa (e.g. Haapasalo & Kadijevich, 2000; Hiebert, 1986; Kadijevich, 1999; Schneider & Stern, 2006), conceptually oriented tasks were generally characterized as non-routine and novel tasks that require the use of understanding of underlying principles or concepts in a mathematical domain, not necessarily involving computations (Hiebert, 1986; Kadijevich, 1999; Kulm, 1994; Rittle-Johnson et al. 2001). On the other hand, procedurally oriented tasks were generally defined in the literature as routine tasks that require the use of previously learned step-by-step solution methods, mathematical computations, algorithms, or formulas. Within this vision, a test was developed by the researchers including the conceptually oriented tasks, intended

to assess the extent to which students comprehend the conceptual underpinnings of length, area, and volume measurement, and the procedurally oriented tasks intended to investigate the extent to which students could apply procedures (routine and complex) related to three domains of measurement. In other words, through the tasks, students were asked to show understanding of measurement concepts and skills by interpreting, applying, and transferring them correctly to different situations. A test composed of 16 constructed-response format tasks (8 conceptually oriented and 8 procedurally oriented) was given to 3 academics from the field of mathematics education, 2 academics from the field of educational sciences, 3 mathematics teachers so as to obtain content-related and face validity evidences. In addition to expert-opinion, a pilot study was conducted with seventh grade students ($N$ = 134) who already received the instruction on measurement during sixth grade, but were not taught the seventh grade measurement content. Based on the reflections and feedback gathered from the experts and the pilot study, the test was revised accordingly. After the completion of regular instruction on the length, area, and volume measurement in the schools, the test was administered to sixth grade students by the researchers. The data analysis process is composed of five stages and illustrated in the Fig. 1. In the first step, a scoring key was prepared and revised by the two academics from the field of mathematics education and a mathematics teacher. In the key, 1 point was assigned for the correct answer/explanation and 0 for both the incorrect answer/explanation and no answer. For the categorization of the students' errors related to .three domains of measurement, in the second stage, the literature-based framework was developed and then each student's incorrect responses were transformed into Word program and listed under the related tasks one by one considering the framework. Afterwards, the newly emerged errors were synthesized with the literature-based framework in order to finalize the categories of errors in terms of three domains of measurement. As an example, the analysis process of the task 1.a is provided in Appendix E. While producing the categories, the researchers analyzed and tabulated the students' responses independently. Subsequently, both sets of data were compared, and through discussion, the disparities were reconciled to reach a consensus. Then, the basic descriptive statistics including frequencies and percentages were carried out by means of Predictive Analytics SoftWare (PASW).

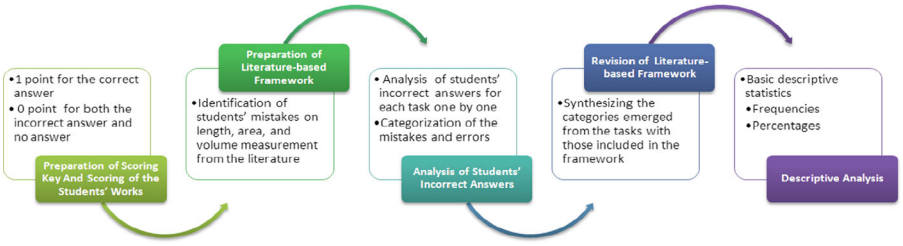



**Fig. 1** Data analysis process

## Results

The results gathered from the analysis of a 16-constructed-response item test (see Appendix C and D) were presented in line with two main research questions.

### Students' Errors and Misconceptions Regarding Length, Area, and Volume Measurement in the Conceptually Oriented Tasks

The first research question was raised to find the errors and misconceptions of students while solving conceptually oriented tasks involving length, area, and volume measurement. In general, the findings revealed students' poor and superficial conceptual understanding in each domain of measurement as the common errors and misconceptions summarized in the Table 1.

Considering length measurement, the analysis of the students' responses to the broken ruler task revealed that a majority of them (60 %) found the length of the broken ruler correctly. About 40 % of the sixth graders had difficulty to grasp the meaning of zero-point and the structure of a ruler and thus, they tried to invent their own strategies. As shown in Fig. 2, a student added all numbers on the broken ruler together and reported that the length of the ruler is 267 cm.

In the second-part of the task 1, more than half of the students (54.6 %) believed that the broken ruler cannot be used for measuring a 2-m-long cloth. The misconceptions and errors made by the sixth grade students while solving the task-1 are presented in the Table 2.

Furthermore, the students' written explanations to tasks 2 and 3 revealed that they had a very rudimentary knowledge about the concept of perimeter. Out of 445 students, only 157 of them (35.3 %) correctly explained why the perime-

**Table 1** The common errors and misconceptions in the conceptually oriented tasks

| Linear measurement | Area measurement | Volume measurement |
|---|---|---|
| Starting from 1 rather than 0 | Believing area is not constant, under partitioning | Counting the square units or faces of unit cubes or only visible unit cubes |
| Believing that a ruler should be longer than the object being measured | Counting the lines around a shape | Double counting unit cubes |
| Believing that cm is not suitable unit to measure in meters | Using units of length/volume measurement | Counting the faces of unit cubes and multiplying that number by 3 because a prism has three dimensions |
| Believing that perimeter is constant when the shape is rearranged | Confusing volume with surface area | Counting the faces of unit cubes given in the picture and doubling that number |
| Using units of area/volume measurement | Believing a shape has more than one surface areas | |

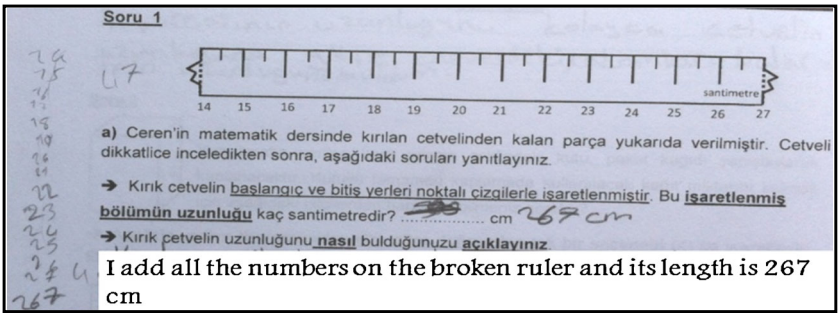



**Fig. 2** A student work task-1 broken ruler

ters of two shapes are equal. The common errors made while solving this task were counting square units (see Fig. 3) which is the indicator of area and perimeter confusion and using units of area/volume measurement while reporting perimeter (see Fig. 4).

According to the findings of the task 3, few numbers of the students (5 %) could be able to correctly explain whether perimeter change under partitioning or not. Most of the students (62 %) believed that since both shapes are made up with the same pieces, their perimeters are equal. The students' misconcep-

**Table 2** Students' errors and misconceptions in task 1 (broken ruler)

| Task | Answers | *f* | % |
|---|---|---|---|
| The broken ruler | Correctly explained | 268 | 60.2 |
| (a) Explain how to find the length of the broken ruler | Incorrectly explained | 177 | 39.8 |
| | Students' errors and misconceptions | | |
| | Counting the numbers on the ruler | 105 | 23.6 |
| | Believing "all rulers are 30 cm long" | 19 | 4.3 |
| | Adding first and last numbers on the ruler | 19 | 4.3 |
| | First number plus last number on the ruler minus 1 | 15 | 3.4 |
| | Reporting the last number on the ruler as its total length | 10 | 2.2 |
| | Adding all numbers on the ruler | 5 | 1.1 |
| | First number plus last number on the ruler plus 1 | 4 | .9 |
| (b) Explain whether the broken ruler can be used for measuring a 2-m-long cloth | Correctly explained | 202 | 45.4 |
| | Incorrectly explained | 243 | 54.6 |
| | Students' errors and misconceptions: It cannot be used, because | | |
| | The ruler is too short to measure the cloth | 112 | 25.3 |
| | The ruler is broken and lost its function | 64 | 14.4 |
| | cm is not suitable unit to measure in meters | 29 | 6.5 |
| | 200 cm cannot be divided by 13 cm | 18 | 4.0 |
| | The ruler is not starting from 0 (zero) | 20 | 4.4 |

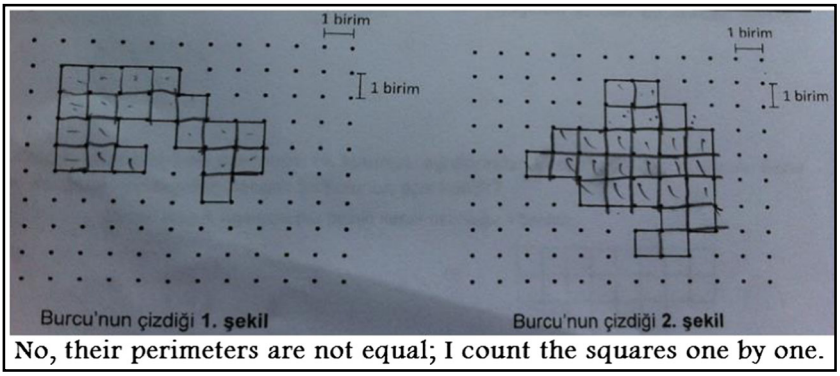



**Fig. 3** A student work task-2: counting square units

tions and errors with regard to the concept of perimeter are listed in the Table 3.

Moreover, the results emerged from the students' responses on the tasks 4, 5, and 6, focusing on the concept of area, indicated a wide range of misconceptions. As seen in Table 4, counting the lines around the shapes (36.2 %) is the mostly committed error in task 4 among the sixth graders. With regard to the idea of area conservation asked in task 5, among 445 sixth grade students, 290 of them could not be able to develop the understanding of area conservation and believed that area changes, if the shape is rearranged. The focus of task 6 was on the concept of surface area. About 70 % of the students could not be able to explain the relationship between the amount of wrapping material and the surface area of a box. Among those students who missed this task, a majority of them (19 %) confused the concept of surface area with volume. The students' misconceptions and errors with regard to the concept of area are listed in the Table 4.

Concerning volume measurement, only 110 of the sixth grade students (24.7 %) could be able to find the number of unit cubes needed to completely

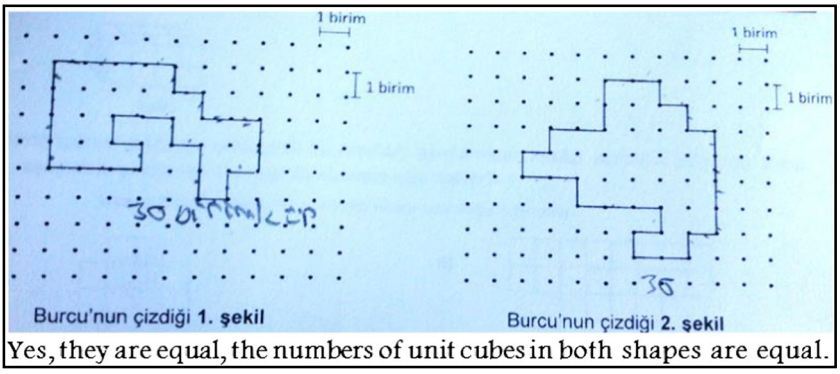



**Fig. 4** A student work task-2: using unit of volume

**Table 3** Students' errors and misconceptions in tasks 2 and 3

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 2: The perimeters of two shapes drawn on dot paper Explain whether they are equal | Correctly explained | 157 | 35.3 |
| | Incorrectly explained | 288 | 64.7 |
| | Students' errors and misconceptions | | |
| | Counting square units | 84 | 19 |
| | Using units of area/volume measurement while reporting perimeter | 50 | 11.2 |
| | Believing that area and perimeter must be equal | 48 | 10.8 |
| | Focusing on physical appearance of the shapes | 48 | 10.8 |
| | Counting dots | 38 | 8.5 |
| | Not counting all lines surrounding the shape | 20 | 4.5 |
| Task 3: Perimeter under partition Explain whether perimeter change under partitioning or not? | Correctly explained | 21 | 4.7 |
| | Incorrectly explained | 424 | 95.3 |
| | Students' errors and misconceptions | | |
| | Believing that the perimeters are equal, since both are made up with the same pieces/ only their pieces are arranged differently | 274 | 62 |
| | Making visual comparison | 85 | 19.1 |
| | Focusing on geometrical features of the shapes | 63 | 14.2 |

fill the box. As given in Table 5, the common error made while solving this task (#7) was counting the faces of unit cubes given in the picture and doubling that number. In the last conceptually oriented task, only 29 of the sixth graders (6.5 %) correctly found out the volume of a prism through its net. Half of the students counted the square units in the net to find out the volume of the rectangular prism which may be considered to be a clear indicator of the confusion of surface area with volume.

Among those students who missed this task (#8), 19 % of them could not determine the dimensions of prism correctly (Fig. 5), and also 15 % of them treated volume as a one-dimensional structure and counted the lines around the prism's net (Fig. 6).

## Students' Errors and Misconceptions Regarding Length, Area, and Volume Measurement in the Procedurally Oriented Tasks

The second research question was aimed to investigate the students' errors and misconceptions while solving procedurally oriented tasks involving length, area, and volume measurement. The analysis of students' responses on eight procedurally oriented tasks also revealed that the procedural knowledge that students have represents only surface-level information. The following table (Table 6) summarized the common errors and misconceptions in the procedurally oriented tasks by domains of measurement.

**Table 4** Students' errors and misconceptions in tasks 4, 5, and 6

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 4: The areas of two shapes drawn on dot paper<br>Explain whether the areas of two shapes are equal | Correctly explained | 148 | 33.3 |
| | Incorrectly explained | 297 | 66.7 |
| | Students' errors and misconceptions | | |
| | Counting the lines around the shape | 161 | 36.2 |
| | Using units of length/volume measurement while reporting area | 67 | 15.1 |
| | Focusing on physical appearance of the shapes | 42 | 9.3 |
| | Believing that if the perimeters of shapes are equal, their areas equal too | 27 | 6.1 |
| Task 5: Conservation of area<br>Explain whether the areas of A and C are the same or not | Correctly explained | 155 | 35 |
| | Incorrectly explained | 290 | 65 |
| | Students' errors and misconceptions | | |
| | C < A, because A was cut and C was constructed from A | 115 | 25.7 |
| | C>A, because C has more sides than A (zigzag's) | 77 | 17.3 |
| | C >A, because C seems to bigger | 36 | 8.1 |
| | C < A, because A is square and C is different | 32 | 7.2 |
| | C = A, because they seems to be similar/same | 30 | 6.7 |
| Task 6: The amount of wrapping material<br>Explain which surface area, the sum of all side lengths, or volume gives the most helpful information in order to find out the amount of wrapping material needed for the box? | Correctly explained | 144 | 32.4 |
| | Incorrectly explained | 301 | 67.6 |
| | Students' errors and misconceptions | | |
| | I need to know the volume of the box, because it is the amount of space covering the outside of the box | 84 | 19 |
| | I need to know the sum of all side lengths, because it is equal to the inside area of box | 65 | 14.5 |
| | I need to know the surface area of the box, because its surface areas will be wrapped (believing the box has more than one surface area) | 42 | 9.3 |
| | I need to know the sum of all side lengths, because multiplying the sum with 6 gives the surface area | 32 | 7.2 |
| | I need to know the sum of all side lengths, because it is easier to find out than others | 27 | 6.1 |
| | I need to know the surface area of the box, because its calculation is more important than others | 20 | 4.5 |
| | I need to know the surface area of the box, because it indicates how many meters/centimeters of material needed | 19 | 4.3 |
| | I need to know the surface area of the box, because its calculation is easier than others | 12 | 2.7 |

Considering linear measurement, most of the students (76.6 %) used a ruler correctly to measure the line segment (task 1), yet nearly a quarter of them either counted the numbers on the ruler or aligned a ruler incorrectly, as shown in Table 9. While calculating

**Table 5** Students' errors and misconceptions in tasks 7 and 8

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 7: Volume of the box<br>Explain how the volume of the box is found | Correctly explained | 110 | 24.7 |
| | Incorrectly explained | 335 | 75.3 |
| | Students' errors and misconceptions | | |
| | Counting the faces of unit cubes given in the picture and doubling that number | 72 | 16.1 |
| | Counting the faces of unit cubes given in the picture and multiplying that number with 3 because a prism is 3D. | 48 | 10.8 |
| | Using incorrect formula for volume (e.g. base × height × 2) | 47 | 10.6 |
| | Counting the unit cubes given in the picture and filling the box (by drawing unit cubes) in a wrong manner (double counting) | 47 | 10.6 |
| | Counting the number of unit cubes given in the picture and multiplying that number with 3 because a prism is 3D. | 43 | 9.7 |
| | Using correct formula with wrong dimension (e.g. 3 × 5 × 3) | 42 | 9.4 |
| | Counting the faces of unit cubes given in the picture and multiplying with 6, because a prism has 6 surfaces. | 28 | 6.3 |
| | Using units of length/area measurement reporting the volume | 8 | 1.8 |
| Task 8: The volume of a prism through its net<br>Explain how the volume is found | Correctly explained | 29 | 6.5 |
| | Incorrectly explained | 416 | 93.5 |
| | Students' errors and misconceptions | | |
| | Counting the square units | 222 | 49.9 |
| | Using the volume formula with wrong dimensions of the prism | 84 | 18.9 |
| | Counting the lines around the prism's net | 64 | 14.4 |
| | Counting all square units then multiplying the total number with 4 | 36 | 8.1 |
| | Reporting the volume with units of length/area measurement | 10 | 2.2 |

the perimeter of a polygon (task 2), 15 % of the sixth graders confused formula for perimeter with area, and also about 10 % of them made addition errors (see Fig. 7).

In addition, given the length and perimeter of a rectangle (task 3), about 60 % could successfully find its width. Among those students who missed

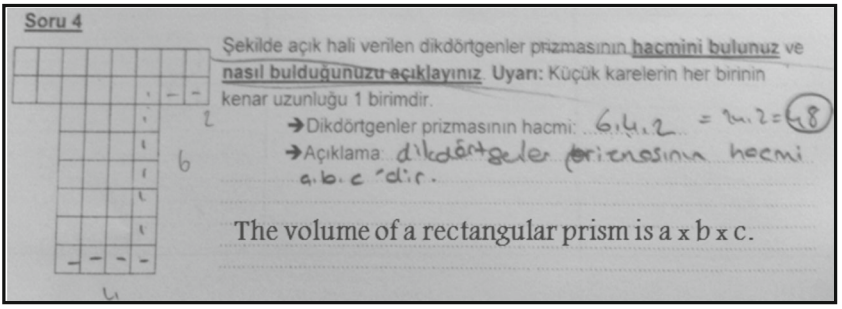



**Fig. 5** A student work task-8: using the volume formula with wrong dimensions of the prism

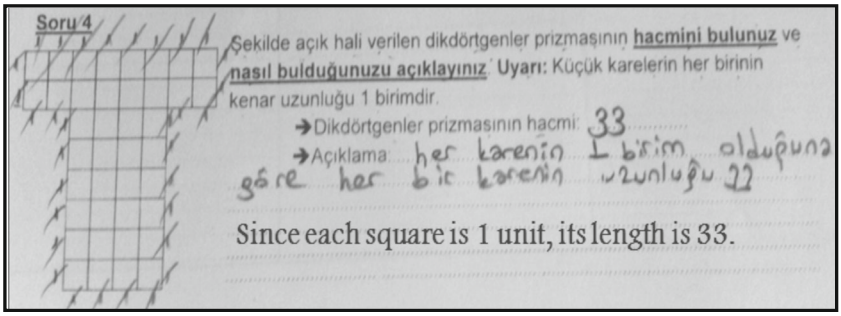



**Fig. 6** A student work task-8: counting the lines around the prism

this task, as shown in Table 7, 12 % of them tried to find out the width as "width = perimeter − (2 × length)" and 9 % of them as "width = perimeter − length." The Table 7 indicates the students' errors and misconceptions with regard to linear measurement emerged from the procedurally oriented tasks 1, 2, and 3.

**Table 6** The common errors and misconceptions in the procedurally oriented tasks

| Linear measurement | Area measurement | Volume measurement |
|---|---|---|
| Incorrect alignment of a ruler | Using perimeter formula for | Volume = length + width + height |
| Counting the numbers on a ruler | Area = length + width | Volume = (length × width) + height |
| Reporting the end point | Using units of length | Volume = length × width |
| Using area formula for perimeter | Surface area= length + width + height | Errors in four basic operations |
| Errors in four basic operations | Using volume formula for surface area | |
| | Errors in four basic operations | |

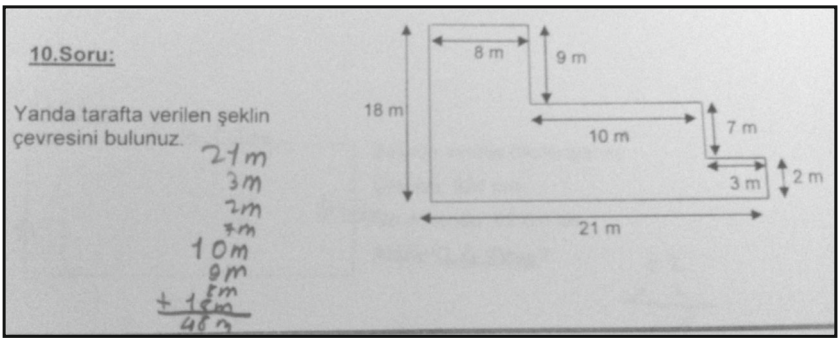



**Fig. 7** A student work task-1: addition errors while calculating perimeter

**Table 7** Students' errors and misconceptions in linear measurement in tasks 1, 2, and 3

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 1: Using a ruler to measure the line segment | Correctly measured | 341 | 76.6 |
| | Incorrectly measured | 104 | 23.4 |
| | Students' errors and misconceptions | | |
| | Counting the numbers on the ruler | 56 | 12.6 |
| | Incorrect alignment | 34 | 7.6 |
| | Reporting the last number matching the end point of the segment (C) as the length | 9 | 2.1 |
| | Adding all numbers on the ruler between the points of C and B | 5 | 1.1 |
| Task 2: The perimeter of a polygon All side lengths were given | Correctly calculated | 278 | 62.5 |
| | Incorrectly calculated | 167 | 37.5 |
| | Students' errors and misconceptions | | |
| | Using area formula | 64 | 14.3 |
| | Using the correct formula, but making addition errors | 39 | 8.7 |
| | Not adding all sides | 25 | 5.6 |
| | Adding the length of the polygon to the width [not multiplying with 2] | 21 | 4.7 |
| | Adding all sides and multiplying that number by 2 | 18 | 4.2 |
| Task 3: Finding the width of a rectangle The perimeter and the length were given | Correctly calculated | 279 | 62.7 |
| | Incorrectly calculated | 166 | 37.3 |
| | Students' errors and misconceptions | | |
| | Width = perimeter – (2 × length) | 53 | 12 |
| | Width = perimeter – length | 39 | 9 |
| | Width = length / 2 | 21 | 5 |
| | Width = perimeter + (2 / length) | 19 | 4.1 |
| | Errors in addition/multiplication | 11 | 2.4 |
| | Width = perimeter + length | 8 | 1.7 |
| | Width = perimeter + (2 × length) | 5 | 1.1 |
| | Width = length × 2 | 4 | 0.8 |
| | Width= perimeter / length | 4 | 0.8 |
| | Width = perimeter × length | 2 | 0.4 |

Furthermore, the results gathered from the students' work on the tasks 4, 5, and 6 related to area measurement revealed a wide range of errors and misconceptions. As seen in Table 8, totally, 158 students could not find the area of a rectangle correctly (task 4) and among these students, 71 of them used the formula for perimeter instead of area, and also 37 of them tried to find out the area by adding length and width together (Fig. 8).

In the task 5, only 30 % of the sixth graders determined the un-shaded area of a rectangular shape in which all side lengths were given. Although various

**Table 8** Students' errors and misconceptions in area measurement in the tasks 4, 5, and 6

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 4: The area of a rectangle<br>The length and the width were given | Correctly calculated | 287 | 64.5 |
| | Incorrectly calculated | 158 | 35.5 |
| | Students' errors and misconceptions | | |
| | Using the perimeter formula | 71 | 16 |
| | Area = width + length | 37 | 8.3 |
| | Using units of length measurement | 25 | 5.6 |
| | Errors in addition/multiplication | 17 | 3.9 |
| | Area = width × length/2 | 8 | 1.7 |
| Task 5: Determining the un-shaded area of a rectangular shape<br>All side lengths were given | Correctly calculated | 132 | 30 |
| | Incorrectly calculated | 313 | 70 |
| | Students' errors and misconceptions | | |
| | Using the perimeter formula | 82 | 18.4 |
| | Area = width + length | 79 | 17.6 |
| | Errors in addition/multiplication | 52 | 11.6 |
| | Calculating the shaded area and reporting it as an un-shaded area | 31 | 7 |
| | Calculating the total area and reporting it as an un-shaded area | 26 | 5.8 |
| | Area = length × length | 21 | 4.6 |
| | Using units of length measurement | 13 | 3 |
| | The un-shaded area = all area / 4 | 9 | 2 |
| Task 6: The surface area of a rectangular prism<br>All dimensions were given | Correctly calculated | 28 | 6.3 |
| | Incorrectly calculated | 417 | 93.7 |
| | Students' errors and misconceptions | | |
| | Surface area = length + width + height | 103 | 23.5 |
| | Surface area = length × width × height | 87 | 19.5 |
| | Surface area = length × width | 76 | 17 |
| | Surface area = (length × height) + width | 51 | 11.4 |
| | Surface area = length + width | 32 | 7.3 |
| | Surface area = (length + width + height) × 6 | 19 | 4.2 |
| | Surface area = length × height | 19 | 4.2 |
| | Surface area = length × 4 | 11 | 2.4 |
| | Surface area = (length × height)/2 | 9 | 2 |
| | Surface area = (length × width)/ height | 6 | 1.4 |
| | Surface area = (2 × width) + (2 × length) | 4 | 0.8 |

types of errors came into sight, using perimeter formula while calculating un-shaded area is the mostly committed error among the students. Besides, the students were also asked to calculate the surface area of a rectangular prism in which all dimensions were given (task 6). Apart from 28 (6.3 %) students, all

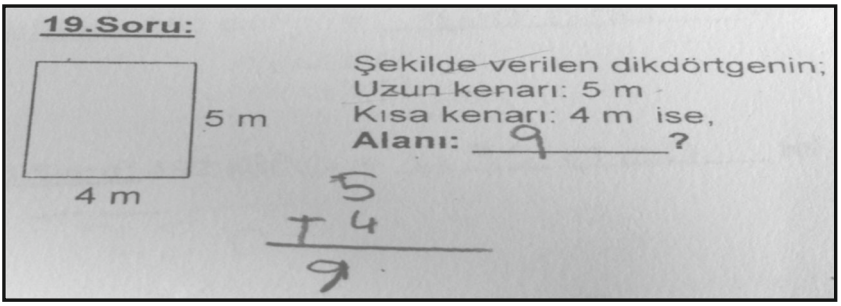



**Fig. 8** A student work task 4: area = length + width

of them missed the task. While 103 (23.5 %) of the sixth graders ignored two-dimensional structure of area and preferred to use the following computation "surface area = length + width + height," 87 (20 %) of them used the formula of volume, and some them tried to calculate surface area as "(length + width + height) × 6" follow added length, width, and height together and then multiple by 6 (Fig. 9).

In Table 8, the students' errors and misconceptions with regard to area measurement emerged from the procedurally oriented tasks 4, 5, and 6 are listed.

In relation to measurement of volume, out of 445 students, only 128 (29 %) of them correctly calculated the volume of a rectangular prism whose dimensions were given (task 7). As seen in Table 9, 317 of the sixth graders generated different kinds of formula for volume ranging from "volume = length + width + height," namely treating volume as one-dimensional structure, to "volume = length × width." The number of students who were successful at finding the height of a rectangular prism whose volume, length, and width were given (task 8) was only 121 (27 %). Indeed, the analysis of their work clearly indicated that the students' knowledge on volume was so fragmented that they could not be applied even in routine problems.

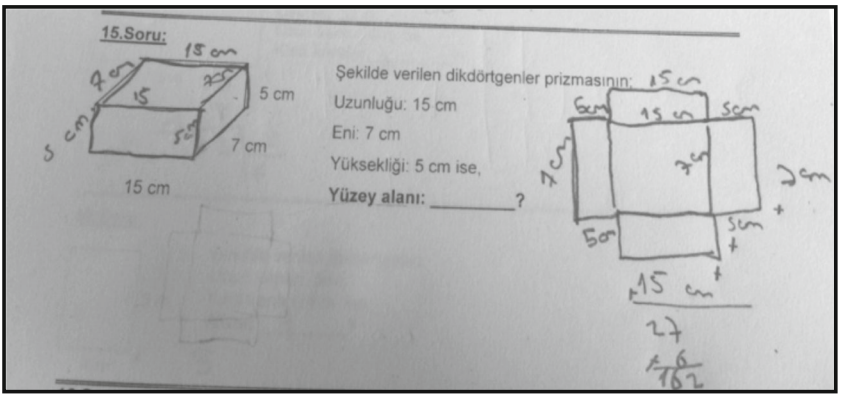



**Fig. 9** A student work task 6: surface area = (length + width + height) × 6

**Table 9** Students' errors and misconceptions in volume measurement in the tasks 7 and 8

| Task # | Answers | *f* | % |
|---|---|---|---|
| Task 7: The volume of a rectangular prism<br>All dimensions were given | Correctly calculated | 128 | 28.8 |
| | Incorrectly calculated | 317 | 71.2 |
| | Students' errors and misconceptions | | |
| | Volume = length + width + height | 107 | 24 |
| | Volume = length × width + height | 81 | 18.1 |
| | Errors in multiplication | 53 | 12 |
| | Volume = 3 × (length + width + height) | 34 | 7.6 |
| | Volume = 3 / (length + width + height) | 18 | 4.1 |
| | Volume = length × height | 11 | 2.4 |
| | Volume = length × width | 7 | 1.6 |
| | Volume = length + width | 6 | 1.4 |
| Task8: Finding the height of a rectangular prism<br>Volume, length, and width, were given | Correctly calculated | 121 | 27.2 |
| | Incorrectly calculated | 324 | 72.8 |
| | Students' errors and misconceptions | | |
| | Height = width × length | 74 | 16.7 |
| | Height = width + length | 72 | 16.2 |
| | Height = width + length – volume | 59 | 13.2 |
| | Height = width + length + volume | 43 | 9.7 |
| | Height = width × length × volume | 31 | 7 |
| | Height = length | 15 | 3.4 |
| | Height = volume / 2 | 12 | 2.7 |
| | Height = width + length / volume | 8 | 1.7 |
| | Height = width × length + volume | 5 | 1.1 |
| | Height = length – width | 5 | 1.1 |

## Discussion and Conclusion

The major concern of the current study was to portray the sixth grade students' errors and misconceptions while engaging in the conceptually and procedurally oriented tasks on length, area, and volume measurement. The findings indicated that the students participated in the study had quite shallow understanding and skills in all three domains of measurement. Table 10 summarizes the students' common errors and misconceptions emerged from the present research study.

Considering the students' errors and misconceptions in the conceptually and procedurally oriented tasks altogether, it is clear that both "knowing *how* to do measure" and "knowing *what* and *why* measure" are so crucial for meaningful understanding of measurement. Indeed, the students' error patterns emerged from the study were rooted both in their available conceptual and procedural knowledge schema constructed previously as well as the relationship (*un*)linked to these schemas. For instance, it was observed that the number of the students who answered the tasks correctly

**Table 10** The common errors and misconceptions made by sixth graders according to the domains of measurement

| Domains Tasks | Conceptually oriented tasks | Procedurally oriented tasks |
|---|---|---|
| Linear measurement | Starting from 1 rather than 0 | Incorrect alignment of a ruler |
| | Counting either hash marks or numbers on a ruler | Counting either hash marks or numbers on a ruler |
| | Reporting the end point on a ruler | Reporting the end point on a ruler |
| | Believing that a ruler should be longer than the object being measured | |
| | Believing that cm is not suitable unit to measure in m. | |
| Perimeter | Believing that perimeter is constant under partition | Using area formula |
| | Counting the square units or dots | Adding only two side lengths for finding perimeter |
| | Using units of area/volume measurement | Errors in four basic operations |
| Area measurement | Believing area is not constant, under partitioning | Using the perimeter formula |
| | Counting the lines around a shape | Area equals to length plus width |
| | Using units of length/volume measurement | Using units of length measurement |
| | | Errors in four basic operations |
| Surface area | Confusing surface area with volume | Using volume formula |
| | Believing a shape has more than one surface areas | Surface area equals to length plus width plus height |
| Volume measurement | Counting the square units or faces of unit cubes or only visible unit cubes | Volume equals to length plus width plus height |
| | Double counting unit cubes | Volume equals to the multiplication of length by width |
| | Counting the faces of unit cubes given and doubling that number | Errors in four basic operations |

decreases as dimension increases, namely length > area > volume, and thus, a few of the students could be able to reason "how length is related to area and volume measurement." In this respect, the findings of this study revealed again that acquisition and coordination of the concepts and skills about length, area, and volume measurement were closely related to each other and either absence or partial understanding of these skills/concepts probably result in poor performance in the domains of measurement (Battista, 2003; Nührenbörger, 2001; Outhred & Mitchelmore, 2000; Stephan & Clements, 2003). Focusing particularly on *length measurement*, the analysis of the students' written responses to the conceptually and procedurally oriented tasks indicated the following errors: (a) starting from 1 rather than 0 while engaging with a ruler, (b) counting either hash marks or numbers on a ruler, instead of intervals, (c) focusing on end point while measuring with a ruler, (d) believing all rulers are 30 cm long—that might be considered as prototypical view of ruler (Ryan & Williams, 2007), (e) mixing units of length with other units of measurement, (f) incorrect alignment, and (g) treating centimeter as a different unit of measurement (i.e. not understanding the relationship between units (cm–m) of length measurement. Similar difficulties experienced by both elementary and middle school students' were also reported in the mathematics

education literature (Barrett et al., 2006; Boulton-Lewis, Wilss & Mutch, 1996; Ellis, Siegler & Van Voorhis, 2001 [as cited in Lehrer, 2003]; Kamii, 1995). Inadequate understanding of zero-point, unit iteration, the structure of a ruler, and relation between number and measurement are the possible reasons behind the errors committed by the students. Regarding perimeter, the analysis pointed out the following common errors and misconceptions as follows: (a) perimeter is constant, when the shape is rearranged; (b) counting the square units or dots for perimeter; (c) using units of area/volume measurement for perimeter; (d) using the area formula for perimeter; and (e) perimeter equals to the total of two side lengths. Tan Sisman & Aksu (2009); Kordaki & Portani (1998) made a similar point with respect to the students' misconceptions and errors related to perimeter. In this respect, it can be concluded that these different types of errors might be attributed to lack of attached-meaning to the concept of perimeter, so relying on only procedures (Moyer, 2001). Moreover, the analysis of students' written responses to the *area measurement* tasks also indicated a wide range of errors and misconceptions given as follows: (a) believing that area is not constant, under partitioning; (b) counting the lines around a shape for area; (c) point-counting for area; (d) confusing area with perimeter; (e) using the perimeter formula for area; (f) area equals to length plus width; (g) using units of length/volume measurement; (h) using the volume formula for surface area; (i) surface area equals to length plus width plus height; (j) confusing surface area with volume; and (k) believing that a shape has more than one surface areas. As stated previously, meaningful understanding of area measurement involves the organization and coordination of various concepts and skills. However, according to the results of this study, it is obvious that the sixth graders neither comprehended the key concepts nor gained the skills of area measurement and thus made different kinds of errors. There are also studies having parallel results with this study. The study conducted by Furinghetti & Paola (1999), for instance, revealed that the linear relationship between area and perimeter was the common misunderstanding among seventh graders. Besides, using the formula for area and perimeter correctly without knowing what dimensions of a shape stand for becomes also evident in the Kidman & Cooper's study (1997) and the Zacharos' study (2006). Another research study carried out by Kamii & Kysh (2006), a few numbers of eight graders could not be able to conserve the area of a shape when it was rearranged. In addition, students' poor understanding of surface area, even in a college-level, were also documented by different scholars such as Cohen & Moreh (1999), Gilbert (1982), and Light, Swarat, Park, Drane, Tevaarwerk & Mason (2007). At this point, it can be concluded that the students of this study had serious difficulties in truly understanding and applying the concepts and skills involved in area measurement. More specifically, inadequate grasp of the spatial structure, of the multiplicative structure, inability to conserve an area, and superficial understanding of length measurement resulted in the abovementioned errors. Concerning *volume measurement*, the students' common errors emerged from the conceptually and procedurally oriented tasks were found as follows: (a) counting the square units, (b) counting faces of unit cubes, (c) counting only visible unit cubes, (d) double counting unit cubes, (e) counting the faces of unit cubes given in the picture and doubling that number, (f) counting the faces of unit cubes given in the picture and multiplying that number with 3 because a prism has three dimensions, (g) volume equals to length + width + height; (h) volume equals to length × width and (j) volume equals to length × width + height, and (k) using units of length/area

measurement. Similar errors and misconceptions committed by students while engaging in volume measurement tasks were demonstrated in previous research as well. Campbell, Watson & Collis's study (1992), for instance, produced evidence that counting the number of individual unit cubes was common strategy among elementary students, without paying attention to the invisible unit cubes. Putting emphasis on enumeration in 3D arrays, Battista & Clements's study (1996) also pointed that most of the fifth graders had lack of structuring array notion, consequently, could not be able to enumerate the cubes in a given solid correctly. According to Sáiz's study (2003), prospective teachers perceived volume as a number produced by multiplying the length, width, and height of an object. In this respect, the findings of the current study clearly indicated that the students could not be able to make sense of the foundational principles behind volume measurement which requires more complex reasoning about the structure of space than measuring two- or one-dimensional regions. In particular, lack of spatial visualization and of meaningful enumeration of arrays of cubes as well as poor understanding in length and area measurement might be the reasons behind the several errors of the students in volume measurement.

Taking the students' errors and misconceptions in three domains of measurement together, the first and foremost implication might be focusing on mathematics curriculum itself. As declared by Schmidt et al. (2002; as cited in Hook, 2004), "specifically the curriculum itself—what is taught—makes a huge difference" in students' achievement (p. 1), there is a close and clear relationship between what students learn and how a curriculum organized in terms of learning objectives, content, implementation, and assessment process (Lehrer, 2003; Smith, 2007). In this respect, the foundational principles of length, area, and volume measurement is proposed to be included in mathematics curriculum as a learning objective, since they are considered to be the vehicles for the development of meaningful understanding in measurement. In addition to inclusion of fundamental principles, an appropriate balance between conceptually dominated (e.g. the notion of thinking in a second dimension for the concept area) and procedurally dominated (e.g. calculating the areas of geometrical shapes) learning objectives should be constructed. With regard to the measurement content, curriculum should move progressively from length, area, and volume measurement as the mostly advised approach to study of measurement (NCTM, 2000; Stephan & Clements, 2003). The measurement content should also be focused on the (*inter*)relations (e.g. how length, width, and height produce volume; the relationship between perimeter and area, etc.) among the domains by considering the cumulative nature of mathematics. Furthermore, Carpenter, Fennema, Peterson, Chiang & Loef (1989) stated that providing teachers with research-based knowledge about student's thinking in a specific content domain positively affects teachers' instruction and students' achievement. At this point, it might be suggested that the research-based information about the students' errors and misconceptions in length, area, and volume measurement may be included in mathematics curriculum to previously inform teachers about possible learning difficulties. It could also be helpful for teachers who want to find the answer to the following question: "What does research say about the errors and misconceptions related to measurement I am about to teach?" (Ryan & Williams, 2007). Moreover, the findings also call for instructional changes. For instance, students' difficulties in grasping concepts of measurement might lie in teachers' way of delivering instruction. Several research studies have pointed out that most of the teachers prefer to introduce concepts

of measurement via formulae instead of using students' conceptions of space and measurement (e.g. Smith, 2007; Thompson, Philipp, Thompson & Boyd, 1994). From Novak's (2009) point of view, it can be asserted that teaching of measurement should become conceptually transparent to students, rather than being as fragmented, opaque, and rule-dominated strand of mathematics. At this point, students' errors and misconceptions should seriously be taken into consideration as an integral part of teaching and also as a valuable source of information about the learning process (Kembitzky, 2009). Neither just explaining nor warning is a guarantee that misconceptions and errors are less likely to happen, rather students are to be helped in the process of construction of new knowledge (Drews, 2005). Presenting intentionally the tasks eliciting false answers as a part of teaching and learning of mathematics is an efficient way to deal with common misconceptions, since such tasks offer opportunities for students to reflect on their learning from their errors (Borasi, 1996). The error-triggering tasks, for instance, the conceptually oriented tasks used in this study, might provide mathematics teachers opportunities to monitor students' thinking process and help them to unravel the patterns of their errors and misconceptions. Furthermore, integration of erroneous examples with measurement instruction has a potential to change the students' focus from performance-orientation (e.g. is the answer correct or not?) to learning-orientation (e.g. why/how is the answer correct or not?). As stated by Melis (2004), erroneous example exercises offer learning opportunities for students to practice and reinforce critical thinking, problem solving, self-monitoring, diverse reasoning strategies, and self-explanation. Additionally, most of the misconceptions and errors made by the sixth graders seem to be linked to each other. For instance, the error "counting the lines around a shape while finding its area" seemed to be linked to "confusing area formula with perimeter formula." Indeed, one of the main reasons behind this chain of errors probably lies in treating "area and perimeter" as well as "surface area and volume" as unrelated and totally different topics. Therefore, compare-contrast tasks (e.g. if the volume of a prism is tripled, is the surface area tripled too?) might improve students' reasoning skills. Another reason related to the chain of errors on spatial measurement might be due to ineffective monitoring and assessment process of student learning. An & Wu's (2012) study revealed that analyzing errors in homework improves teachers' understanding of students' thinking as well as the strengths and weaknesses of instruction. In this respect, paying explicit attention to monitor students' learning process and to analyze how and why they make errors can help to overcome the construction of the error chain in spatial measurement. The following table (Table 11) might be given as an example of students' erroneous answer analysis and possible reasons.

Another noteworthy finding to emerge from this study was that no matter what domains of measurement asked, "counting" was the common strategy while solving conceptually oriented tasks. For instance, while finding the length of the broken ruler, most of the students counted the hash marks; for finding the area, they counted the lines around a shape and they also counted the unit squares while finding the volume of a box. The reason behind "counting fits all domains of measurement" misconception might be due to inadequate understanding of the key concepts of measurement such as relation between number and measurement, accumulation of distance, unit-attribute relation, etc. Furthermore, the findings also revealed that some of the students did not pay attention to what the task (problem text) is talking about and the necessary

**Table 11** An example of students' erroneous answer analysis and possible reasons

| | Question | Indicator of errors misconception students' answers | Possible reason(s) |
|---|---|---|---|
| Length measurement | Broken ruler<br>Explain whether the broken ruler can be used for measuring a 2-m-long cloth | The ruler is too short to measure the cloth<br>A ruler should always be longer than the object being measured | Inadequate understanding of iteration, additivity, the structure of a ruler |
| | | The ruler is broken and lost its function<br>If a ruler is broken, the remaining is useless | Inadequate understanding of iteration, additivity, the structure of a ruler, zero-point |
| | | Cm is not suitable unit to measure in meters<br>A centimeter ruler only measures the objects in centimeters | Inadequate understanding of unit size relationship, additivity and iteration |
| | | The ruler is not starting from 0 (zero)<br>A ruler should always start with zero (zero should always written on the ruler) | Inadequate understanding of zero-point, the structure of a rule |
| | | The ruler starts with 13<br>If a ruler starts with a number other than zero, it cannot be used for measuring | Inadequate understanding of zero-point, iteration |

mathematical operations executed. As a consequence, they tried to reach the answer through meaningless calculation attempts (e.g. area = perimeter + length; height = length + volume). These ineffective and unsuccessful efforts of students' could be one of the indicators of their limited knowledge repertoire on measurement probably due to teaching for execution of operations to get the right answer. At this point, designing inquiry-oriented mathematics instruction that highlights the links between measurement concepts and skills provides students with meaningful opportunities to learn through comparison, contradiction, reasoning, reflections, and communicating ideas (Stipek, Givvin, Salmon & MacGyvers, 2001). Besides, inquiry-oriented teaching materials may enhance measurement instruction. For instance, ruler construction activities might be designed to help students explore the structure of ruler (e.g. numerals, intervals, hash marks; the relationship between them). In addition, a mathematics teacher could make breakable ruler by cardboard or wood and use this breakable ruler while teaching such foundational concepts as zero-point, unit iteration, etc. Another example providing opportunity to understand how iterated units of length are represented in rulers might be to design and use the strange rulers (Dietiker, Gonulates & Smith, 2011). In conclusion, the findings of this study revealed a wide range of misconceptions and errors committed by sixth grade students while solving the conceptually and procedurally oriented tasks on spatial measurement and might be considered as one of the valuable contributions to removing the barriers to students' learning of measurement.

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