THE EFFECT OF INTERACTIVE DIAGRAMS ON SECONDARY STUDENTS' UNDERSTANDINGS OF SELECTED MATHEMATICAL REPRESENTATIONS BASED ON VAN HIELE THEORY AND REPRESENTATION THEORY

by

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A dissertation

Submitted to the University at Albany, State University of New York

in Partial Fulfillment of

the Requirements for the Degree of

Doctor of Philosophy

School of Education

Department of Educational Theory and Practice

2003

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Acknowledgements

I would like to thank to all of my professors and my teachers who prepared me for every inch of this phase and for the future. From these, Dr. Vicky Kouba, and Dr. Audrey Champagne, since they have shared their experience with me and Dr. Karen Swan since she shared her childlike interest for knowledge with me, my Turkish professors since they let me fly when I needed it most, and my parents since they pushed me hard and expected the best from me so that I could expect the best from me were the best of all.

I owe my special thanks to my advisor and my friend Vick, since her artistic as much as academic sides inspired and empowered me in every step of this very spatial experience.

My co-workers Mary van Benschoten and Susan Sherwood were my English and culture pals to whom I could ask any question and get the answer for. Also the challenging and thought provoking discussions with them shaped my interests and the direction for my future.

I can never thank enough to the high school, the two special mathematics teachers I worked with, and to the 70 students who stayed even after countless hours of testing required for my study. The mathematics supervisor of this school believed in my research from the beginning. I want to use this opportunity to thank him, too.

I owe my gratitude who gave me permission to use the mathlets; Alberta Learning from Canada, and also Zalman Usiskin to use the van Hiele test.

I want to thank to my fiancée Ender Gurgen from my heart for being there in every step of this inspiring, confusing, perplexing, mind bearing journey I fell into.

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Finally, from many countless special friends I earned in USA and my old friends from Turkey with my old department SCED, I learned how to laugh at myself and continue at hard times. I learned to believe in myself. THANK YOU..

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ABSTRACT

This study reports an investigation of the effects of the selected combination of mathematical representations within a mathlet (an online mathematical interactive diagram) on differential van Hiele level students' understanding of quadratics. Theory of van Hiele (van Hiele, 1986) and Representation Theory (Janvier, 1987) established the baseline to infer the role of the cognitive processes; insight, intuition and translations within the structure of the representations on students' understandings as well.

The participants of the study were 9^{th} and 10^{th} grade students from two MATH AB classes of one teacher and two MATH ABh classes of another teacher in a local high school (n=70). The van Hiele test preceded author prepared pre and posttests, and the intervention used two different mathlets at a computer lab inside the school library. Interviews with selected students from each van Hiele level (low, inbetween and high) followed the posttest (n=17).

The results indicated that there is a significant interaction between effects of the number of mode of representations and effect of the students' van Hiele levels on students' understandings based on the gains from a mathematical performance test. Students with inbetween van Hiele levels experience 3 mode representations in a negative way while they favor 2 mode representations with their positive gains. Interviews with different van Hiele levels demonstrated that students differed by their use of mathematical terminology, the confidence in using it, the preferences among representations, the way they dealt with counter intuitive translations between different representations in the mathlets.

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Implications of the study give valuable insight to the future use of mathlets inside mathematics classroom, and students' cognitive processes required by mathlets.

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CHAPTER I: INTRODUCTION TO THE STUDY

Introduction

Advances in information technology are having profound influence on the practice of mathematics education. Technology has motivated rethinking of mathematics teaching, the redesign of existing curricula, and consideration of new ways to teach and assess mathematics (Jonassen, 1999; NCTM, 2000a). Information technology provides students many opportunities including (1) the investigation of the characteristics of complex three and four dimensional figures (e.g. Mobius figure, four dimensional cube, topological shapes, etc; (2) the analysis of the large sets of data using spreadsheets, and statistics packages; (3) exposure to multiple representations, (4) modeling physical situations using graphical calculators, (5) solving complex numerical problems, (6) conjecturing, (7) manipulating virtual objects; and (8) generating matrix transformations that would lead to fractals.

Four of the NCTM standards for mathematics instruction in geometry stress the importance of working with two or three dimensional figures, mathematical representation systems, appropriate transformations, visualization, and geometric modeling for Pre-K through grade 12 schooling (NCTM, 2000).

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The results of these changes reflect in the nature of testing. The future of high stakes testing as stated in the Capstone Report (1997, cited in Duran, 1999) describes a number of possible technology- based assessment innovations for the National Assessment of Education Progress (NAEP) such as inter-active demonstrations, problem representation, and synthesizing complex textual and figural information displayed on a computer screen. This report suggests that representations will be a focus of NAEP mathematics assessments in the foreseeable future.

Interpreting representations, generating meaning and making connections between representations is a challenge in a world filled with images, symbols, and patterns connoting multiple meanings. The ability to attach meaning to representations or recognize patterns is called intuition or insight by the mathematician van Hiele (1986). Mathematics educators who are interested in the construct of intuition and insight assume that if students are exposed to many patterns, the rate at which they recognize patterns will increase. In a typical classroom practice, pattern recognition is taught by having students draw diagrams and sketch graphs representing equations. Confrey, Castro-Filho, & Maloney (1998) have suggested that if the internet or computers were used to teach pattern recognition, less time would be lost to incomplete, uncertain, and poorly detailed graphical examples.

Interactive mathematical representations, which complement students', own drawings and representations of mathematical concepts and tasks provide simultaneous changes observed in less than seconds. These mathematical representations take the form

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of Interactive Diagrams. In this dissertation, Interactive Diagram refers to mathlets (JAVA™ applets; JAVA is a trademark of Sum Microsystems) that are designed to address key ideas in mathematics via interlinked simultaneous representations systems with reciprocal changes associated to them (Confrey et al, 1998; Castro-Filho, Confrey, Wilhelm, Meletiou, 1999). Via mathlets students can visualize the patterns generated by data hence making connections becomes much easier. Furthermore, students' nature of interactions with the mathlets predicts changes in understanding. These interactions may take the form of not only student to student (learning in groups, in pairs, etc.) but also student to teacher (as discussion) and within student (as metacognitive awareness), and especially in the form student to mathlet (as making sense of the representations and parameter manipulations inside a mathlet). In the mathematics education research literature, interactive mathematics representation primarily addresses dynamic software such as Geometer's Sketchpad and Maple. There is little research on Interactive Diagrams (i.e., stand-alone JAVA applets), the mechanisms by which their use promotes understanding or the features of applets that are effective in promoting understanding.

Rationale

Because development of the skills of interpretation and generation of symbols has such great educational importance, the potential for interactive diagrams to contribute to the development of these skills and the need to better understand how interactive diagrams influences learning, I chose to investigate the effectiveness of Interactive Diagrams in the learning of quadratics. While each student has the capacity to learn,

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learning styles differ. The van Hiele Theory (van Hiele, 1986) which is comprised of five consecutive but not developmental levels of geometric thinking and learning offers a means for investigating a relationship between cognitive aspects of learning and students' work with mathlets. Furthermore because van Hieles' research on intuition and insight has implications for the interpretation of Interactive Diagrams, VanHiele levels of geometric thought are an important variable in my study, the van Hiele levels of geometric thought are an important variable in my study. Since the research literature already sought the effectiveness of mathlets on students' mathematical understanding, this study rather builds on that line of research and searches if there is any relationship between the differential effects of representation structures within an Interactive Diagram (ID) and students' mathematical understanding with them. While these representation structures (modes) may vary; the wide variety of them reflects the ones in which main representations are graphical, tabular, and symbolic. Hence, mode of representations became another variable for my study. The van Hieles and theorists working in the area of representations (Janvier, 1987) point to the intuition and insight in investigating the students' nature of interactions with IDs such as mathlets. That is why I included both the van Hiele Theory and Janvier's constructs of Intuition and Insight as another meaningful corner of the study.

Summary

The following instructional study has been an investigation of the effects of interactions with two (graphical & tabular) versus three (graphical, tabular, and symbolic) -mode representations in online interactive diagrams on the development of students'

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understanding of mathematics specifically quadratics. I have tested the idea that students' interactions with mathlets (which are restricted to the ones involving two and three-mode representations) will increase the rate at which students develop understanding of mathematics concepts and the quality of their understanding. My hypothesis for the study is based on psychological theories related to representations and mental processes with representations, translations among different modes of representations, and the roles of interactivity, intuition and insight in the development of understanding. Because the aforementioned constructs are so closely related to van Hieles' levels of geometric reasoning, I hypothesize an interaction between students' levels of geometric reasoning, the number of modes of representations present in a mathlet and student's gain scores after working with a mathlet.

Because of their availability, ease of use and specialty in demonstrating the translations in a quicker way, I have chosen to use mathlets (mathematical JAVA applets) as the interactive diagrams. The mathematics topic I chose to study is quadratics. The choice of quadratics was based on the topic on the curriculum at the time the study was being carried out and on the teachers' recommendations of which topics they thought could best be supported by the use of mathlets.

Translations are key elements of a mathlet and play a key role in identifying the prime factors of the insight and intuition a student needs to develop. In the line of this statement, I have studied the role of cognitive processes intuition and insight on student's making sense of the mathlets.

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CHAPTER 2: REVIEW OF THE LITERATURE

Interactivity

Interactivity as a process is a complex construct with high cognitive connotations. (Rose, 1999, cited in Cezikturk, Cirik, & Kahveci, 2001). In our 2001 Mathematics and Science Education with Technology Conference (MSET) paper, we propose a typology of components for interactivity. The components are social, physical, technological, and intellectual. *Social* components of interactivity include student/student, student/teacher interactions both between individuals and in groups. *Technological* components include software/software, software/computer, human/computer interactions. *Physical* components include student/physical object, student/virtual object interactions. Finally, *intellectual* components refer to those interactions occurring in the mind such as metacognitive awareness. We proposed that only virtual reality environments would be a good example of having everything under one roof. However, we also believe that a teacher can make any interactive learning experience a more interactive experience with the instructional design choices. The typology is meant to be inclusive and other than the virtual reality environments, any mathlet that is developed would lack at least one of more of the components above. Recognition that interactivity is multifaceted is essential to designing a study of the effects of interactive diagrams.

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Mathlets and Interactivity

An interactive diagram is a small-scale interactive learning environment and it uses the principles of a subset of components of interactivity. Interactive diagrams that I refer to in this study are called by many different names including Java applets", "mathlets", "javabeans", "virtual manipulatives", or "simulations" and have different attributes. Confrey et al (1998) & Castro-Filho et al. (1999) support this definition by saying that IDs are JAVA™ applets designed to address key ideas in mathematics and science.

Wiley (2002) defines IDs as a form of learning object; any entity, digital or non-digital, which can be used, re-used, or referenced during technology-supported learning. He proposes a taxonomy of interactive diagrams. In his taxonomy, java applets are in the *generative-presentation learning object group, which* highlights the attributes of java applets that distinguishes from the other learning objects. Unlike Wiley (even though he mentioned that NSF highly regards java applets as the only form of IDs), I used the term ID to refer to this specific sub group of his taxonomy only. Wiley argues that having the flexibility of reusable component objects (we may refer to different representations within), or some flexibility for extra-object dependence (if the object needs any information from other learning objects), and **having medium level potential for inter-contextual reuse** (one can use a mathlet for different contexts, and mathematical topics) **and high level potential for intra-contextual reuse** (mathlets are specific mathematical representation by themselves, hence many different uses of one java applet are possible and encouraged) places generative-presentation learning objects into the one lower

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position than simulations in the hierarchy of the learning objects taxonomy Wiley proposes (2002) .

I want to make myself clear that in any part of this study, I do not argue that mathlets are the only forms of IDs but for the purposes of this study, I will continue referring to mathlets as the forms of ID that this study will investigate. Mathlets provide an ideal context for ID research because they are free to use, they enable multiple representations in a compact one frame/page environment, and they enable translations between representations in this compact setting.

Mathlets as interactive learning environments, promote various types of interactions. The teacher can be a facilitator or a lecturer and decides some part of the social interaction. Students can work individual, in pairs or in small groups and may determine the collaborative nature of the social interaction. The students' metacognitive awareness and thinking about their own thinking results in the intellectual nature of the interaction. The students' other interactions with the mathlet can be categorized under a technological component, which includes both technical understanding and how the technology is used for generating meaning.

Technicality of a mathlet

A mathlet is a short JAVA™ program attached to the source html code of a web site and which functions from there. Hence, mathlets work under any Internet browser (as long as they are not incapabled by the browser options) and most of the mathlets can be

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re-used for slightly different purposes and for various reasons. Mathlets enable the building of interactive and dynamic diagrams. Many uses of these diagrams can be found in Internet as games, puzzles and instructional purposes. Please see APPENDIX A (example mathlet web sites).

A standard mathlet has at least two or three different representation modes embedded in it. The most general cases include graphical, tabular, and symbolic representations as different modes of representations. The student has the capability of a specified set of parameter manipulations, and can observe the translations between and within representations immediately. Even though the sizes differ, most of them would not be bigger than the monitor size to be able to be visualized in the same frame. This enables translations to be observed within the same frame even though sometimes a new frame may be embedded inside the existing frame. Structures may vary from mathlet to mathlet but structures would be preserved within a mathlet. The diagram is continuously re-drawn keeping intact the mathematical relationships that have been underlined.

Logistics of a mathlet

Even though the websites for mathlets are built with JAVA™, any teacher and any student can use them without knowing the JAVA™ programming language (Chyi, 1999). Some of the most important characteristics of geometry such as constructions, proofs, applications, and even parameter manipulations, can be observed, compared and contrasted in seconds with various modes of representations on a screen. This provides a potentially powerful instructional tool for mathematics teachers. A possible application of

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a mathlet may involve changing parameters in a situation, dragging the mouse to change the locations of a point, observing linked changes in the results of the situation, adding auxiliary lines or structures to the original system of the task all of which can be done in seconds. Most students today already experience these kinds of manipulations through playing computer games. As a plus, mathlets enable **precision**, and increase the **accuracy** of classroom-wide graphical examples.

Confrey et al. (1998) point to the restrictedness (pre-set boundaries) of a mathlet in comparison to a dynamic geometry software. The students can choose from a set of already defined actions, hence teachers do not need to deal with possible disturbances or confusions resulting from the use of more complex programs.

Regarding the time saving opportunities related to them inside a fully developed mathematics curricula, Interactive Diagrams (IDs) may be a promising way to increase the possible advantages of computers for mathematics education. Indeed, many popular mathematical software packages have recently incorporated the use of interactive diagrams including Geometer's Sketchpad, Cabri Geometer, Graphical Calculator, Maple. This dissertation will focus on the stand-alone Interactive Diagrams as in the form of JAVA™ applets. This restriction is made because there is existing literature on dynamic softwares even though, relatively less research has been done specifically on mathlets. Furthermore, the highly cost-effective advantage of mathlets when compared to dynamic softwares which need to be continually revised asks for specific attention. It does not mean that any mathlet is time independent, even though they are certainly topic

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dependent. For each topic chosen, a teacher needs to look for a good applet on the Internet.

Portability and simplicity are other advantages of mathlets. Even with these advantages, mathlets have accuracy problems due to pixel details. The zoom function seems to fix this problem since it can give a corrected visualization of the same points. Also, its restrictedness may be taken as a problem especially if the teacher favors more of a discovery class. Evaluating the existing ones in terms of their representation of the specific mathematical concept, topic or unit, teachers might look for several to be used interchangeably, since it might be more beneficial to use a combination of them within a well organized lesson. Since the mathlets are mostly on the Internet without any necessary payment, a teacher or instructor may find more useful ones with the changing instructional needs or so.

Like all technologies, mathlets have some limitations, too. Research indicates that the limitations of using mathlets include that they may run differently on different machines, and that although common web browsers support mathlets, not all platforms do. The cost-effectiveness of mathlets, however suggests that these problems will be resolved in the future.

To be brief, the advantages of mathlets over dynamic software are following:

a. They are compact, they enable most of the capacities of a specific program in a small dialog box on the computer screen.

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b. There are less factors to deal with

c. Detailed research is needed in this area

d. They are ready to use

e. No programming experience is needed,

f. They enable learner control without the loss of control of the learner

g. They enable parameter manipulations with flexible options

h. They enable change in the locations of a point by dragging, moving, copying and pasting

i. They enable observing linked changes in the results of the situation/scene or in the sub structures (dynamic reciprocal changes system)

j. Add and remove auxiliary lines or structures in to the scene in seconds (flexibility of the structure)

k. A mathlet is a dynamic representation system, hence, the student has the chance to see the multiple representations at once.

Empirical research on mathlets

The relatively small literature nonetheless has reported some interesting and attractive results concerning mathlets. Wilhelm, Confrey, Castro-Filho, & Maloney (1999) used clinical interviews with 9th graders to see the effect of mathlets on students' mathematics reasoning. They used an interactive diagram, which they named as "Bank Account". Using Bank Account, students were able to predict, model and check their ideas in the most efficient way possible. The researchers found that **mathlets helped students develop and intuit the concept of rate of change.** The bank account mathlet

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included two graphical representations, as well as some verbal and symbolic representation on the same screen. Students were able to make changes in their initial balance, add and subtract money from their account and see the dynamic displays of a daily transaction graph as well as a balance graph. Students were given three tasks to work on. The tasks were for predicting the changes in different representations due to changes in reciprocal ones.

Klein & Hanisch (1998), and Shwarze (1999) emphasized use of mathlets for visualization and interaction. Klein & Hanisch (1998) used 70 highly sophisticated reusable Java beans (JAVA™ applet components). These Java beans are programs for mathematical and geometrical utility components like vectors, matrices, triangles, spheres, 2D/3D graphics, editable curves, function parses, etc. In the future, we may have access to Java bean lists (like lesson plan lists now) so that a teacher or an instructor could choose or alter specific beans for specific purposes of instruction. Either static or dynamic interactive tools and demonstrations may be a good way to promote interactive learning.

Cognitive processes required for a mathlet

Regardless of the simplicity of mathlets, there is an array of cognitive processes a student needs to develop and use to make effective use of mathlets. A student first would need to **visualize** the mathlet to see both the static diagrams at any second, but also to understand the notion of dynamic geometry, **interact** with the mathlet to fully utilize its representation components, and identify the individual parameters to make use of them in

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the meaningful manner. Then the student would need to **analyze** the mathlet to identify the underlying mathematical relationships, interconnected representation systems, **synthesize** all the parts to **make a conjecture**, and **investigate** the emerging patterns with where each pattern leads to. The student would finally use these patternistic understanding to **intuit** for similar and different kind of mathlets. But also would need to use this **insight** for further explorations of the same topic under consideration.

Sometimes student would need to **judge** the correctness of the representation in a mathlet and also identify the restrictions of a mathlet so as to make appropriate and accurate generalizations. Accurate drawings are needed for correct reasoning according to Kortenkamp & Richer-gebert (1998). They mention Interactive Geometry Systems (ICGs), and argue that these enable students to consider **drawing as a dynamic entity.** With dynamic modeling, dragging, macro-level repetitive tasks, and locus (tracing the movement of a point) students can **identify** and **perform** mathematical or geometrical tasks.

Bennett (1997) argues that, we must not underestimate the power of dynamic geometry for simply **creating** good static ones. Besides supporting visualization, these static variations of geometrical figures as well as their interconnectedness, may enhance our understanding of geometry and students' thinking with different representations. Balacheff & Kaput (1996) argue that **dynamic visualization** *is a new experiential realism.* They state that this change is occurring at the level of mathematical experience. Students would need to shift their experience with mathematics to a more experiential

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level in which they can use the parameters and representations to be manipulated as inputs and the changes on the screen as outputs of a mechanism

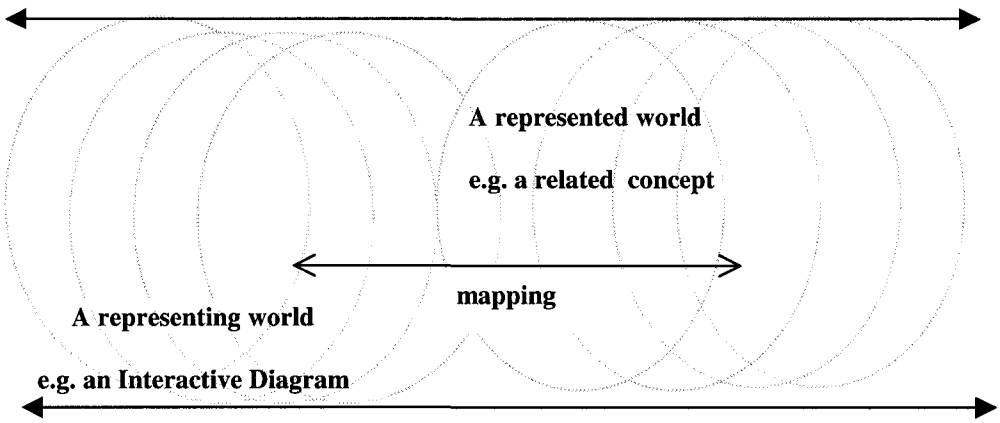
Jones (2001) uses the term "functional dependency" for the context of dynamic geometry referring to the robustness of a figure under drag. We can use his terminology to include different representation systems inside an ID which are dependent upon other representation systems and which function together. Here Jones states, "the things that do not move are the geometrical relationships that have been constructed." (p.72). Hence, the student would need to analyze an ID to identify these geometrical relationships.

Mathlets as mathematical representations

Any mathlet can be thought of a sole mathematical representation of a geometrical or mathematical relationship as a well as a composed representation of a combined group of **mathematical representations as graphical, tabular, symbolic (formulae), pictorial (as long as the mathematical characteristics of specific shapes are conserved) or even verbal.** These types of representations are also named as "*external representations* " other than the "*mind's eye" internal representations* of students (Tabachneck-Schijf, Leonardo, & Simon., 1997). Palmer (1977, cited in Kaput, 1987) states that every mathematical representation has two parts; **a representing world,** and **a represented world (Figure 1)**. These two worlds function separately from one another as separate systems, in the form of a mathlet and its corresponding mathematical concept or entity. **Children's ability to make the connections between the represented world and the representing world as well as among many types of representing worlds is directly linked to the development of their intuition or insight (Janvier, 1987).** In

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other words, children can grasp mathematical representations if their intuitive understanding is enhanced. Remembering the babies and how they learn, children who are exposed to more representations would be less vulnerable to different examples of representations. It is a two-sided coin; students need intuition to make sense of representations but also use of representations help them develop intuition.



As each layer of a telephone line is a mapping from one world to another and they carry a lot of information, a representation carries information through its own channels.

Figure 1. Structure of a mathematical representation

The use of multiple representations then might enhance intuition in a mathematics classroom. This idea is supported by the studies of Tabachneck et. al. (1997), Behr, Harel, & Post (1992), Cheng (1999) and Brenner et. al. (1997). Sternberg and Ben_zeev (1996) argue that any representation is restricted in some information and excess of some other information. Using multiple representations would help to reduce these imbalances and build better cohort examples. This is supported by Thompson's (2002) view on

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mathematics shown by program software both being more and less than the mathematics that is intended to be "seen" by students. Ainsworth (1999) identifies three main functions of multiple representations as to *complement, to constrain, and to construct.* One representation is used to constrain possible misinterpretations in the use of the other. Also, representations that carry equivalent information still support different inferences. By exploiting the combinations of the representations, learners would less likely to be limited by the strengths and weaknesses of any single one. And sometimes, partial redundancy of information supports new interpretations of the represented domain. Ainsworth also states that distributing information over such partially redundant information may be both less complicated and also less demanding for translation and integration.

There is abundant evidence of research on the translation between representations being very difficult (Tabachneck, et.al, 1994). Sometimes familiar representations are used to exploit inherent properties of one representation to constrain interpretation of a second. Ainsworth (1999) argues that in case of simultaneous representations, learning to translate between them is more of a bi-directional process. Tabachneck et.al. (1997) and Ainsworth (1999) both state that to maximize the effects of multiple representational environments, the environment should carry the translation process itself, not leaving it to the student. And this is achieved via dynamic linking of representations as in the form of Interactive Diagrams. For Ainsworth, this familiarity process should be stepwise for the student meaning that for novice learners, it may be full linking at first, and then gradually relinquished linking since the learner would get the easier. Finally she concludes that;

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-if multiple representations are to support different information processes then translation should be discouraged.

-if multiple representations are used to constrain interpretation, then translation should be automated.

-if multiple representations are used to develop deeper understanding, then translation should be scaffolded. (p.150). Hence, the role of translation depends on the intent of the developer of the multiple representations.

Intuition/Insight and mathlets

A mathlet is a representation of a mathematical concept in which students' understanding is highly affected by the insight a student has. If this relationship between the represented world (math concept) and the mathlet are clearly identified and highlighted, then there is a high chance of the transformation of this knowledge to other representations the student has as well! The insight to link and to search for better representations of the same mathematical concept may be affected by the use of mathlets in the classroom because such use may serve as a conceptual amplifier (Lesh, 1987). Jones (2001) added that these mathlets could be thought more of a "cognitive reorganizer" than an amplifier.

There is a theory of learning and mathematical thinking, which gives a high importance to intuition and how it is related to understanding of some structures. van Hieles (1986) see their theory as a theory on the learning of structures. They argue that some pedagogies lay more stress on the learning of facts more than structures. Although,

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facts lost are lost forever if a part of a structure is forgotten, the remaining part facilitates recall of the lost one. van Hieles argue that this is also linked to the presence of "insight" when a person acts adequately and with intention in a new situation. In "Structure and Insight", van Hiele makes the following argument;

> "*A person who acts with intention does not act at random, he acts according to the structure he perceives, corresponding to his mental structure, the structure of his expectation*" (p.24).

Regarding his stance, we could argue that any kind of mathematical thinking requires some sort of structural analysis and understanding. Each new structure is understood by the degree that the previous related structures are understood. He points out that this would urge for a certain degree of intuitive thinking in one's mathematical agenda. Rambally (1982), Thompson (2002) point to the visual aids for fostering the development of intuition, insight, and relate abstraction to concreteness. Realization of complex figures asks for already developed insight in geometry. And this can be achieved by the interactive systems. Bennett (1997) identifies this as "gleaning some insight while moving parts of the figure around" within the geometric sketchpad program. The time required to make a variety of figures by hand to illustrate cases of a geometric configuration is beyond anybody's time priorities, besides the precision problems one would have with each one of these cases. Instantaneous and precise variations is a key word while talking about the graphs driven from interactive programs. Bennett continues with stating that whatever value dynamic geometry would have, we must not

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underestimate it for simply creating good static figures. Castro-Filho et.al. (1999) confirmed that mathlets help students to develop intuitive ideas by their bank account ID research.

Burton (1999) asks many mathematicians and mathematics educators to study the value of "intuition" in mathematics. He defines intuition as visual, based on a physical model or on some special examples, holistic/integrative as opposed to detailed or analytic. More than 90 percent of mathematicians studied stated that they believed in the value of intuition or that van Hiele's terms "insight". Hersh (1998, cited in Burton, 1999) claims that intuition is an essential part of mathematics. It is related to seeing and picking up the connections in mathematics, sometimes, taking the form of pattern matching. Reflecting on the value of intuition, there are some researchers who favor adding or including more space for the nurturance of intuition inside the mathematics curricula (Wilder, 1984; cited in Burton, 1999), as well as a few who argue against it.

Connections can be recognized much well through repetition and revisiting the same knowledge domain from different perspectives and views. If insight is linked with successful encounters with effective representations in various situations, then we can say that the repetitive encounter promotes better usage of mathematical representations. In this way, mathlets might make it easier for students to realize that some problems go hand in hand with some representations and vice versa. This idea is in congruence with the existing literature as well. Adiguzel & Akpinar (2001) concluded that computer based

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instruction rich in multiple representations improves students' performance and provides them with the cognitive tools needed to develop problem solving skills.

Let's use an example to talk more about structures. The following is a number sequence with a structure behind the sequence of numbers.

2,4,8,16,...

...32,64,128 or 18, 22, 30, 32,.. one could extend the given sequence (structure), it could be a part of a different structure(s) as of the first or second, or different, there may be sub structures inside this sequence (doubles of the last digit, +2, +4, +8, etc.., and it could possibly be isomorphic to another sequence as well. One can see many possible structures underneath, also, with structure to which this sequence belongs to, for example the even numbers (by using the properties of structures given by van Hiele, 1986). His ideas on the use of structures and systems are good to build on for our purpose.

> " *The change from one structure to another more suitable structure is not very easy. The first structure is chosen because of its inviting character, because of the knowledge that this structure is appropriate for solving many problems. The necessity of the switch can only be understood after a conviction that the first chosen structure does not contain the solution*".
> (van Hiele 1986, p.131).

In the above example with the given number sequences, seeing the first structure does not guarantee moving to the second one. Students may have many difficulties seeing

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another structure. "The ability to think and act with a given structure guarantees a certain safety."(van Hiele, 1986, p.132). What if the safety is not there? Then it is understandable to expect some degree of discomfort by the students. Increasing the number of examples though will increase the level of comfort with the new ones since the new possible structures are seen and different possible ways of extending structures are established. Besides the educational value of repetitive tasks or multiple representations, students still differ with respect to their cognitive readiness for complex ideas.

van Hiele levels and mathlets

van Hiele Theory

Pierre van Hiele and Dina van Hiele-Geldof are known for their theory of the development of thinking in geometry. According to them, students pass through five stages of reasoning, from recognition to rigorous thinking (Burger & Shaughnessy, 1986; Mayberry, 1983; Pegg, 1985; Senk, 1989; Fyykholm, 1994; Swafford, Jones, & Thornton, 1997; van Hiele, 1986, King, 2002) (Table 1). These consecutive but not developmental stages are reflected on the level of functioning with texts and materials as well. Two fundamental concepts for achieving the movement between these levels are *Structure* and *Insight.* King (2002) frames this as the third postulate of the van Hiele Theory: **Ideas at a higher level result from the structure at a lower level.** This differentiates van Hiele Theory from other theories of mathematical thinking (Pegg, 1985; van Hiele, 1986). His theory often compared to the Piaget's theory. Peg and Guiterrez (1998) gave a clear distinction between these two theories.

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An important part of the roots of van Hiele's work is based upon Piaget's ideas and perceived problems with Piaget's theory. In particular, Piaget's structures d'ensemble, where learners are seen to pass through a series of stages that are closely linked to age parameters was discompted. Instead, van Hiele attributes growth to strong social-cultural features which are linked to a teaching/learning plan that has more sympathy with Vygosky's ideas than with the biological maturation basis of Piaget (p.278).

van Hiele was a mathematics teacher. His theory is unique in that it was developed to find ways to change instruction to accommodate the students who are having problems with mathematical thinking. As van Hiele states, there are ways to ascend from one level to the next and the teacher can help the pupil find these ways (van Hiele, 1986, p.vii).

Pierre van Hiele argues that the main problem for students, who cannot succeed in geometry, is the *lack of insight* (or the recognition of structure and pattern in the given mathematical expressions or geometrical tasks.). They argue that students pass through five consecutive but not developmental stages of reasoning (visualization, analysis, abstraction, deduction, and rigor)(Table 1). For a more detailed explanation of each level please see Malati (un dated) & Fuys, Geddes, Lovett, & Tischler (1988).

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Table 1

Descriptions of van Hiele Levels

van Hiele Levels	Descriptions
Visualization	The students reason about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components.
Analysis	The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.
Abstraction	The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept.
Deduction	The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions, and theorems.
Rigor	The student can compare systems based on different axioms and can study geometries in the absence of concrete models.

Here, similarities of the cognitive processes required for each level and the ones that I described for mathlets is highly recognizable. It suggests for a relationship between the students' van Hiele levels and student's interaction with mathlets. To understand if van Hiele levels could predict the nature of students' interactions with mathlets, or vice versa we may need more understanding of the theory of van Hieles as well as the research literature tied to van Hiele levels.

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Research with van Hiele Levels

Following van Hiele's theory, studies have sought to determine the hierarchical nature of the levels, and the ways to identify the levels of students (Burger & Shaughnessy, 1986; Mayberry, 1983; Prevost, 1985 & Usiskin, 1982). Some have measured the geometric abilities of students as a function of van Hiele level, and some investigated the effects of instruction on students' functioning and development within levels (Senk, 1989). Others searched for the effect of the training of mathematics teachers on van Hiele level, on students' achievement and teachers' relative teaching experiences (Swafford et.al., 1997).

The notion of being at different levels of thinking for different geometrical concepts has attracted a lot of attention from researchers. Burger and Shaughnessy (1986) investigated the discreteness of the levels using interview tasks concerning triangles and quadrilaterals. At the higher levels, it was found that the discreteness of the levels diminished. However, the original descriptions of the levels were confirmed. **They also suggested that learners might be inbetween levels** and they may be oscillating between levels while in transition. (Burger & Shaughnessy, 1986).

Since the model consists of five consecutive levels of thinking, some researchers investigated its developmental meaning. Hoo & Khoh (1995) revealed that **there were no significant differences between secondary and junior college students on the van Hiele Levels.** It was interesting to see the equal difficulty in attaining the 5th van Hiele level of thinking even in preservice teachers and students with mathematics majors

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(Burger & Shaughnessy, 1986; Mayberry, 1983). The fifth level was not observed in any of these students via interviews.

Hoffer (1981, cited in both Senk, 1989 and Mayberry, 1983) identifies these five levels as recognition, analysis, ordering, deduction, and rigor. It is supposed that students pass these levels in order but differentially in time. Hence, in a classroom, according to van Hiele, the teacher, the text and the students may be functioning at different levels. This leads to the conclusion that the mathlet would have a Van Hiele level as well.

Interviews were not the only way to investigate students' van Hiele levels of thinking. Studies also investigated van Hiele levels in large-scale subject groups via test results (Senk, 1989; Pegg & Davey, 1989). Results revealed the relative difficulty of identification of students' van Hiele levels of thinking with interview and test procedures. **There were disagreements concerning students in transition between levels (inbetween levels) and students having some intermediate level problems in that they had correct answers to 4th and 2nd levels without having a correct answer at the 3rd level** (Mayberry, 1983; Senk, 1989). Mayberry (1983) identified 7 % of the teachers he studied as at undefined levels and no teachers were at level 5. Mason & Shell's (1988) research gave similar results with 38 % of teachers functioning at less than level 4 and 8 % not even in the lowest van Hiele level (cited in Swafford et al, 1997).

Swafford et. al. (1997) reported that analysis of lesson plans of teachers involved in the study revealed significant shifts in their goals and expectations to a higher van

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Hiele level. Fifty percent of teachers gained two levels after the intervention. Swafford used the interview questions developed by Mayberry as well as four of the five for each level in multiple choice items developed by Usiskin (1982). Using this test they identified 10 out of 39 participants who could not be assigned into any one of the levels. Unlike some of the other researchers however, Swafford et al were able to detect participants functioning within the 5^{th} level; rigor.

Cezikturk (unpublished) investigated the levels of students' geometric thinking and the use of Java applets in a learning environment. Cezikturk found (although it needs to be redone due to subject constraints) **there is an interaction between higher levels of geometrical thinking and the positive effects of the usage of mathlets. In other words, there was a positive learning gain for students with higher levels (above level 2) of van Hiele geometrical functioning and not for students at levels 1 and2.** This is in conjunction to the definitions of those levels, since the visualization level seems to be the only level where we can find no kind of regard to the components or properties of a geometrical figure. At the analysis level, the analysis is only informal rather than systematic, as in abstraction or deduction.

Borrow (2000) studied the students who are in transition from level 2 to level 3 with a microworld. She concluded that microworld helped students to attain level 3 with their increased sophisticated geometric thinking.

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Remembering these 5 levels of mathematical thinking, the reasoning van Hiele suggests that better learning occurs when the two worlds have the similar van Hiele levels representational and conceptual. If we take a close look up to the levels described by them:

In the first van Hiele level (visualization), a student just recognizes whether there is a structure or not. In the **second level (analysis)**, a student starts to see properties of that structure, and differentiates between similar structures, or within the structures more generally. In the **third level, (abstraction)** the student logically orders the structures in order to decide which one is the best or most suitable and identifies the defining properties to make that structure. It is hard to do a similar analysis of with the **fourth and fifth levels (deduction & rigor)**, and actually van Hiele himself acknowledged that it was much harder to know when students were at those levels. Traditional instruction has been mostly for the students who are at the third level of thinking or lower.

Nunokawa (1992) proposed that the van Hiele levels were related to Vygostky's zone of proximal development. His theory argued that recognition of figures at the first level can be taken as informal and situated knowledge about those figures, and transitions from the first level to the third level can be seen as the **transition from informal knowledge to formal knowledge**.

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Studies investigating the ways to identify the levels of students were two-fold. Some were trying to find a valid and reliable way to measure students' level of functioning and some were trying to understand the differential effect of various concepts on van Hiele Levels. The hierarchical nature of the levels was supported by the findings of Mayberry (1983). The study also revealed that the match between students' level of thinking, the level of instruction and of the text material is a necessary condition to achieve student understanding. *A student cannot function adequately at a level without having had experiences that enable the student to think intuitively at each preceding level and if the language of the instruction is at a higher level than a student's thought processes are, the student will not understand the instruction.* Furthermore, it was found that a student could be at different van Hiele Levels for different geometrical concepts.

A study on the predictive capability of the van Hiele Level of students on writing geometric proofs indicated that although the predictive validity of the van Hiele model seems supported, students do not need to be in levels 4 and 5 to be able to write geometric proofs successfully (Senk, 1989).

All of the above research was dealing with 2 dimensional (plane geometry) concepts. Some researchers like Saads & Davis (1995) enlarged the area of research by including 3 dimensional concepts such as cubical blocks and polyhedra. This study also confirmed the hierarchical nature of van Hiele Levels with 3-dimensional geometry. The study also revealed van Hiele levels were interacting with students' spatial abilities while communicating mathematically.

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Gutierrez & Jaime (1988) used the van Hiele Levels up to 4 (1-4). They also identified different processes of reasoning as characteristics of several van Hiele levels as recognition, definition, classification, and proof. They argue "Students can rather be considered as reasoning on a van Hiele level n only when they show mastery of the processes integrating such level." (p.28).

Gutierrez & Jaime (1998) used 309 students in grades 6 of primary to 4 of secondary. Their model of assessment was based on the idea that (a) every student uses different levels of reasoning depending on the kind of problem being solved and its difficulty, so it is necessary to assess the progress the student makes at each Van Hiele level, and (b) the progress through the Van Hiele levels is continuous, so the students are not assigned to a level, but a "degree of acquisition" of each level of reasoning. They state "As application of the above mentioned theoretical viewpoint, we have designed a test where the items are not intended to assess a certain Van Hiele Level, but the use of some of the process integrating the levels." (p.45).

In the research study done by Yerushalmy (1997), the technology supported the students by helping them to discuss the representations as "objects"." Students were the designers of the representations in this research project. This object referral seems to help students who have problems with the dimensions over 3 as well as those who have difficulty visualizing the dimensions. Then it also becomes much easier for a teacher to explain it.

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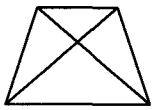
The only experimental study was performed by Mason. He (1989) studied the misconceptions of gifted students 4th through 8th grades. In his experimental study, the two groups of students differed from the point of their intervention. One group attended a summer camp on geometry, other group received instruction in geometry via LOGO. He observed significant differences between pretest and posttest results of the two groups regarding their van Hiele levels. Kieren & Olson (1983) carried a similar kind of research. They used the level structure to analyze students' acquisition of concepts and reasoning abilities in the LOGO environment.

van Hiele (1986) gives the following example to explain the Van Hiele levels with a concrete example; e.g. Stages in the study of the rhombus

1st level...This is a rhombus; the pupils are shown other geometrical figures and are asked if they also are rhombuses.



2nd level...Something is noticed about the diagonals and the angles.



3rd level...Pupils exchange their ideas about properties of a rhombus

4th level...Construction of a rhombus with certain vertices and sides are given

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5. level…Properties of a rhombus are summarized and memorized. Different types of rhombuses in nature can be identified.

van Hiele (1986) then suggested the progress from one level to the consecutive (in-between van Hiele levels, may go to the next upper level by) one may be facilitated with the utilization/application of the following phases of learning:

Information, pupils get acquainted with the working domain;

Guided orientation, they are guided by tasks (by teacher or by themselves) with different relations to the network that has to be formed;

Explicitation, They become conscious of the relations, they try to express them in words, they learn the technical language accompanying the subject matter;

Free orientation, they learn by general tasks to find their own way in the network of relations;

Integration, they build an overview of all they have learned of the subject, of the newly formed network of relations now at their disposal;

These learning stages can be useful in translating van Hiele's ideas and their connections with the use of mathlets inside a mathematics classroom. I will use his theory to give possible implications of mathlets to use inside a mathematics classroom.

Representations, van Hiele levels and mathlets

A good representation should allow the learner to quickly obtain a sense of the overall topology of the network of concepts and some means to locate where in the

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structure any particular concept belongs (p.120, Cheng, 1999). A possible indication of van Hiele's theory could be that, with just a slight difference in the representation for the same thinking levels he proposed, it could give us a very different view on the same thinking levels. Similarly, **a representation can be a representation from one representation to another as any mathlet would be.** I have mentioned earlier, a representation is a mapping between two structures or two worlds as any representations in a mathlet would be. Yerushalmy refers to representations as objects (1997).

On the different level, Lesh (1987) argues that the diagnosis of students' learning difficulties may involve presenting an idea in one representational mode and asking the students to illustrate, describe, or represent the same idea in another mode. Mathlets use this feature as an underlying structure. In case of problems, the processes, which are stronger than the others, may be used to strengthen the others. Hence, for a child with this kind of problem, **the situation may be reversed**, or an indirect process by inserting another translation process within may be tried (Lesh, Post, & Behr, 1987).

Most mathlets enables this to the student as simply changing the direction of the mapping and see the changes in the original structure with parameter manipulations within the second structure while the arrow of the mapping is now reversed. Here, the student can be ready for either or both of the mappings (translations). According to van Hiele this would be related to the familiarity of the structures to the student and also the student's readiness level as in the terms of van Hiele levels. If the first or second mapping would be counter intuitive to the student (i.e. if the student has not been exposed enough

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to the each structure and translation before), this shift would be highly difficult to make. If not, then there would be no problem.

Lesh (1987) names a Cartesian coordinate system representation of an algebraic problem as a powerful conceptual amplifier. He argues that the meanings a student attaches to such "conceptual amplifiers" in one context can serve as "free access meaning" which helps them to generate new ideas when the same representation is used in another context. He calls this a dynamic representation system. His ideas support the notion of a mathlets.

McCoy (1994) found students' difficulty in linking graphical and tabular forms of representation to algebraic forms (symbolic) of representation (cited in Brenner et al, 1997). One may think that enabling students with the applications first may make students much lazier and may lead to no deep thinking at all. However, Lesh (1987) argues that this may possibly save time of the students who would be losing too much time on the first order thinking otherwise, and this may even lead to higher order thinking in them. Although he does not support his claim with empirical findings, his idea deserves at least an analysis. In a classroom situation, a mathematics teacher could arrange the learning environment so that the student still could do some of the applications by hand, but also see others in the interactive diagram light, in a much sooner time and with as much precision.

Cunningham (1991) states that

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"Perhaps the most important issue for mathematics education is how visual and symbolic learning complement each other. At the actual course level, there are dual aspects to the problem of visual versus symbolic presentation. First, what material is best introduced visually, and what is best introduced symbolically? For material that needs a visual presentation, when (if ever) should this be treated symbolically? What makes the visual treatment eventually inadequate and requires symbols?..Conversely, for material that is best introduced symbolically, when (if ever) should this be treated visually? Perhaps visualization gives added value to these topics through development of intuition, an overall understanding of a complex problem, or the ability to see the topic's principles in an application. For example, Mathematica notebooks merge textual, symbolic, and graphical information in a way that allows the individual student to explore each in the order that fits his or her curiosity and learning needs best."(p.75).

Mathematica is a software program designed to carry higher order mathematical visualizations, topics, concepts, etc. It requires more software knowledge than Maple, Graphic Calculator (a software program embedded inside Macintosh Computers that recently most high schools use to carry dynamic software capabilities) or Geometrical Sketchpad. It is essential to note that saying of Cunningham translates for mathlets in general as well! Cunningham's points are also important while considering student's

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preferences for specific kind of representations than others. Apparently Mathematica makes good use of this principle and learning occurs.

Lesh's chapter appears in the book of Janvier (1987), which gives a table of the translation processes regarding the representation theory (Table 2). Each cell corresponds to the name of the translation process from the row to the column. For example, when we try to translate the given mathematical representation from a table to a graph, then we are occupied with the plotting. Same -mode representations mostly occur when we want to

Table 2.

Translation processes

		Modes of representation				
	To / **From**	**verbal descriptions**	**tables**	**graphs**	**formulae**	**pictures**
Modes of representation	**verbal description**	same mode translation	measuring	sketching	modeling	sketching
	tables	reading	same mode translation	plotting	fitting	demonstration
	graphs	interpretation	reading off	same mode translation	curve fitting	generalization
	formulae	Parameter recognition	computing	sketching	same mode translation	drawing
	pictures	interpretation	data gathering	geometry	symbolization	Same mode translation

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compare two similar representations within the same mode of representation system. For example, replacing a formulae element with another formulae leaves us with a bigger formula to deal with. This is a same -mode representation, but still a different representation in a sense. Due to the geometrical figures, I have added a column and a row on from /to pictures to the existing table. This way, it is prospected that it would be easier to talk about the different types of representations in a mathlet.

It is interesting to realize that most of these terms appear within a mathlet or in the tool line of the dynamic software like Cabri Geometer and Geometer's Sketchpad. Hence, most of these features are either enabled by those softwares or easy to produce with dragging a mouse, using the enter keystroke, or by a chain reaction algebraic notation change. Janvier purposefully avoids same mode translations, thinking they are as not important as others. I think **these same mode translations are necessary and important to make clear if the student's difficulty with translation is due to the nature of the translation itself rather than its mode.** Any kind of word problem is a good example of this. You can change the wording and it may still mean the same thing but this new representation has some new features that enable the student to see what the problem is about, subtracting and not adding for example. Please note that in Table 2, the yellowd highlighted translations are the translations of 2-3 -mode representations in which the representations are tabular, graphical and symbolic.

Then Janvier puts some arrows to indicate that in the actual instruction or learning of mathematics, the translation

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"table(formula)" is often carried out as "table(graph(formula)))"

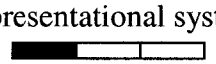
and

"formula(graph)" as "formula(table(graph)))".

He argues that this indirect translation is usually found much easier to accomplish than the previous. This is highly supported with Kaput's (1987 & 1985) view on the choice of some representations over others. Janvier mentions that indirect processes are substantially different than direct ones. Indirectness may be thought, due to the difficulty of the direct translation processes within some modes of representation as in the above examples. "Our research suggested that processes were best developed in (symmetric) pairs, in our case, "graph(verbal description (interpretation) and "verbal description(graph" (sketching)." (Janvier, 1987, p.29). Symmetric here refers to the processes' place in the above table with respect to the diagonal from top left corner to the bottom right corner.

Translation (dis) abilities influence both mathematical learning and problem–solving performance, that strengthening or remediation of these abilities facilitates the acquisition as well as the use of elementary mathematical ideas (Behr, Lesh, Post & Wachmuth, 1985; Post, 1986, cited in Lesh et. al., 1987). In that manner, for a student to understand an idea like "1/3" is to,

recognize the idea embedded in a variety of qualitatively different representational systems. (e.g. one thirds of a cake, 6/18, [(x+2)/(3x+6)],



)

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manipulate the idea within the given representational systems (e.g. [(x+2)/(3x+6)] being equal to 1/3 when x = -2 and also being equal to the $(y^3/3(y^3)$

translate the idea from one system to another (p.36)

(e.g. lim n->([[(n+1)/(3n-8)], or [(x/3)(d(ln x/2)/dx)])

Here an ID could be a way to carry all these processes at once to the student.

Boulay, Cox, Lutz, & Romero (2002) carry a project which they call it as "CRUSADE", and they investigate the effects of multiple representations to inform the design of effective programming environments with JAVA™. They state that there are conflicting and confusing literature on the effects of multiple representations on student's learning, especially stressing the importance of cognitive style differences of students etc. They also claim though, that multiple representations if designed well may serve as an advantage rather than a disadvantage even with students with various individual characteristics, since still the students could choose between which representation to follow and the translations would matter in that sense. Their ideas carry a different insight and they need to be considered in a much detailed manner later on.

Integration of the literature

I am trying to find out how mathlets may be helpful in understanding mathematical thinking. In this regard, representation theories are helpful for me to understand the ways mathlets may represent different mathematical domains and the ways they manage it. In a different representation system, I am reminded that I am interested in

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mathematical thinking and I should try to make use of a base theory of mathematical thinking, if possible. Then taking van Hiele would be a good idea since, he is also into the value of Intuition inside mathematical thinking. Now, I am in the representation system of the van Hiele Theory of the same problem I was referring to from the beginning.

Another perspective is that of "systems separation". System separation can be thought in terms of the dual process of two systems at the same time without any possible confusion. What we want to achieve is ideal case but we suffer from not providing the possible environment for students to make use of. Let's think of two representation systems for a mathematical task (as a graph and a parameter table). Then suppose that the student is required to make the translations as much as possible to understand the whole task. The level of thinking required here is more than visualization, analysis, or abstraction. What the students needs to do can be explained with the last two levels of van Hiele Theory; deduction, and rigor. These levels ask for functioning within systems and comparison of different systems. Hence, a good representation task would make use of the last two phases of students' van Hiele level of mathematical thinking. If a match between students' level of thinking, the level of instruction and of the text material is a necessary condition to achieve student understanding. Mathlets might help a lot in this problem. Then one of the next steps could be the identification of possible van Hiele levels of different types of Java applets or in general interactive diagrams.

Any **ID has two distinct characteristics; types of representations and the nature of translations involved**. The teacher can base the decision to use one in the

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classroom on the students' lack of insight with some representations and the learning problems associated with specific translations.

Lack of insight (van Hiele, 1987) may be a major drawback for students to function in upper levels. Visual representations in the form of mathlets may help learners to overcome this drawback and develop their intuition. As a metaphor, an interactive diagram or a Java applet may enable the learner to see the structure of a task or a conceptual map as an unfolded cube, so that the next time the learner faces a similar unfolded cube, identification of it as a cube would be much easier.

Can the mathlets help the students in building more insight/intuition in to the mathematics? I think yes they can. We have already demonstrated that visualization helps build more intuition, and Java applets promote visualization. Dragging, changing the loci, changing the parameters and seeing the resultant changes require some degree of intuition, anyway and that would be built with some sort of experience with the highly visual and dynamic figures.

How can a student's level of thinking interfere with the possible level of thinking required by an interactive diagram? Above in the literature, we have realized that for some levels, such as above level 2, students seem to be favoring mathlets. Can this be due to the systems approach that we have mentioned earlier? Or can this be due to the structure of the representation specific? Well, these questions need to be thought about more deeply. I think much more important point is the degree of control that should be

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given to the student to achieve a better understanding with the Interactive Java applets. If the student only does the changes asked to do, and if there are not "why" questions asked, I am afraid we would have the same kind of classroom that we did before mathlets.

The studies in interactive diagrams go back to the beginning of 1980s. Rambally (1982) designs an interactive computer graphics system, and discusses its utilization and overviews the timely advances in computer graphics, back then. Recent study related is dated 2000 by Caroline van Auken Borrow, searches development of 6^{th} grade students' geometric reasoning within a computer microworld with constructivist paradigm at hand. Her design is a teaching experiment. Hence, she uses in depth interviews before and after the program and case studies and group discussions to gather data and investigate the whole phenomenon. This is a preferable design procedure for different purposes as going into the thinking processes that students experience while they are engaged into the specific program and graphics eventually. This study on the other hand will focus not only on the process but will also investigate some interrelationships between theory and the practice as in the form of the place of van Hiele Theory and representation theory in the students' understanding of the interactive diagrams and use of them.

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CHAPTER 3: METHODOLOGY

Method

The study used mixed methods to ensure the highest possible understanding of the situation. The qualitative approach was used to see if students with inbetween van Hiele levels experience Interactive Diagrams differently from the students with high and low van Hiele levels. Whether these students' mathematical understanding inferred from their performance on a test of quadratics affected by their van Hiele levels or by the nature of the translations among modes of representations involved within a mathlet was the main point of the investigation. A quantitative approach was used to get a generalized version of the van Hiele level effect and the number of different modes of representations on students' mathematical performance.

Research Questions and Variables

1. How does the number of modes of mathematical representations in interactive diagrams affect ninth and tenth grade students' gain on tests of quadratic equations, after one class period of using a mathlet to review content?

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2. How does the van Hiele level of the students affect ninth and tenth grade students' performance gain scores on tests of quadratic equations, after 1 class period of using a mathlet to review content?

3. Is there a combined effect of the number of different modes of mathematical representations in mathlets and the van Hiele levels of students on ninth and tenth grade students' gain scores on tests of quadratic equations, after 1 class period of using a mathlet to review content?

4. How do students with different van Hiele levels experience mathlets?

5. How do notions of "structure" and "insight" as defined by van Hiele and Janvier function for students with inbetween van Hiele levels when students are working on translations among different modes of representation in mathlets that review quadratic equations content?

6. How do students process same mode and different mode translations while working with an interactive diagram?

The study mainly aimed to investigate the differential and combined effects of the following Independent Variables on students' mathematical understanding inferred by students' mathematical performance with Interactive Diagrams (Figure 2):

Input Variable 1... van Hiele level of the student [high, low, in-between]

Input Variable 2...number of different modes of representation [2→tabular and graphical, >2 → tabular, graphical, and symbolic]

Input Variable 3...Teacher [1, 2]

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Input Variable 4...Last year mathematics grade

Dependent Variable...Performance gain score

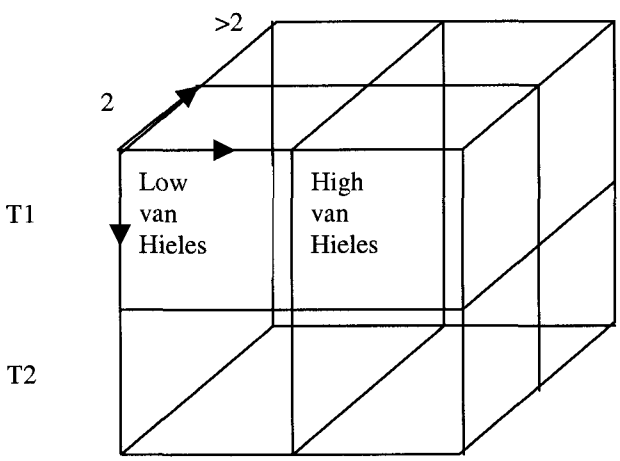
Participants

High school seems to be the grade level where students would have sufficient computer technology and Internet experience to make effective use of Interactive Diagrams. Whereas Grade 8 students are highly busy with high stakes testing, grade 9 and grade 10 students appear much more relaxed in terms of their learning than their 8th grade counterparts. Thus, the participants were selected from grade 9 and grade 10 mathematics classrooms. The target population was all secondary students in the USA. The accessible, available population was the 9th and 10th graders in an urban district in upstate NY. In the school district chosen, four classes (two classes of two mathematics teachers) represented the subjects of the study. One teacher was teaching a math AB and the other teacher was teaching a MATH ABH course. These two courses were parallel in format and in algebraic content and were taught to grade 9 and grade 10 students respectively. This age group difference was due to the one group of students being in accelerated (9th grade ones).

This high school district had 1299 students in 2000-01 year. The school staff consisted of 93 teachers and 15 other professionals. 26.3 % of the students were found to be eligible for free lunch, and 3.8 % of students were English deficient. The ethnic diversity of the school consisted of 18% Black students, 5.5 Hispanic, and 2.1 American

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Indian from the same school year (The NewYork State School Report Card, March 2002).



*Each block is a unit of analysis of students. *Each class was thought of 2 groups with respect to the students' van Hiele levels and hence there were 8 groups.
*The dimensions of the cube are three variables of the study: number of different modes of representations (2, >2), van Hiele levels of the student (low, high) and teacher (T1 &T2).

Figure 2. Quantitative methodology model

Design of the study

The study used a pretest and posttest quasi-experimental design followed by interviews and augmented with observations (Table 3). Interviews were done with students to explore the relationships of van Hiele levels, transitions among representations, and thinking with interactive diagrams in general. Researcher did an observation of the students to check their being on task. The researcher observed each student and helped him or her to be on task again.

The sequencing of the design was as follows. Both teachers have taught the topic of quadratics for at least one week. The teachers then came to a consensus of what to teach and how and they followed these informal guidelines. At the time of the study, teachers agreed that both classes were almost at the same point in their review of the

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quadratics. The van Hiele test, followed by the quadratics pretest, mathlets intervention, and the posttest were administered to the sample in the next four days one day at a time. Hence, the study took four whole class hours. Since, the class hours were each time every day, the study took almost 4 days to finish the procedure not including the interviews.

Their own teachers acted as facilitators and one to one lecturers for specific problems by students. Sometimes the researcher helped to the students who were having problems due to technical problems or due to the context. As it would be expected still some students could not pay 100 percent attention to the intervention due to being at that age, which most likely should be treated as a normal classroom behavior. Due to the availability of subjects, random selection of subjects did not seem to be feasible; therefore, a Quasi-experimental design was used. The intervention classes were videotaped for further observation. Students filled out an activity sheet in which they were required to fill out the vertex, x-intercepts, y-intercept, min/max characteristics for a set of 10 quadratic equations ranging from $y=x^2$ to $y= 3x^2-2x+5$. They were asked to rate their observations also. Students were given two different applets in the morning and afternoon classes. The morning classes of the two teachers were given the "Slider Graph Applet" which had two modes of representations; tabular, and graphical (see applet descriptions). The afternoon 6^{th} classes of the two teachers were given the Quadratics standard Applet, which had three modes of representations, embedded in it, tabular, graphical and symbolic (see applet descriptions).

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Any experimental research carries some problems regarding the validity of the study. In this specific study, the threats to the internal and external validity were addressed as following. Instrumentation turned out to be a problem, but in a different way than expected. The students and teacher thought pre and posttests were too difficult. Here the researcher takes the whole responsibility of un-informed educators who do not know the realities of the math classroom. The test already developed by Usiskin (1982) was used to avoid any problems due to validity and reliability of the test. Senk (1989) also used these van Hiele level identification tests in his research for many years.

Table 3

Quantitative design methodology for the study

grp	teacher	class	Van Hiele levels (IV1)	pretest	treatment (IV2)	posttest
1	1.	1	1,2	Ach.test1	2 type representations	ach. Test2
2	1.	1	3,4,5	//	2 type representations	//
3	1.	2	1,2	//	>2 type representations	//
4	1.	2	3,4,5	//	>2 type representations	//
5	2.	3	1,2	//	2 type representations	//
6	2.	3	3,4,5	//	2 type representations	//
7	2.	4	1,2	//	>2 type representations	//
8	2.	4	3,4,5	//	>2 type representations	//

The reliability and the validity studies of these tests were already carried out. Since everything should be completed in one year, mortality was not a major problem. However, students who did not bring permission slips back chose not to come to the

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classes for the testing or the intervention. It was important to note this. Among the threats to external validity, pretest-treatment interaction might have produced a concern, but, since the pretest was not on the Interactive Diagrams but was on the subject topic, this effect may be overlooked. In general, since IDs on the Internet are highly varied over many topics in mathematics, generalization of findings to all IDs and/or topics is also a problem. However, it is hoped that this research can provide a baseline for further studies that it will open up new horizons for this line of research.

Instruments and Material

The interactive diagrams that were used (Figure 3) for this research were the two java applets selected for their content specifications and also for their capability of multiple representations regarding quadratics. The web site authors were contacted to get their permissions (APPENDIX B) to use the web pages for this specific research. The permissions were granted and will accompany the appendixes. Both of the mathlets were made for the grades that were the subject of the study. The following brief descriptions preceded the IDs selected in their corresponding web sites.

The Slider Graph applet: This applet that draws the graph of a function that can include references to the "constants" a, b, and c. The values of these constants can be adjusted using the sliders at the bottom of the applet. The applet responds to the mouse actions click, shift-click, click-and-drag, and right-click-and-drag. I've added another buttons to the limit control panel. The "Equalize Axes" button

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adjusts the x- and y-limits so that the scales on the two axes

are the same.

(http://www.albertaonline.ab.ca/resources/mathapplets.htm)

The Quadratic Functions in General Form applet: HOW TO USE THIS APPLET

The slider area

The sliders change the parameters a, b and c in the quadratic function $y = ax^2 + bx + c$.

- Drag a slider to change the value of a parameter.
- The left and right arrow keys can be used to change the selected parameter [light blue].
- The up and down arrow keys can be used to increase or decrease the selected parameter. (Use this feature for fine adjustments.)
- If the arrow keys don't respond, click the cursor in the applet frame then try again.
- Drag a parameter scale to "move" it. (ie. Change the range but maintain the scale "length".)
- Drag towards or away from a parameter thumb to "re-scale" the scale.
- Select the corresponding parameter button to enter a value for a parameter.

The graph area

- Drag the vertex to move the parabola in the coordinate system.

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- Drag a location along the axis of symmetry of the parabola (other than the vertex) to move the parabola only horizontally or vertically.
- Drag a location along the parabola (other than the vertex) to change the width of the parabola.
- THE SHIFT KEY : Click and drag in the graph area to move the entire coordinate system.
- THE CONTROL KEY : Click and drag in the graph area towards or away from the origin to change the x and y axis scale.
- THE CONTROL-SHIFT KEY : Click and drag in the graph area towards or away from the :
  - x-axis to change the y-axis scale.
  - y-axis to change the x-axis scale.

Other

- Use the "RESET" button to restore the applet to the initial state.

(http://www.members.shaw.ca/ron.blond/QFA.GF.APPLET/index.html)

To be more specific, we may need to analyze the two applets for what they are capable of and for what purposes they function.

Description of mathlets

a)The Slider Graph Applet

This applet has two interrelated parts (Figure 3 and Figure 4). The top left part of the frame is where the graph is drawn and the bottom part is for adjusting the values of constants a,b,c of a quadratic equation ax^2+bx+c. They are interrelated because any slight

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change in the values of the constants causes a corresponding change in the graphical representation. For this applet, the causal change is only one way and to manipulate the graph with the aim of seeing the related constants is not possible. The ranges for the x and y axes are predefined but can be changed with respect to the user's choices. One needs to push the "set limits" button to save the changes. Unfortunately the axes only show some integer values instead of all, hence the user needs to make value judgments on the basis of other identifying factors such as the graph shape, closest identifiable number on the parabola, or approximation. The little box in which the quadratic equation in standard form is given, does not work as flexibly as one might want and it requires specific syntax rules followed (e.g. when students typed "y=" it gave an error message as much as whenever there was a blank character between the terms of the equation.) However, the slides for the values of a, b, c constants at the bottom of the frame works pretty well with the graph. And it is possible to see the exact value of the constant altered with the slider on the right of the slider. A hidden trick of the graphical representation in this mathlet is zooming into the graph and to the specific points or parts of the graph. For this purpose, one needs to left click the parabola analyzed. Right click and drag enables to move the parabola anywhere possible. Other than these little tricks for effective usage of the applet, one other importance of the applet comes from its way of giving the transition from tabular representation by a slider to a graphical representation above. Any incremental change is possible to observe. This enables the learner to investigate what produces which graphs and in what ways. And it only asks for a few of mouse clicks, drags or key strokes.

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The difference between the classroom representations using paper- pencil work and this slider graph applet would be the applet's capability of demonstrating the "Δt" incremental changes on the existing/given/default graph so that the student could get the pattern without a need to see various examples at once. We could make the analogy here by relating how this works to a special camera with which observer would be able to see what it would be impossible to see with naked eyes. A camera, which captures how a bullet comes out of the apple "Δt" after it is shot into the apple deciphers the whole process for learner, hence the learner can use his/her intellect for more sophisticated problems like why the curve gets twisted with fourth degree of x. In other words, teachers, educators or students themselves may use this applet to give them a head start for a higher level of cognitive functioning.

b) Quadratic functions in general Form Applet

This applet has three interrelated parts (Figure 5 and Figure 6). The upper left part has three formulas functioning together the discriminant, axis of symmetry, and the half the distance between a root of the quadratic function and the axis of symmetry. All the values in these formulas are reflected in the bottom left graphical representation. One can observe the axis of symmetry if it is other than the y-axis by its specific green color, and the discriminant by seeing the two roots (two x-intercepts), one root (one x-intercept) or no roots at all. The distance (d) is reflected as the closest distance from one root to the axis of symmetry with separate colors as well as in case of two x-intercepts of the graphs. The right piece is a slider for the values of the constants a,b,c from a standard quadratic function.

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Unlike the Slider Graph Applet here, the sliders are in upright position, but they both represent the same thing. One can also use "SET A", "SETB", "SETC" buttons to ensure the sliders position and specific values for a,b,c constants. The "Reset" button is to set the a,b,c constants to their default values. The graphical representation points to the vertex of the graph with a red point on the graph. One can drag the vertex of the parabola and get different parabolas conserving the width and shape of the old parabola. The axis of symmetry is also demonstrated on the graph with x=s next to the bottom right position of the relative axis of symmetry. It is also possible to move the graph by dragging any point in it. This time new parabolas differ in their width, shape, but in a different way.

The exact equation of each parabola can be seen at the top left portion of the formula (symbolic representation). Here one can observe the exact value and notation of the constants and decide if these values are the intended ones or not and change them accordingly. Hence, this part also is similar to the right hand portion of the sliders in the previous applet.

One main difference between this mathlet and the previous one is the addition of the symbolic representation part as a catalizer between the tabular and the graphical part so that the relationships of constants one another and to the formulas that define the shape, vertex, axis of symmetry and the position /existence of the roots to are explained with a different form of representation.

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Thinking about the focus ability of eye to one thing at a time, one might ask the question of why one would need two representations and how these might work together. This may depend on the preference of the teacher/instructor as well as the preference of the learner. However the main point is that each form explains one another and each form needs one another to make sense. Demonstrating to the learner all these three at once, asks the learners' decision of the priority of each representation form and it is sure that it changes both from problem to problem and from one context to another. The learner's ability to prioritize and acknowledge all representation forms could benefit from this applet's use if used properly. The learner would need to master each representation form separately at first and then make pair wise connections and then finally the triple connections. The remaining understanding of how these applets work is one of the underlying questions of this research.

The number and types of different modes of representation and the nature of the translations within and between these representations are distinctive features of an interactive diagram as. Table 4 explores the possible number of different ways of having representations in an interactive diagram by using my extended way of Janvier's types of mathematical representations. Combinations ($_xC_y$ represents the combinatorial of x with y. or the # of possible combinations of x with y size groups) show the number of possible ways of arranging something within another group. Here, we have 5 different types of representations and table 4 shows that combinations of two and three representations would be the most frequently found versions of these. Thompson (2002), mathematics

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software designer states the three main approaches of software and the second and third approaches are actually the 2 mode and 3 -mode representations differentiation.

Table 4.

Possible representation type combinations in a mathlet

V for verbal
T for tabular
G for graphical
F for formulae (symbolic)
P for Pictorial
V, T, G, F, P where # = $_5C_1$=5
T & V, G&V, F&V, G&T, F&T, F&G, P&T, P&V, P&G, P&F where # = $_5C_2$=10
V&T&G, V&T&F, V&G&F, T&G&F, P&V&T, P&V&G, P&V7F, P&T&G, P&T&F, P&T&F where # =$_5C_3$=10
V&T&G&F, T&G&F&P, V&G&F&P, V&T&F&P, V&T&G&P where # = $_5C_4$=5
P&V&T&G&F where #=$_5C_5$=1

Even though, the choice of the IDs was determined by the mathematical topic at the time that I was able to carry out the research, Table 4 supports the case of why I have chosen an ID with two or moredifferent modes of representations instead of only one.

The van Hiele Theory favors gathering data on the students' level of functioning through interviews. However, after van Hiele's primary research efforts to identify the levels, other researchers developed surveys and tests that can be used in this type of research. A large-scale subject selection is thought to be necessary to avoid the possible

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consequences of the use of these surveys instead of the interviews to identify students' van Hiele levels. An instrument developed by Usiskin (1982) and then used by Senk (1989) to determine van Hiele levels of students was the instrument for this study. In addition, mathematical performance tests were designed for the math topics explored in the selected IDs (see APPENDIX C & D).

The van Hiele test developed by Usiskin (1982) has been the subject of a few psychometric critiques in the literature (Crowley, 1990 & Wilson, 1990). This 25 item multiple-choice test is questioned for having some discrepancies concerning its reliability and overlaps in its subtests (each subtest includes five items and these items correspond to one van Hiele level). Usiskin, while acknowledging the low reliability scores, stressed that the original purpose of the test was to test the van Hiele Theory, and that this was fully accomplished. Usiskin and Senk (1990) argued that the test is a good predictor of van Hiele theory and hierarchical levels. They further asked the more than 100 people who used the test for related research purposes to share their data with them, and they checked the consistency of those results as well. They argued that nothing in these studies seem to have a totally different student population then theirs. The pros and cons of this debate are beyond the scope of this report. It is sufficient to note that Usiskin's test is the shortest test available that is widely known and accepted and it is the only test, which can be administered to groups. The test is graded according to the following rule: a student is assigned to a level if he answers all the questions at the previous level correctly and fails to answer only 1-2 items in the assigned level. Hence, for passing a level at least 3 items need to be correct from that level.

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Table 5.

Table of specifications for pretest and posttest development

NAEP framework Performance Indicators	Conceptual understanding	Procedural Knowledge	Problem Solving
1. determine the effects of changing parameters on the graphs of functions	5a, 5b	7a, 7c	17a, 17b
2. translate among the verbal descriptions, tables, equations, and graphic forms of functions	1a, 1b	8, 21	4d, 13b
3. apply ideas of symmetry in sketching and analyzing graphs of functions	10, 19d	7b, 15	4e, 12
4. judge the reasonableness of a graph produced by a calculator or computer	3, 20	21, 13a	22, 9
5. apply quadratic functions in the solution of problems	6, 19a	11, 19c	14, 18
6. approximate the roots of polynomial equations	4a, 4b	2, 4c	16, 19b

Pre- and post-tests of understanding quadratics were designed by the author to identify students' mathematical performance with the topic of the quadratic equations.

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NAEP 92 & 96 frameworks were used for the table of specifications (see Table 5). NY State MST (NYS, 2002) standards were used to identify the performance indicators of what the students would need to learn from the topic. The test was developed with this table of specifications in mind. An expert panel consisting of test developers, math teachers, and educators were contacted via e-mail for their feedback regarding pre and posttests. Tests were revised with respect to the feedback of the expert panel (EA and B). In Appendix E and F the ideal answers to the questions are included.

Also, an interview protocol was developed by the researcher for the specified qualitative part. The proposed interview protocol followed Wilhelm et al. (1999) approach. These researchers used three tasks to guide them through the interview questions. The tasks for this research were based on Roseann Allen's Discovery learning approach worksheet for quadratics. The format was tabular and the equations reflected the ranges of the IDs studied.

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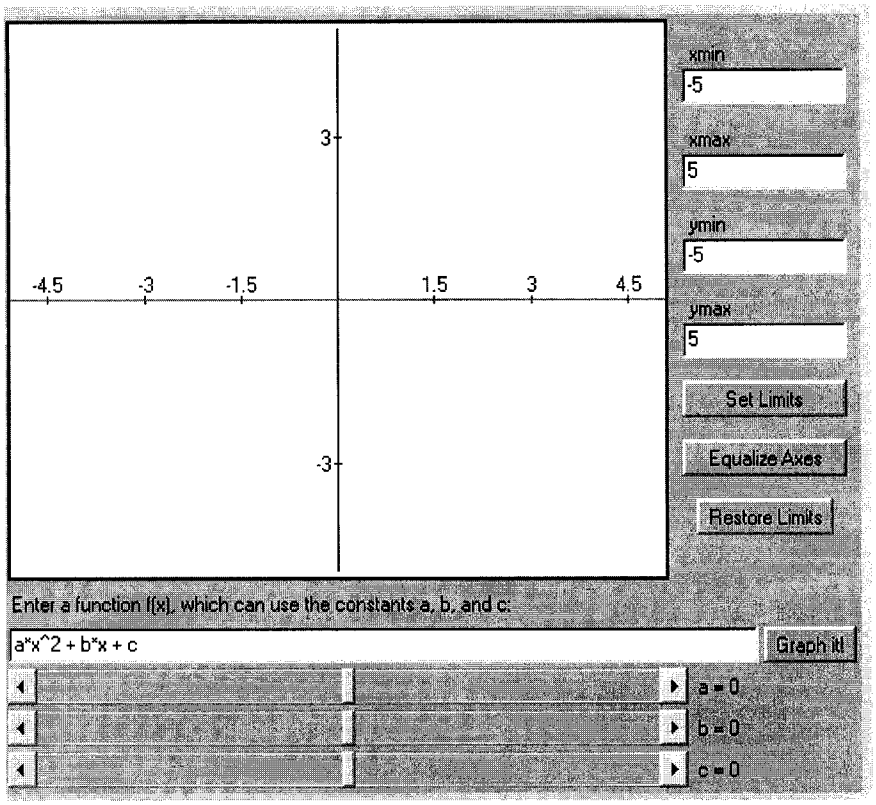


Fig.3. Slider Graph Applet default screen

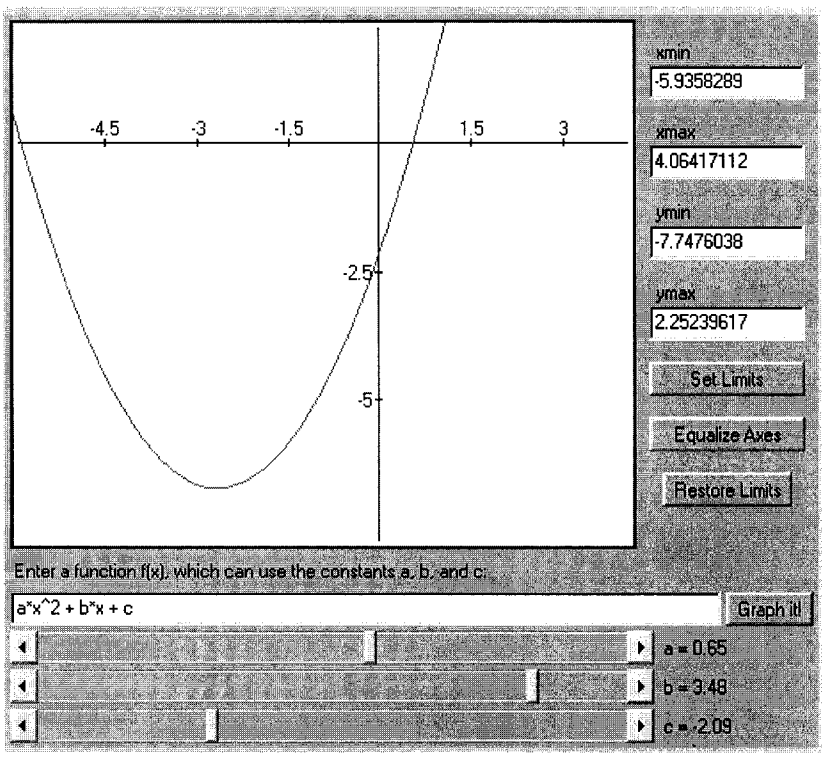


Fig 4. Slider graph applet $y=.65x^2+3.48x-2.09$.

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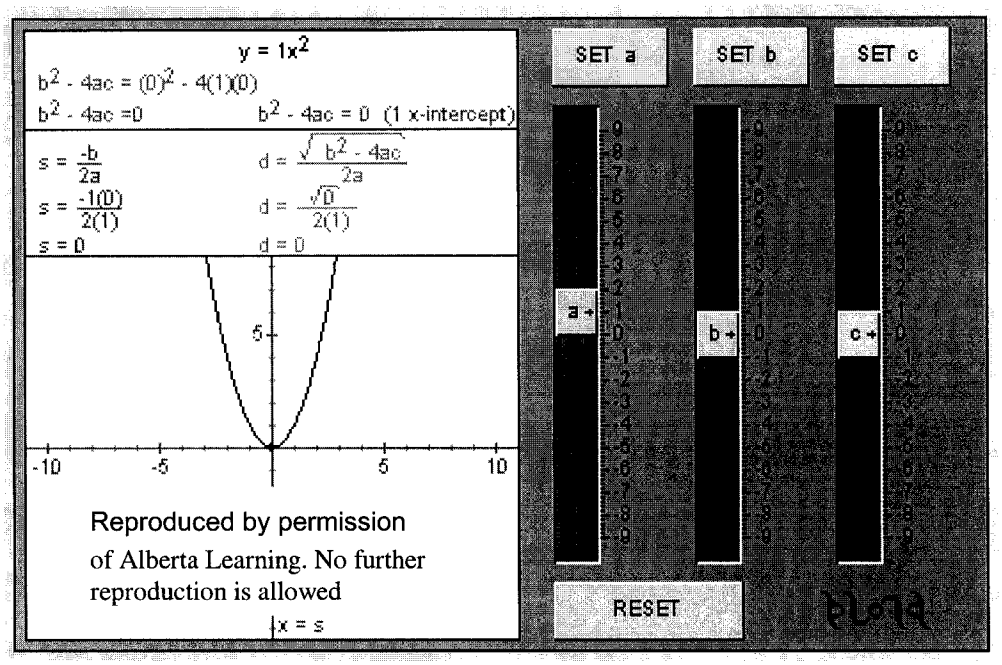


Fig 5. Quadratics in standard form applet default screen

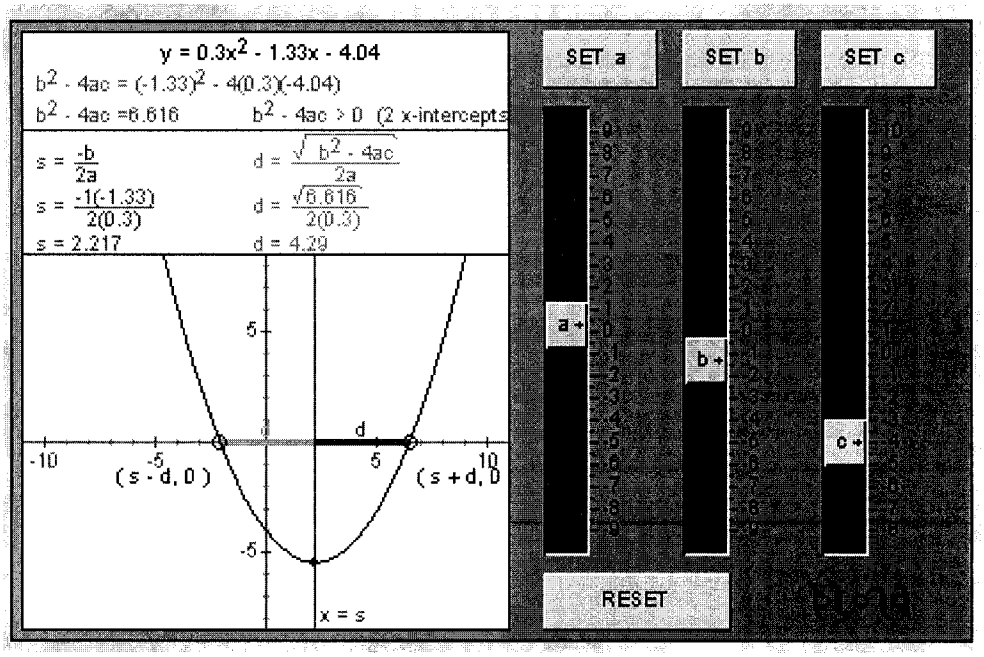


Fig 6. Quadratics in standard form applet for $y=.3x^2-1.33x-4.04$

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Interview tasks and protocol

For interviewer...

Hi, my name is ..... I will be your interviewer for the next half an hour. This interview will be used for investigating how thinking with the Quadratics Function Applet works for you. Please answer all the questions based on your thinking and try to think aloud. The researcher would use the information for identifying the characteristics of applets that hinders or ease learning of mathematics, actually for this case Quadratics (the equations which are like $y= ax^2+bx+c$).

I want you to log on into the computer with your ID and open the web page:

http://www.members.shaw.ca/ron.blond/QFA.GF.APPLET/index.html (they need to be sure the syntax is correct since, the page does not come up in case of syntax errors)

For students...

Quadratic Functions in General Form applet lets us to specify the equations by dragging the right hand side table for the a,b,c coefficients of a general quadratics function which is written as $y= ax^2 +bx+c$.

Task for the student...

Please try the following equations one by one and point (on the screen) x--intercepts, y-intercept, vertex and axis of symmetry while doing it.

$y =x^2$

$y = -3x^2$

$y = .5x^2$

$y = x^2+2$

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$y = (x+2)^2$

$y = 2x^2+3x+5$

$y = 2x^2-3x+5$

$y = 2x^2-3x-2$

$y = 3x^2-5$

Set of interview questions:

- How do these set of equations differ graphically?
- As with each equation you specified, what changes occur on the screen?
- What different parts you see on the screen?
- How do changes on the screen on different parts work together?
- What is the relation between these parts?
- What happens when when you change something at here and then see a change there? (pointing to the coefficients a,b,c or graphical representation or to the symbolic part where they can see the "d" distance between the roots (x-intercepts of a parabola) and x= s as being the axis of symmetry...)

Now please focus on the graphical representation frame and click on the left hand side of the mouse and drag a point inside the frame.

- What kind of changes occur within the graph only?
- What kind of changes occur within the symbolic part above?
- What kind of changes occur within the whole applet?
- to see what kind of changes occur inside the graph frame?
- Then make them focus on the symbolic part while they are making again the same changes with dragging a point.

Ask at the end, what do they think about this experience? How they would like to use it if they would.. In what ways..

-How frequently you use Internet? In what ways? For what purpose?

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-Would you search Internet for findings mathlets, too?

THANK YOU FOR ALL YOUR HELP>.

Set of probe questions:

- What do you mean?
- How did you get that?
- What do you get out of this?
- How do you make sense of it?
- Can you point on the screen where are you looking at?
- What are you thinking about as you look at those?

Procedure

The whole quantitative data collection procedure took a week for each class after the student and parent permission slips were returned. The van Hiele test (APPENDIX G) was administered prior to any other data collection (see Table 6). Students' van Hiele levels were identified. The next day, pretests were given. The following day, the intervention took place in the school library in which there were 20 computers (15 PC and 5 Macintosh). Some PC computers were problematic due to the missing Java virtual machine program, which enables any mathlet to function. The macintosh computers took too much time opening the mathlets and so could not be used. Students then shared the remaining computers with their peers. This enabled a cooperative learning to take place. An activity sheet is used with students during the intervention (APPENDIX H). The day after interventions (there was a one day elapse in time due to the unavailability of the

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library to each class at the same time between interventions), posttests were administered for the following day. Pre and post tests were developed with 25 min of time at mind but then teachers warned for a need for longer time. Student scores were computer by taking the scoring rubric (APPENDIX I) Hence, whole class period was devoted to these tests. Within one week after quantitative data collection, student interviews were initiated. These took almost 2 weeks time due to the lunchtime availability of at most 4 students at a time (APPENDIX J).

Two videotapes were used to interview two students at a time.

Table 6

Timeline of the research procedures

Timeline	**Procedure**
Day 1	van Hiele test was administered to the students of the 4 classes of two teachers Van Hiele levels were identified (low, inbetween and high)
Day 2	Pretest was administered
Day 3	Morning classes used the mathlet with the 2 mode representations and afternoon classes used the mathlet with 3 mode representations
Day 4	Posttest was administered
Following weeks	Interviews with the 17 selected students of which 6 high, 6 low, and 5 inbetween were carried.

The Usiskin test was administered to identify students' van Hiele level. Each class group was considered as divided into two groups with respect to students' van Hiele Level as being high (levels 3,4,5) or low (levels 0,1,2). Researchers referred to the level of the student where no other level is inferred as the "0" level. It is expected that there

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will be students with in-between van Hiele levels (students who can not be identified as either low or high), however these group of students did not exceed 10% of the whole sample. Some of these students can be grouped as either high or low van Hiele, with the division we have for this research (0,1,2 as low, and 3,4,5 for high) according to whether or not they were able to answer all low level questions but not all of the high level questions. The Usiskin test acknowledges the inbetween van Hiele students with identifying them as students who answer the questions from a higher level while they miss some questions from a lower level. Inbetween students were also identified for this study.

Each teacher taught according to two different learning environments: 1. the interactive diagram chosen with only 2 different modes of representations (Slider Graph Applet), and 2. The interactive Diagram with more than 2 different modes of representations allowed (Quadratics applet with standard form). Both teachers taught using each mathlet to control for teacher effect.

Treatment applets were decided with the hope of distinguishing the differential effects of mathlets with two different types of representations and mathlets with multiple representations. This differentiation explores the most common differentiation of the mathlets already available in the java applet pool of Internet (see table 4, and remember Thompson, 2002).

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Remembering the literature, for a group of students inside every classroom, the van Hiele levels can not be determined exactly as anticipated. Those students became the subjects of the qualitative part of the study as well. Their situation was unique as they are in the transition period or inbetween levels. These students were the focus of the interviews. or inbetween levels. To be on the safe side, the same number of students were interviewed from high and low van Hiele level groups as well, nonetheless I doubled the number of students with using two teachers classes. Being inbetween students means that the students can answer the questions of the higher levels but not some of the lower levels. Neither teachers not the students knew the students' van Hiele levels, nor students knew their van Hiele levels to control for the Pygmalion effect.

The confidentiality of the students was assured by keeping their test answers in locked boxes and after 1 year, they would be destroyed. The classes were videotaped while the intervention was taking place (both the computer screens and the students' general behavior). The Usiskin test and the pretest were administered in the class period before the intervention to allow time for intervention. One class period was allowed for the intervention and one for the posttest. After the quantitative part was over, those students within the transitions were determined and **were interviewed for their understanding with the interactive diagram.** This way, the retention could be achieved. The interviews were administered by the help of the one trained interviewer during the lunch period. Interviewing all three – high, low, inbetween would give me **eighteen students' interviews**. However, only seventeen students could be interviewed. Interviews were videotaped. Due to the difference in synchronicity difference between

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computer screens, the interviewers were given specific directions on making sure of all the student behavior was noted by their voices. Same anonymity procedures described above will be carried here as well. This is supposed to bring the process of learning with interactive diagram at least from an angle specific to my study. This angle could be explained as not only understanding how the students' cognitive readiness (by van Hiele level) and the mode of representation acts together in general but also how these two factors effect thinking specifically with individual differences of students.

The activity sheets were collected after the intervention. This activity sheets were designed as another way of keeping students on task. This sheet included the web site address of the interactive diagram being used. Students used their school ids to log onto the computers and this option has diminished while in the study.

The quantitative approach was used to develop a general idea of how IDs' representation types and the students' van Hiele Levels affected students' mathematical performance. Pre- and post-tests were administered to test for changes in the students' mathematical performance that could be attributed to the intervention or use of interactive diagrams. The research literature suggests that one of the best ways to deal with confounding variables is to control them. Hence, teacher effect (using two teachers and their classrooms respectively) and students' general achievement (8^{th} grade student math scores) were controlled by related data.

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Analysis

Since the pre and posttest scores were very low, reliability procedures were applied to the tests. Both, item total and split half reliability coefficients were identified. ANCOVA procedures from the statistical package of SPSS were used. Students' last year mathematics grade was used as a covariant and van Hiele levels and the number of modes of representations were used as two independent variables where gain scores acted as the dependent variable. The variables of the study were intervention, van Hiele level, pretest, posttest, gain scores (posttest-pretest score) previous year's math achievements etc. Even though, there would be some possible changes at the van Hiele levels of students, for this study we chose to focus only on the possible change in the students' mathematical performance level. van Hiele himself suggested that these changes would be due to the instruction, if any. Instead of the students' cognitive level, the mathematical understanding, which was inferred from performance on a quadratics test, was chosen as the focus of the study. In addition to the ANCOVA procedures, some descriptives of the date were noted. Since the time between the pre and posttests was only one class hour, and this was devoted to the intervention, repeated measures analysis was not considered. Out of the two analyses that could be conducted with the data at hand, ANCOVA was much more suitable to the data at hand as the way we collected, and also it gave a much clear analysis of the independent and dependent variables with the possible confounding variable. The dependent variable; the gain scores was calculated as the difference between posttest and pretest scores. For the qualitative part of the study, I checked all the interview data and look for the recurrent themes in the processes students use. Observations were gathered on from the activity sheet and the results were descriptively

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analyzed. I used one video recorder, to get both the screen to identify the students' actions with mouse and the feedback they got and they gave out of it. First a description of the interviews was written from watching the videotaped interviews. From these descriptions, themes, categories were decided, and used to give a coherent understanding of the qualitative data. The emerging categories, and specific teacher and student behaviors were noted. Again, triangulation was used to make sense of the all the data from different perspectives, by categorizing the data into meaningful pieces, and then analyzing the possible relationships and importance of categories.

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CHAPTER IV: RESULTS

The results section is organized around the answers to each research question. However before then, the descriptives of the data was given to ensure of the necessary framework of the data to make sense of the analyses following. The qualitative data description is available at the APPENDIX J, too.

Quantitative Results

Distribution of van Hiele levels

Out of 96 students from total of 4 classes of two teachers (2 classes each), sample included 83 students overall. This decrease was due to the students who did not bring permission slips and who preferred not to come to the school for the intervention period. Out of 83 students 10 of them did have problems with permission slips (either parent, student, or both said "NO"). Hence data became 73 students. Out of these 73 students, 3 of them did not enter either pre or posttests. Hence final data became 70 students. (Table 7).

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Table 7

Descriptive statistics of van Hiele levels

	Van Hiele Level			
Teacher	High	Inbetween	Low	totals
Abernathy*				
2-mode group	11	3	8	22
3-mode group	16	2	4	22
Bork				
2-mode group	5	4	5	14
3-mode group	2	3	7	12
totals	34	12	24	70

* Names are pseudonyms

The number of students was higher for Teacher1(Abernathy) than Teacher 2 (Bork), but I lost also more number of students due to permission slips with Bork. 2-mode group stands for the mode of treatment, and 3-mode group stands for the 3 -mode representations.

Interactive Diagram with 2 mode of representations involved (slider graph applet) was administered at the first period and the Interactive Diagram with 3 modes of representations involved (quadratics in a standard form) applet was administered at 6^{th} period to the subjects. Students of Abernathy were accelerated students hence, from grade 9 mostly whereas, students of Bork were 10^{th} grade students. This differentiation is also reflected in the course name they take as Math AB-H for accelerated and Math AB for non accelerated students. The topics are the same. Hence, the results should be treated in a way that we should not forget to emphasize the age difference, and also the motivation difference between these two groups of subjects. In between students generated almost

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$1/8^{th}$ of the whole sample of students, hence they reflected the van Hiele classification as well. Table 6 also indicates that the higher level students were more than the lower van Hiele level students. High van Hiele included 3,4,5 levels, and low van Hiele included 0,1,2. Inbetween students were (as explained in the literature referred to the van Hieles) the students who could not answer all the questions of low level while they answered some of the high level. The distribution given in Table 6 also confirms van Hieles predictions of the van Hiele levels of the students.

Instrument reliability and validity

The tests designed by the researcher had a somewhat lower reliability than hoped for. This could be due to two things, due to items characteristics or due to the sample characteristics, or both. In my case, it turned out to be the both. Following paragraph, demonstrates how I carried out the tests statistics and how I identified the construct and content validity of the questions in addition to the table of specifications I explained in methodology section.

Pre and posttests were analyzed for reliability. Since the structures of the tests were very different than conventional forms, the scores were standardized to achieve the same metric to compare them. Z-scores were computed for all items of pre and posttests. Two internal consistency estimates of reliability were computed for the pre and posttests; coefficient alpha and Spearman-Brown correlated coefficient. For the split-half coefficient, the tests were split into two halves as equivalent as possible. In splitting the

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items, I took into account sequencing the items. The first half included items 1,3,5,7,9,11,13, 15, 17, 19, 21 and the second half included the remaining ones of the 21 items (Table 8 and 9). Dividing by half otherwise would not work, since there were clear indications of tired students who gave up for the second part of the test.

Table 8.

The results of the split half analysis for pretest items

Statistics for	Mean	Variance	Std Dev	N of variables
Part 1	0	20.9911	4.5816	11
Part 2	0	17.478	4.1807	10
Scale	0	55.3244	7.438	21

Reliability Coefficients	21 Items		
Correlation between forms=	0.44	Equal-length spearman brown=	0.6111
Guttman Split-half=	0.6093	Unequal-length spearman brow	0.6115
Alpha for Part 1=	0.5236	Alpha for part 2=	0.4754
11 items in part 1		10 items in part 2	

Table 9.

The results of the split half analysis for posttest items

Statistics for	Mean	Variance	Std Dev	N of variables
Part 1	0	24.8735	4.9873	11
Part 2	0	15.7024	3.9626	10
Scale	0	62.0839	7.8793	21

Reliability Coefficients	21 Items		
Correlation between forms=	0.5442	Equal-length spearman brown=	0.7048
Guttman Split-half=	0.6929	Unequal-length spearman brow	0.7052
Alpha for Part 1=	0.6135	Alpha for part 2=	0.4035
11 items in part 1		10 items in part 2	

Cronbach's standardized alphas were computed as 0.65 for pretest and .70 for posttest by the SPSS analysis. To understand the item characteristics, item analyses were

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conducted for the total 21 items of each test. Items with less than 0.1000 corrected item total correlation (see Figure 7 and Figure 9) was suspected of elimination and re-run the scales with new set of items. From each test, 3 items were suggested to be deleted. From pretest; items 3,5, and 14 and from posttest items 3,6,and 8 were suggested to be deleted and item analyses were re-run. The new scales' reliability coefficients were .68 and .72. More deletions caused a drop in the reliability coefficient. Inter item correlations are also computed for both pre and posttests, these can be observed at the Appendix K.

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Squared Multiple Correlation	Alpha if Item Deleted
ZPREQ1	.0000	51.1267	.2236	.3991	.6409
ZPREQ2	.0000	49.3591	.3534	.4343	.6261
ZPREQ3	.0000	53.9302	.0268	.2785	.6623
ZPREQ4	.0000	52.1084	.1535	.3341	.6486
ZPREQ5	.0000	53.5836	.0506	.2364	.6597
ZPREQ6	.0000	48.8147	.3943	.4324	.6214
ZPREQ7	.0000	49.8272	.3186	.4720	.6301
ZPREQ8	.0000	51.1992	.2184	.3417	.6414
ZPREQ9	.0000	48.9286	.3857	.4061	.6224
ZPREQ10	.0000	51.0822	.2268	.4553	.6405
ZPREQ11	.0000	49.5527	.3389	.2752	.6278
ZPREQ12	.0000	52.2707	.1420	.3092	.6499
ZPREQ13	.0000	50.5315	.2668	.3532	.6360
ZPREQ14	.0000	53.3688	.0654	.2118	.6582
ZPREQ15	.0000	51.0545	.2288	.3764	.6403
ZPREQ16	.0000	52.5277	.1239	.3439	.6518
ZPREQ17	.0000	50.7563	.2504	.3751	.6379
ZPREQ18	.0000	52.6623	.1145	.2694	.6529
ZPREQ19	.0000	49.9428	.3100	.3264	.6311
ZPREQ20	.0000	48.0395	.4534	.5463	.6144
ZPREQ21	.0000	51.4965	.1970	.3491	.6438

Figure 7. SPSS output for Item analyses (item total statistics) of standardized pretest items

However with any test construction, one needs to be careful of deleting any items which could result in less than desired content and construct validity. While constructing the tests, I have used the table of specifications to achieve construct validity. Here instead

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of deleting the items the reliability analysis suggested and since these items were different for both pre and posttests, I have carried out factor analysis to see if the items work for a similar cause overall.

The dimensionality of the 21 items of pre and posttests were analyzed using maximum likelihood factor analysis. These criteria were used to determine the numbers of factors to rotate; primary hypothesis that the measure was unidimensional, scree plot, and the interpretability of the factor solution. The scree plot (see Figure 8) indicated three factors to be rotated due to the first steepness in the plot. Scree plots are the generally agreed references to decide the number of factors in a test. Factor analysis itself gives a set of factors as well, however sometimes factors from factor analysis can be too much or complicated a little, that is when the scree plot is used. The first number of dots before the steepness begin in the plot gives the possible number of factors for the factor analysis rotation. The factors are identified after the scree plot and the rotated factor analysis with the number of factors identified from scree plot.Then a content analysis was carried from the test items to see if they categorize into some meaningful categories.

Consequently, three factors were rotated using a varimax rotation procedure. The rotated solution yielded three interpretable factors; conceptual understanding of discriminants, conceptual understanding of graphing, and conceptual understanding of the relations between concepts like discriminant, coefficients, vertex, min/max etc. Posttest items gave almost similar results but with slight changes in the list of the items for each factor.

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	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Squared Multiple Correlation	Alpha if Item Deleted
ZPOSTQ1	.0000	54.8584	.4203	.5315	.6689
ZPOSTQ2	.0000	58.4562	.1718	.3395	.6925
ZPOSTQ3	.0000	59.9279	.0747	.2434	.7013
ZPOSTQ4	.0000	57.0355	.2680	.3490	.6835
ZPOSTQ5	.0000	57.7731	.2178	.3331	.6882
ZPOSTQ6	.0000	60.1116	.0627	.3384	.7024
ZPOSTQ7	.0000	58.5140	.1680	.4589	.6928
ZPOSTQ8	.0000	61.1143	-.0019	.1898	.7082
ZPOSTQ9	.0000	56.4313	.3097	.3336	.6796
ZPOSTQ10	.0000	56.8101	.2835	.3500	.6821
ZPOSTQ11	.0000	56.7309	.2890	.3791	.6815
ZPOSTQ12	.0000	58.9557	.1386	.3799	.6955
ZPOSTQ13	.0000	54.1313	.4725	.5285	.6637
ZPOSTQ14	.0000	58.1901	.1897	.4468	.6908
ZPOSTQ15	.0000	54.4689	.4482	.5074	.6661
ZPOSTQ16	.0000	56.7340	.2888	.3751	.6816
ZPOSTQ17	.0000	55.9549	.3428	.4539	.6764
ZPOSTQ18	.0000	57.2307	.2547	.5107	.6848
ZPOSTQ19	.0000	54.5684	.4410	.6045	.6668
ZPOSTQ20	.0000	53.2883	.5340	.5783	.6576
ZPOSTQ21	.0000	59.3090	.1152	.3808	.6977

Figure 8. SPSS output for Item analyses (item total statistics) of standardized posttest items

Rotated factor analysis (Figure 10) gave 3 factors of which had the following items in it. The items from Figure 5 with the highest loadings were selected to decide which items belongs to which factor. Results indicated that;

Factor 1; 5,6,7,8,9,11,19,20,21

Factor 2; 1,10,15,16,18

Factor 3; 2,3,4,12,13,14,17

A through analysis of the items from pretest gave the indicated the following clusters of conceptual understanding

Factor 1: graphing and forms of quadratic functions

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Factor2: Principles relating following quadratic concepts(vertex, axis of symmetry)

Factor 3: x-intercepts, solutions of a quadratic equation

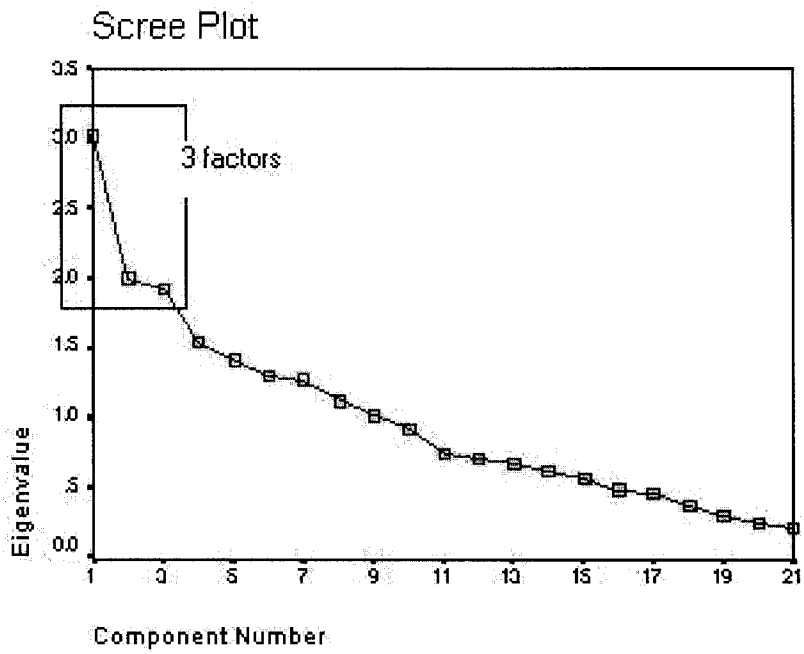


Figure 9. Scree plot of the eigenvalues for pretest factor analysis

Hence, the possible deletion of items would cause this balance to loose, the deletion of the items were disregarded. Rather these items were considered where the students were having problems most, hence they have been kept for further analysis. APPENDIX K includes inter-item correlations for pre and posttests. s

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	Initial eigenvalues			Extraction sums of squared loadings			Rotation sums of Squared loadings		
Factor	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %
1	3.03	14.427	14.427	2.318	11.038	11.038	2.001	9.531	9.531
2	2.007	9.555	23.982	1.242	5.916	16.954	1.423	6.775	16.306
3	1.932	9.199	33.181	1.212	5.769	22.723	1.348	6.417	22.723
4	1.55	7.38	40.561						
5	1.413	6.727	40.561						
6	1.3	6.189	47.288						
7	1.279	6.092	53.478						
8	1.123	5.347	59.57						
9	1.021	4.862	64.917						
10	0.911	4.338	69.779						
11	0.741	3.527	74.116						
12	0.713	3.393	77.643						
13	0.675	3.215	81.036						
14	0.619	2.948	84.251						
15	0.57	2.712	87.199						
16	0.489	2.328	92.239						
17	0.46	2.191	94.43						
18	0.379	1.803	96.233						
19	0.309	1.471	97.704						
20	0.263	1.253	98.957						
21	0.219	1.043	100						

Figure 10. SPSS output for rotated factor analysis for 3 factors with pretest

To come up with above factors was interesting since, there was also some other part of the data which was indicating a topical lack of understanding throughout the activity sheets they were provided. They were given empty (see appendix for activity sheets) cells to fill out on the characteristics of each quadratic equation, which they saw with the applet at hand. For 1 st period students this applet was with 2 -mode representations and for 6^{th} period students, it was with 3 -mode representations. The format of the table they filled was suitable for an analysis of lack of understandings with some concepts and it turned out to give the following information. Each row and column was analyzed for common problems and then the rows and columns with higher number of problems than 3 was identified and noted.

Table 10

Activity sheet table for lack of understanding with quadratic concepts

concept		1st period (n)	6th period (n)
discriminant	T1	7	6
	T2	11	11
x-intercepts	T1	9	10
	T2	8	11
y-intercept	T1	3	6
	T2	2	6
min/max	T1	2	0
	T2	1	6
vertex	T1	5	6
	T2	1	7
relations	T1	4	2
	T2	5	1
equations	T1	1	8
	T2	0	11
(a,b) & (b,a)	T1	1	5
	T2	0	0

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The three factors from pretest, reflected the almost the same problems with the problems with activity sheet answers (Table 10). Of course, pretest was a test hence, students needed to not to look anything as an aid or do not get any help even from their teacher, but with activity sheets, they could get help from their teacher, their peers, their notebooks, and foremost the from the applet. The looked carefully, one could see the pattern of the same focus of problems from same teachers and the overall problems with the concepts of discriminant and the x-intercepts or solutions of a quadratic function. Hence, even though the three factor rotation revealed three items to be deleted, I proceeded with them since, the seemed to have some valuable information on the lack of understanding and also, I did not want to jeopardize the equal loads of each table of specification cell.

In addition, the observations with the set of equations did not give as much as data as intended. However, the words students used shows parallel instances of what they said in interviews. But also, students of Bork made interesting comments like "looks like a reflection".

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Factor Matrix			
	Factor		
	1	2	3
Zscore(PREQ1)	0.23	0.331	-1.72E-02
Zscore(PREQ2)	0.399	0.295	0.327
Zscore(PREQ3)	1.47E-02	-0.126	0.324
Zscore(PREQ4)	0.139	0.114	0.497
Zscore(PREQ5)	9.99E-02	8.64E-03	-0.304
Zscore(PREQ6)	0.415	6.78E-02	-3.52E-02
Zscore(PREQ7)	0.482	7.45E-03	7.30E-02
Zscore(PREQ8)	0.356	-0.263	-0.256
Zscore(PREQ9)	0.453	-3.32E-02	-0.306
Zscore(PREQ10)	0.266	0.358	-2.58E-02
Zscore(PREQ11)	0.41	-9.99E-02	-9.85E-02
Zscore(PREQ12)	0.25	-0.113	0.137
Zscore(PREQ13)	0.345	-0.149	0.426
Zscore(PREQ14)	4.02E-02	5.74E-03	9.53E-02
Zscore(PREQ15)	0.252	0.593	-0.15
Zscore(PREQ16)	9.61E-02	0.29	-0.292
Zscore(PREQ17)	0.316	4.72E-02	0.328
Zscore(PREQ18)	6.84E-02	0.342	-7.21E-02
Zscore(PREQ19)	0.399	7.36E-02	-0.137
Zscore(PREQ20)	0.67	-0.237	-2.40E-02
Zscore(PREQ21)	0.363	-0.388	-0.185
Extraction Method: Maximum Likelihood.			
a 3 factors extracted. 7 iterations re			

Figure 11. The rotated factor matrix from the factor analysis

Effects of modes of mathematical representations, van Hiele levels, the teacher

To answer the research questions 1-3, the effect of van Hiele levels, the effect of different number of representations in the mathlets and the combined effect, I used ANCOVA performed with SPSS statistics software. Because van Hiele assumes that students' van Hiele levels would be affected by the quality of instruction, and because this would be directly related to the teachers' instructional decisions, I tried to factor out the teacher effect by including the teachers in the study analysis. To control for general ability and achievement in mathematics, the students' previous-year grade was used as a covariate. The sample for this study met the homogeneity requirements for performing

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ANCOVA. The homogeneity test results can be seen at the APPENDIX L. ANCOVA results are indicated with the tables and figures that are positioned in the rest of this chapter. These results indicated that the only significant interaction was between the number of different modes of representation in a mathlet and the van Hiele levels when students' last year mathematics grade is controlled. Since, there was teacher included in the model, and nothing came out significant of this variable, we can conclude that the teacher effect was controllable.

ANCOVA results indicated that only the interaction between v3 (modes of treatment; 2 mode vs. 3 -mode representations in an ID) and van Hiele levels of the student (low, inbetween, high) is significant at $F(3, 41)=3.076$, at $p= .038$, and the partial $\eta^2= .18$ (effect size) suggests a strong relationship between number of modes of representations in an ID vs. van Hiele level of the students controlling for last year's math achievement and teacher (Figure 12).

Since, the levels of the two variables cause 6 groups to identify which ones are producing the significant difference SPSS contrast procedure did not work. Instead, we have tried to see which groups made the difference by plotting the interaction in a couple of ways. Here v3 stands for 2-3 -mode representations for 1 and 6 respectively, and First teacher is Abernathy and Second Teacher is Bork.

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Tests of Between-Subjects Effects
Dependent Variable: Zscore(GAIN)

Source	Type III Sum	df	Mean Square	F	Sig.	Eta Squared
Intercept	0.007	1	0.007	0.044	0.836	0.003
ZMATH_AC	2.054	1	2.054	2.476	0.123	0.057
TEACHER	0.178	1	0.178	0.626	0.434	0.018
V3	0.004	1	0.004	0.008	0.933	0.001
LEVELS	0.226	3	0.075	0.166	0.912	0.177
TEACHER * V3	0.035	1	0.035	0.426	0.517	0.010
TEACHER * LEVELS	0.238	3	0.079	0.602	0.617	0.042
V3 * LEVELS	1.503	3	0.501	3.076	0.038	0.182
TEACHER * V3 * LEVEL	0.002	2	0.001	0.001	0.999	0.000

1. TEACHER
Dependent Variable: Zscore(GAIN)

TEACHER	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Bitley	0.111	0.237	-0.367	0.588
Fisher	-0.078	0.218	-0.519	0.363

a Evaluated at covariates appeared in the model: Zscore(MATH_ACH) = 2.83E-02.

2. V3
Dependent Variable: Zscore(GAIN)

V3	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	0.043	0.199	-0.359	0.444
6	0.000	0.249	-0.503	0.503

a Evaluated at covariates appeared in the model: Zscore(MATH_ACH) = 2.829230E-02.

3. TEACHER * V3
Dependent Variable: Zscore(GAIN)

TEACHER	V3	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
Bitley	1	0.141	0.305	-0.476	0.757
	6	0.081	0.357	-0.639	0.801
Fisher	1	-0.055	0.267	-0.595	0.484
	6	-0.108	0.345	-0.805	0.589

a Evaluated at covariates appeared in the model: Zscore(MATH_ACH) = 2.83E-02.

Figure 12. Results of the ANCOVA

Figure below illustrates clearly where the groups of interaction differ. For levels low and high, for both treatments there is an increase in the mean gain scores whereas, for inbetween van Hiele level students the mean gain drops for a huge amount and even from a positive gain to the negative mean gain score.

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Please see the APPENDIX M for a table of the values of the means of the raw pre and posttest scores with respect to the modes of representations and the Van Hiele levels as high, low or inbetween. Please also note that the raw scores have a skewed distribution that is why the standardized scores are used in the above ANCOVA analysis procedure. In that table, the arrows indicate if there was a increase or not, and if there was a decrease. The decrease of the inbetween students from 2 -mode representations intervention to the 3 -mode representation embedded mathlets is very interesting result of this study.

To further the analysis, I have plotted the same plot but for each teacher. As expected from the ANCOVA results, teacher change does not have an effect on the impact of the interaction of v3(mode of treatment) vs. van Hiele levels of students. Even though one teacher's classes slightly lower means than the

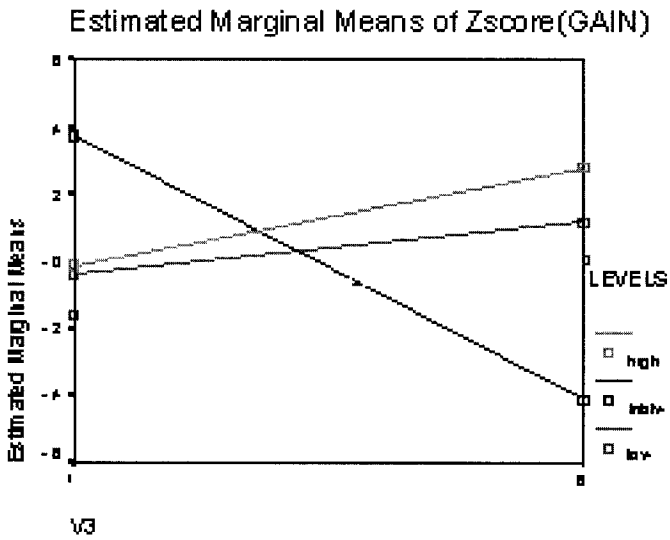


Figure 13. Interaction plot of v3 vs levels

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other, we can state that only for inbetween students the effect of the number of modes of representations reverses and indicates that their mean gain scores gets significantly lower with the 3 -mode representations embedded Interactive Diagrams then 2 mode embedded ones (see figures 14, 15, 16) .

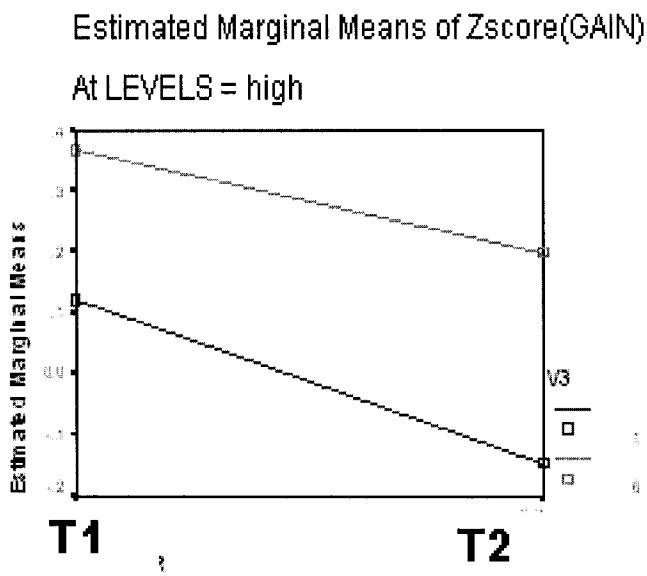


Figure 14. Interaction plot of V3 vs. teacher for high van Hiele students

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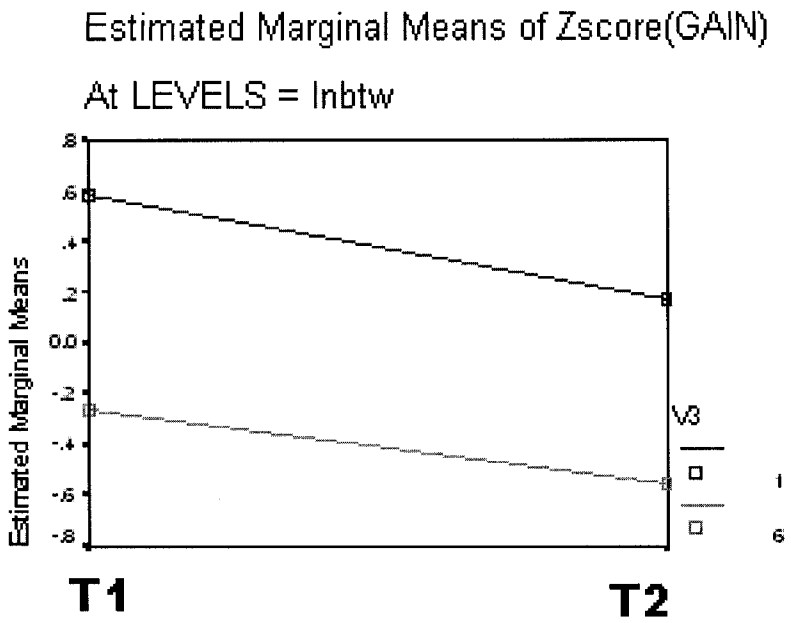


Figure 15. Interaction plot of v3 vs. teachers for Inbetween van Hiele students

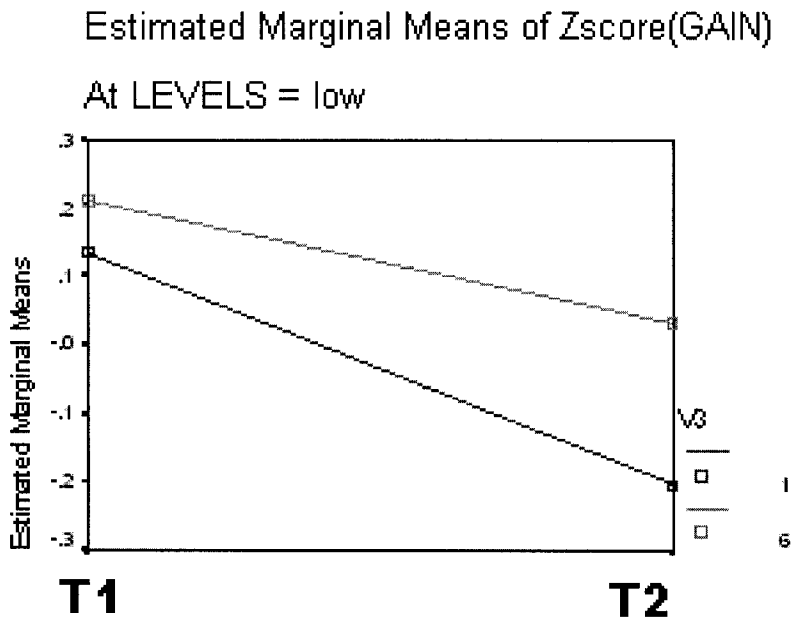


Figure 16. Interaction plot of v3 vs. teachers for low van Hieles

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To make sense of the diagrams above, students' gain scores were computed as the difference between posttest and the pretest score. For each teacher, the box plots at Figure 18 shows mean gain for teacher 1 (4.29 with 9.48 standard deviation) being higher than mean gain for teacher 2 (-.7593 with a 9.22 standard deviation) and even negative gain for teacher 2 (see Table 10).

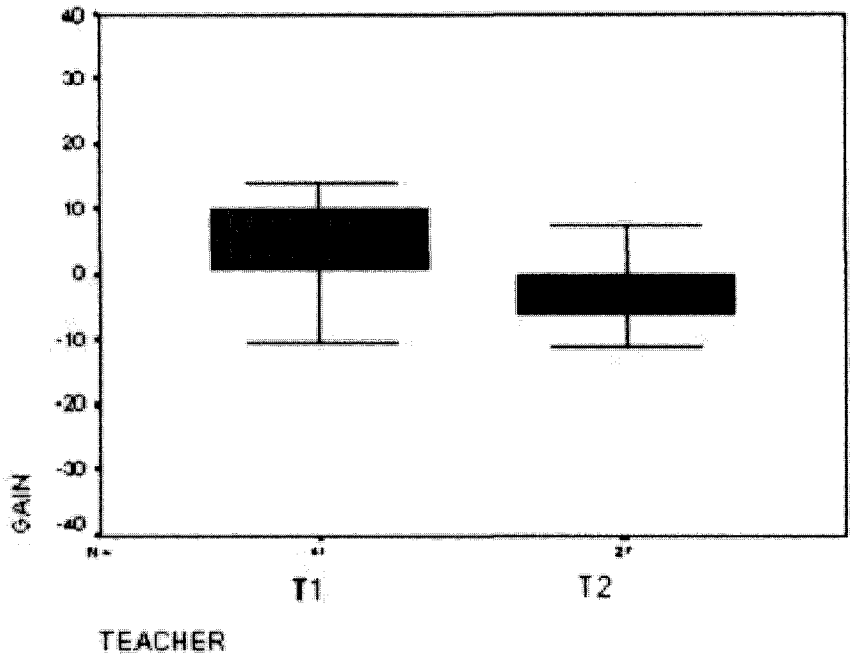


Figure 17. Box plots for mean gain scores for the students of each teacher

Table 11.

Means and standard deviations for Gain score of each teacher's classes

	Abernathy		Bork	
	M	**SD**	**M**	**SD**
Gain	**4.2927**	**9.4771**	**-.7593**	**9.2231**

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Figure 17 and Table 11 indicates that Inbetween students (students with transition from lower levels to higher levels of van Hiele) had negative mean gain whereas low and high van Hiele students had positive mean gain score. We will analyze the results for significance hence, right now it is better to give the results descriptively. Low students had lower mean gain (1.1132, with 8.18 standard deviation) than high van Hiele students(4.4533 with 9.09 standard deviation).

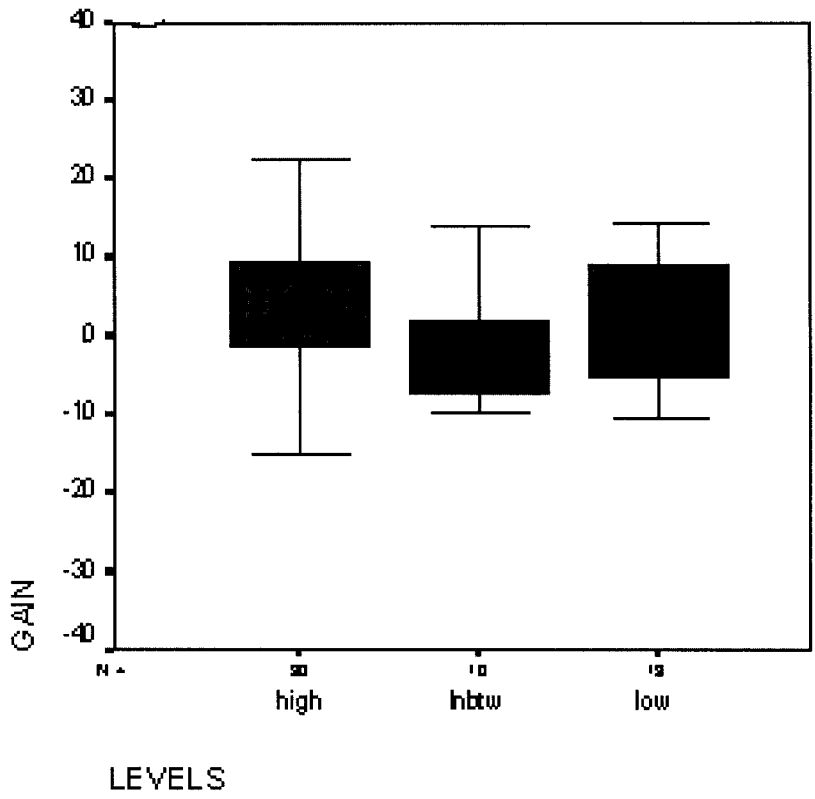


Figure 18. Boxplots for gain scores with respect to the van Hiele levels of students

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Table 12

Means and standard deviations for gain scores of each van Hiele level students

	Low		Inbetween		High	
	M	SD	M	SD	M	SD
Gain	1.1132	8.1793	-1.5500	14.2837	4.4533	9.0901

A two-way contingency table analysis was conducted to evaluate whether students with different van Hiele levels (low, inbetween and high) were differentiated into different modes of intervention (2 mode and 3 -mode representations) even. Insignificant Pearson χ^2 (3, N= 81) = .79 , Cramer's V=0.099 results indicated that the students were evenly distributed to the two intervention modes. In other words, the two groups were equally represented regarding the van Hiele levels of the students.

Table 13.

The bivariate correlations between van Hiele levels and last year's math achievement

		DUMMYLEV	MATH_ACH
DUMMYLEV	Pearson Correlation	1	0.428 **
	Sig. (2 tailed)		0.002
	N	61	52
MATH_ACH	Pearson Correlation	0.428 **	1
	Sig. (2 tailed)	0.002	
	N	52	66
	** Correlation is significant at the 0.01 level (2- tailed)		

A dummy variable from the qualitative variable levels (low, inbetween and high) was recoded to produce the new variable dummylev (0=low, 1=inbetween, and 2=high). This variable was used to compute correlation coefficient. Table 13 indicates that there is

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a significant relationship (at p=.001 level) between van Hiele levels and last year's math achievement.

4.1.2.Qualitative data analysis and results

The three questions qualitative part of the study aimed to investigate were varying van Hiele levels student's experiences with mathlets, the role of structure and insight for the inbetween students while working on translations and the translations between same mode and different -mode representations.

Students' experiences with the mathlet led to some categories as following. Inbetween students' description of the changes made use of an array of informal terminology; going down.,wider, moving up and down, etc. Low students were using mostly similar words as of inbetween students; "flip over", "diminished" , High students used the same words but not in isolation, they preferred to make whole sentences besides they have made use of the mathematical terminology associated with them, like "vertex", "x-axis", "function", etc. They used the facts of coefficients being negative or positive, "graph changes quadrants", "a reflection of it"etc.". The ignorance of some parts of the mathlet was most observed among the inbetween students. The symbolic part appeared to be the most ignored representation. While the problems with the conceptual understanding of the mathematical equations got less with the higher van Hiele levels, their confidence levels seemed to have an increase. Inbetween students tend to have more problems due to part whole relationships within the mathlet. With the low van Hiele

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students part to whole complexity was observed at a slightly lower level as well. While low students saw only parts of the graph and graph itself as a whole, high students were able to see the difference and how they were connected. The focus of the experience with mathlets for students seemed varies; inbetween students were the ones with most diverse preferences of the representation types whereas high students preferred mostly graphical representation, while low students did not even mention about their preferences. Problems due to unfamiliar representation came surprisingly from all but more from the high van Hiele students.

High students were the ones who saw the relationships and who tried to rephrase them;

"as you slide tabular values up and down, the formulas change upwards, and then the graph changes accordingly",
"the effect of change in c affects the place of vertex."
""how the parabola change with fluctuating a.b.c"
"moving c adjusts graph on the y axis"
"when you change the settings, graph changes and causing this part to change (symbolic)"
"Equation defines the parabola"

Low van Hiele students used the following sentences to describe the relationships happening inside the mathlet.

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"formula part is just taking the problem and dissecting more"

"it adjusts into that"

"they correspond with each other"

"as you can see the stuff, you can set numbers and see the results"

Inbetween students' arguments were lacking depth and some of them even di not want to answer that question. They stated things like "I do not know, program does it, etc."

"as the parabola moves, the numbers (referring to the slides) go up and down"

"the equations are changing, the set numbers are changing as well"

They were able to give explanations mathematically for the things happening. And sometimes they used the similar informal words as low and inbetween students. Hence, even though different students' experiences were different still, it was possible to see some common elements within each level of van Hieles.

Inbetween students stated that If they knew how it was functioning before, they would have done better. As they were reluctant to use mathlets, they were also reluctant to see the changes according to the translations. This may be explained by field dependence, or the preference of one sort of representations/structures for inbetween students. In other words, inbetween students may be more field dependent and this may be causing them to have problems of not being able break the mathlet into meaningful parts and work from there, or to differentiate the relevant material inside the mathlet from

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the not relevant. Low and high van Hiele students may be thought as field independent since, they seem to have no problems within either the parts of mathlet (for the low van Hiele students) or the whole of the mathlet (as for the high van Hiele students).

Most students mentioned the problem of not being familiar with the mathlets and not having enough time to play or experiment with them. If we think of time as an important base piece of the development of insight, unfortunately these students did not have that chance. However, they certainly showed some early indication/tendency of identification of the characteristics of mathlets as of being able to drag red dots, expecting some changes with some actions, or realizing a bigger frame than the ones held in classroom while studying graphical systems (since they were able to see the possibilities of a,b,c,s and how big these numbers can get and how big and small the parabolas can be if zoomed into and out of a frame, these students certainly were able to get some nuances of insight or the early buds of insight with these mathlets.
Regarding the answer to the lat qualitative question, inbetween students' translational abilities seemed better with the translations of same -mode representations. Their skills in visualization helped them to make sense of the graphical representation especially by wordings like "moved up and down", "shifted", " bigger, wider, positive, negative, etc." For the translations between different -mode representations as of making sense of the how the tabular parts affects graphical and symbolic part, they showed no indication of an idea.

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CHAPTER V: DISCUSSION AND CONCLUSION

Summary

This instructional study has been an investigation of the effects of differential Van Hiele level students' mathematical understanding with two vs three- mode representations embedded mathlets. Here the three- mode of representations appeared to be tabular, graphical and symbolic, and two mode representation mathlet included only two of these; tabular and graphical.

The hypothesis of the study is based on psychological theories related to representations and mental processes with representations, translations among different modes of representations, and the roles of interactivity, intuition and insight in the development of understanding. Because the aforementioned constructs are so closely related to van Hieles' levels of geometric reasoning, I hypothesized an interaction between students' levels of geometric reasoning and student's facility in using interactive diagrams as learning tools. My ultimate indicator of facility is the students' improvement on a test of mathematical understanding. Because of their availability, ease of use and specialty in demonstrating the translations in a quicker way, I have chosen to use mathlets (mathematical JAVA^{TM} applets) as the interactive diagrams.

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I have assumed the idea that the nature of students' interactions with mathlets will increase the rate at which students develop understanding of mathematics concepts and the quality of their understanding. I investigated the interactions among a students' van Hiele level (high, inbetween, and low), the number of representations in a mathlet (two vs. three) and students' gain scores on a quadratics test after the use of the mathlets

The mathematics topic I chose to study is quadratics. The choice of quadratics was based on the topic on the curriculum at the time the study was being carried out. The subjects were the 9^{th} and 10^{th} grade students of two mathematics teachers who were teaching MATH AB and MATH ABH classrooms of an innercity school district.

The results indicated that low and high van Hiele level students' who used a mathlet with three modes of representation achieved a higher gain score than low and high level van Hiele students who used a mathlet with only two modes of representation. For the inbetween van Hiele level students, the reverse was the case, in that the group with only two modes of representation had a higher gain than the group with three modes of representation. In fact, the sdtuents who used a mathlet that had three modes of representation had a negative gain score.

The inbetween van Hiele level students' preference (inferred from higher gain scores) for the 2 mode representation mathlets may be explained by their perplexing status with their own place in learning as well as the complexity of the mathlet and the

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possibility of the counter intuitive nature of at least 2 translations within a 3- mode representation mathlet.

Out of three van Hiele level students, inbetween students outperformed low and high students for 2 mode representation mathlets, whereas the low and high van Hieles outperformed inbetween students for the mathlets with 3 mode representations. van Hiele defines inbetween students as the students who can answer some of the high level questions but can not answer some of the low level questions. Regarding the van hiele specifications for levels, these students are actually the ones who fluctuate between abstract- concrete & formal-informal thinking dualities. My study confirmed the previous literature on van Hiele Theory regarding the distribution of the levels and the students' level of learning specified by that level. On one hand, the difficulty of the pre and posttests enabled analysis of students' behavior due to not having a possible ceiling effect. On the other hand, the low range and the skewed distributions restricted the depth of analysis.

The study is also an investigation of the role of the cognitive processes intuition/insight on students' making sense with mathlets. Qualitative analysis of the interviews with students indicated that less confident inbetween students tend to use wording with less mathematical terminology than the high and low van hiele students. They tend to ignore the representations and translations that they are not familiar with. They focus on various representation modes but they are highly reluctant to make translations to the other modes.

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Overall inbetween students seemed to have the most problems with the translations between representations while they favor more of the same mode representation translations. A gear metaphor (see Figure 20) is helpful in explaining the importance of the structure of the representation as well as the structure of the mathlet as van Hiele suggests. It points to the fact that in a representation system structure (in our case of mathlets) the translations, which require thinking in the counter intuitive direction, may impede/hinder learning. In multiple representation systems the counter intuitive thinking is indispensable because each wheel or gear is used to make sense of the whole structure of the representation. Here the counter intuitive direction refers to the backward thinking required from a translation of which the reversed more intuitive way of thinking would be forged inside a normal classroom setting or in a textbook (eg. instead of a translation from a tabular to graphical, a translation from a graphical to a tabular representation).

Discussion and conclusion

I have mentioned 6 questions in the beginning of the study. The first three have been answered up to a degree by the ANCOVA results as I mentioned at the results chapter. The first question was related to the effect of the number of different modes of representations in an Interactive Diagram on the students' mathematics performance on Quadratics test. My research techniques could not find a significant relationship. There seems to be a slight difference between pre and posttests after the intervention, but this difference is not significant. Most of the students who were interviewed indicated that

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whatever we did would not help them. However, they also mentioned that if they had been informed as to how the mathlets functioned prior to the actual use of the mathlet and also if they were more capable of the topic, their learning gains would have been higher. Students' self reports are always problematic, however they do reveal something about the students' preferences. Interviews informed us that students prefer to play/experiment with the mathlet besides classroom activities. With some students, I even got the idea that they do not like to see the Internet (their safe heaven) to be included inside the classroom that much. That was an interesting information to get from them.

The second question focused on the effect of van Hiele level of a student on the performance after they worked with IDs. The ANCOVA results gave no indication of a significant effect however, the interview data we had suggested some degree of relationship. Students within each van Hiele level used quite similar mathematical terminology. Low van Hiele students have a tendency to use trivial wording for understanding how coefficients work within mathlets. High van Hiele level students have a tendency to highlight the relationships while making sense of the applet. Inbetween students on the other hand, indicated a tendency to getting confused quickly and frustrated with the overall complexity of the mathlets. Inbetween students are the ones who fluctuate between formal and informal thinking and as much as concrete vs abstract thinking according to van Hiele Theory. The levels are consecutive and these students are thought to be intransition from informal to formal and from concrete to abstract thinking. These students can function as formal/abstract thinkers in some parts of the system

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whereas they function differently in the others. This fact also explains their situation with the part to whole problems as explained with the gear metaphor.

The third question was answered well by our ANCOVA statistic and also by the degree of the information possible to get from interviews. There is a significant combined effect of the number of different modes of representations vs van Hiele levels of the students. Here one needs to be careful while generalizing the results, since the significance p was at level $p<.05$ and in any statistical test, there is an error term, and a possibility of seeing a relation when there is not (type II error). Meanwhile, our effect size (.18) eases our concerns since it is in the large effect size range (this effects size is the degree of association between the two variables, hence should not be confused with the other effects sizes). Furthermore, we saw that inbetween students had negative effects on their performance with IDs in which 3 -mode representations are embedded. Interviews also indicate that some students prefer some representations over others. The students stated that where they focused on the frame of the ID and inferences were made on which representations they ignore, or so. This result is a little bit surprising. I was thinking that inbetween students should be affected most but not in a negative way. Hence, not the degree of effect but the way they are affected was surprising. However, in the literature review, van Hiele referred to lack of insight for the students having problems with mathematical thinking. Also, he mentioned that one structure is chosen for the inviting character and familiarity with the structure. If that structure does not lead to the solution, then the student would look for another structure. Here, inbetween students are the ones who still did not formalize the relationships between concepts, who have not

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started to use abstraction in understanding, and with whom the characteristics of the quadratic concepts still need some more understanding. According to van Hiele levels structuralization, they are not supposed to feel confident about how they think. They try to make sense of the one representation mode they know. However, making conjectures based on the principles or relationships between concepts is a higher van Hiele level requirement hence they are supposedly incapable of yet. If we think about the interviews, they prefer some representations to others. But why is this not the case with low van Hiele students. They have preferences. Their lack of insight may be worse than inbetween students. In spite of all these, they know what they know, and they are pretty confident with it, hence they use it. In the interviews, some low van Hiele students used and preferred one mode of representation over others repeatedly even after some warning with the questions.

Familiarity is an interesting concept when dealing with the type or mode of representations. Regarding what we know on the structural characteristics of representations, IDs are both a different representation, and also a combined piece of representations. A student either sees the first or the latter. In case of a inbetween student this familiarity may take the form of a graphical representation, since it is the representation which uses universal conventions. One interviewee who came from Germany was a good example of this. Use of an interesting terminology did not hinder her playing and eventually her understanding with it. However, she was not confident from the beginning. For some inbetween students, familiar representation became the tabular one even though it was not a conventional one.

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The fourth question was on the experience of students with different van Hiele levels. In the above paragraph, I tried to explain it as well. Here, the term "experience" takes my foremost attention since IDs bring a new form of mathematical experience to the students. They were mathematicians for a few minutes. Once in a while, they get out of the structure of the interview, and they carried out their own investigation in quadratics. I have heard sayings like ' Oh, not only a,b or ,c but they all does", or "drag vertex, only vertex moves". The way they experience mathematics led them to carry on mathematical experiments. Even low van Hiele students did that. They learned something even though it was not probably as technical or mathematical as high van Hiele students or the way we learn at class. All of them stated that they use the Internet one way or another, but only a few of them argued that this could be helpful and they could search for more if they are given a chance. For low van Hiele students, their initial start level is less mathematical than high van Hiele students, but they still try. And they come up with similar conclusions as the high van Hiele students but with less mathematically correct terminology. Inbetween students on the other hand, seemed to be fluctuating as much as the a,b,c values were in the slider applet. They are in a transition as van Hiele suggests. And this transition both pushes them forward to learn more, to investigate more, and holds them back too. This complexity added with the complexity of the 3 -mode representations may be the problem they are facing within our study.

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The fifth question, stressed the need for a further understanding of the insight a student uses, to make sense of the structure of the representations and the translations between them.

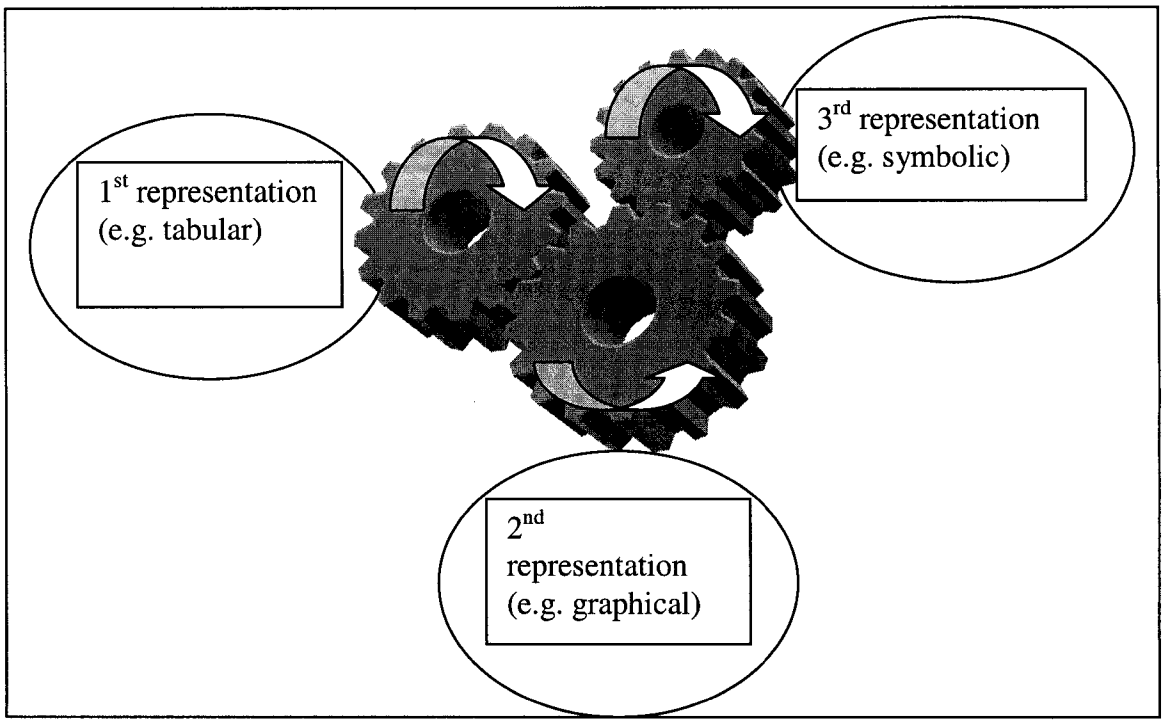


Figure 19. The gear metaphor for the structure of structures for intuitive thinking and for representations within a mathlet with 3-mode representations

My study also gave valuable insight into the role of insight in mathematical understanding with mathlets. I want to explain my thinking with a metaphor I designed above (Figure 19). The direction of the arrows indicate the direction of the translations. Please note that not all three wheels correspond but they eventually correspond due to the wheel inbetween. I do believe that representation systems in a mathlet makes use of this idea, i.e. any mathlets is a representation system of some number of representations

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which are tied to each other either via indirect links or via direct links. The way student thinks with them may be more complicated, but the results of this study leads to the other way Students needed to switch their thinking backwards whenever there was a problem with one mode of representation. With two-mode representations, this switch is only once, since if we think of each mode of representation as a gear then, one forces to think in the counter intuitive way while the gear forces in intuitive way. However, 3 mode representations, this switches doubles, with one gear would work as counter intuitive and two working in intuitive way. While low van Hiele students can function with only one familiar and convenient mode of representation as graphical mode in our case, inbetween students lacks this advantage since they are in the process of building an abstract understanding of the mathematical topic behind, they are puzzled with complex thinking processes, and their functioning with formal concepts and relationships is not fully developed yet as van Hiele (1986) explains.

I believe that we can think of three representations in an ID as three structures as van Hiele (1986) and Janvier (1987) suggests. In an ID, these three representations are required to work in a cooperative way. To be able to do this, sometimes they even go in a counter intuitive way as it may happen with some students' understanding. Even though the student may function well in the case of only one or two representations, some discrepancies may occur when 3 representations are working together. In a three-mode representation mathlet, at least two of the representations would go in an intuitive way, but only one in a counterintuitive. If there are only two representations the involved student would not have any problems because choosing only one of them would be easy

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to do, and non problematic. In the case of three representations, students may choose from two intuitive ones. If the student would have a preferable representation, then it would not be a problem, the student may go with that. If the student did not decide yet or is in the process of understanding or solving these two types of representations, then there would be a problem. And it may well be the case with inbetween students. The idea of being high and low van Hiele student with higher mathematical performance gains may be due to the different reasons. For high van Hiele students, it may be a capability of functioning with more than 2 mode representations, where as with low van Hiele students, this may be due to the intuitive, non-intuitive confusion. From the interviews, that some students came to a point in which they found the need to decide what the representations were representing, and sometimes they made slight mistakes. As, one or two students were mentioning about the wrong conclusions on the role of different coefficients of the standard form quadratic equations. Some students would need more equations to see the pattern. They tend to see the small patterns within small ranges and generalize it falsely to the bigger range. Time may be another concern here. There may be a frustration of inbetween students with the task because of its complexity and because of their willingness to experiment or do many trials. This is actually supported with students' own self report within interviews where they mentioned that they would do better if they had more chances to work with the applet.

The last research question focused on the differences between students with same mode and different mode translations. Some of the interview questions were designed to reveal the differences. Some questions asked for working backwards, from graphical to

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symbolic, and tabular. Other questions focused on the way one understands how dragging the vertex, and dragging any arbitrary point on the parabola results in changes in the other modes of representation. Again some students made false judgments on the dragging results. They tended to give their answers in the same terminology they used for the first part of the interviews for understanding how the coefficients (a,b,c) works within an quadratic equation to result in the changes associated within the parabola. At the point when the students realized that the previous terminology would not work to describethe dragging results, they changed to different explanations. Or they got confused with both their understanding of the context and the terminology.

Limitations

The subjects of the study were taken from two levels of the same course, Math AB and Math ABH students. Their grade levels were 10^{th} and 9^{th} grade respectively. Their age levels were also differentiated by 1 year, being the accelerated students being younger. Results reflect their situation; one needs to be careful while generalizing the results into whole population of 9^{th} and 10^{th} graders.

There were some students for which, I did not have the data of their number or, the number they gave did not match with what I had, hence some discrepancies occurred, hence I did lose some data.

For van Hiele levels identification, I tried to make my procedures parallel to the van Hiele procedures however, some decisions regarding the data had to be made. One

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of these decisions was while identifying the level of the student with respect to at least 3 corrects from previous level questions. The part if they could answer three from previous level correct but not all of them, was not clear with van Hiele procedure. In other words, a student whose answers were reflecting 3 from 3rd level, and 3 from 5th level, this student was identified as inbetween, but for a student whose answers were correct for only 3 from level 4 and was able to give again 3 correct answers from level 5, student's level was identified as 5th level.

The tests designed by the researcher could be revised, and they could be made a little easier since students and teachers mentioned that the test were more difficult than the students were ready for. First of all, students did not have enough time for figuring out the topic of quadratics, hence the research gave the results almost for first time learners of quadratics, and also quadratics is another topic in which students have relative difficulty of understanding why and how it works and conceptual understanding of the set of formulas. Hence, the results reflect that, too.

The study gave a good direction of where to proceed next. We need more understandings of the representations, how they work, why they work. But also, we need further understanding of the students' thinking. This interview protocol gave me some valuable insight into what to conclude but it lacked some valuable details as well. During the procedure, even though the teachers were very kind and understanding to devote at least 5 class hours for the project, time was still a factor. Students mentioned that they would function better with the applets if they knew the applet before, or if they

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had more time to spend on understanding the applet. Regarding the realities of a mathematics classroom, study made use of many hours than one could ever ask. Hence, the best case scenario may be to do the research with individual students outside of the classtime.

The sequencing of the use of mathlets may very well produced an unwanted effect, since some students mentioned that they would prefer to be familiar with the mathlets first. However, if we did the other way around, we still could not be sure if it was due to sequencing or not. Hence, it may be another future research idea.

Overall, I do believe that this study gives conclusive evidence of van Hiele as a factor for measuring students' performance with an Interactive Diagram. What is nice about this study is that, these IDs could be any IDS. They represent a huge range of 2 mode and 3 mode representation IDs. However, they do not represent IDs with complex behavior, or IDs with one complex representation in which it is hard to differentiate between the different modes of representations. Those kind of IDs require more investigation.

Recommendations for future research

When I started this research, I had simple intuitive feelings on the way IDs should work with students from different van Hiele levels within an ID. Some of them turned out

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right, some of them needed more investigation, and some of them turned out astonishingly wrong. The conclusions I made were all clouded with the literature review this study used. However, different perspectives make different explanations. I would like to see some future research on the same or intersecting topics with different perspectives. I think this could give us a better understanding of the all factors and their relative roles inside the process. Subsequent studies in this line of research should take into consideration of following:

1. A Much larger sample, use of same level classes if possible.
2. More schools if possible.
3. Much certain and effective procedure of getting permission slips.
4. The achievement tests could be revised for better reliability.
5. The replication of the study with a larger sample.
6. Use of different mathlets.
7. More time for students to spend with the mathlets
8. More opportunities for students to explore mathlets alone in their free time.
9. Studies that follow up students over time and individually with mathlets.
10. With a different sequencing of the mathlets (e.g. first being exposed to mathlets for a very long time on individual basis, and then investigating how the students think with those mathlets)

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APPENDICES

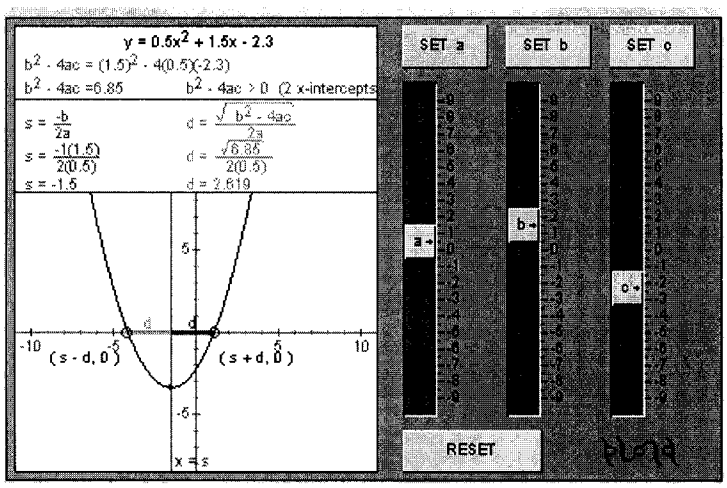
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APPENDIX A

Example web sites for mathlets

Even though there are many sites when somebody searches topic wise on Internet, some may be found under familiar organizations and names. For example, NCTM has a portal, which includes a set of Pre-K to 12th grade and varied topical interactive math java applets (see http://illuminations.nctm.org/mathlets/index.html). Students and teachers can use these applets stand-alone or they can incorporate the I-math investigations (tasks developed around these applets for mathematical understanding) into their mathematics instruction.

Fig.1. Quadratics Applet for $y = 0.5x^2+1.5\,x - 2.3$



Another site that is mentioned in the NCTM illuminations is website of Journal of Online Mathematics and its applications (JOMA). JOMA (see www.joma.org) enables developers to see the areas that needs the java applets to be developed as well as facilitates the searches of the instructors on specific topics. This online journal defines mathlets as "small, interactive, platform-independent tool for teaching math". Most of the applets on JOMA are peer reviewed. The criteria for reviews are based on three main areas other than the applets' essential characteristics as of contributing to the learning of mathematics and the correctness of the

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mathematics involved. These three areas are interface, content and technicality. The clarity and attractiveness of the visual appearance, the navigational facilities of the mathlet for student, and the necessity of the interactive features of the mathlet are considered as the interface characteristics. The user friendliness of the mathlet, clarity of the topic behind, level required from students, and the possible debugging are content wise decisions to take while choosing a mathlet. Technical questions mostly roll around the following issues; context, platform, plug-ins, cataloging, suitability for the classroom.

Still another site to mention should be the Interactive Mathematics Miscellany and Puzzles page developed and maintained by Dr. Alexander Bogomolny has been active since 1996 (see http://www.cut-the-knot.org/content.shtml). This evolving page is a source of not only some topical interactive activities but also a lot of challenging and intriguing puzzles and games. Any learners are welcome to the page for any reason; to be puzzled, to be dazzled, or to learn an old paradox of mathematics and how it is applied to the other disciplines or games.

Another web site, which includes various mathlets for a set of mathematics topics, is the web site developed by Tom Leathrum (see http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/). These are mostly college level introductory calculus or higher algebra level topics but they may still be used in conjunction with other web sites.

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The example applet above is taken with the permission from Alberta Learning. The mathlet is a unique example of standard form of quadratics and it is a multi-representational applet for grades 9and 10. Figure 1 shows the applet in its default screen, where only $y=x^2$ is graphed on a coordinate system. It demonstrates also the discriminant, axis of symmetry, how far the roots of the polynomial are from axis of symmetry and from each other. The student can change the slider values for a,b,c coefficients and can see the resultant changes in the accompanied graph. Figure 2 is an example of the graph for a=0.53, b=1.5, and c=-2.18. The graph of $y=0.53x^2 + 1.5\ x - 2.18$. The student can move along the slider of "c" and can see the shift in the graph upwards for a c greater than c= -2.18. Similar onscreen demonstrations are available for students to explore for all possibilities of c, b and a (for this applet, the ranges are given limitless, some others may differ). Please note that the variety of the mathlets are beyond guessing on the Internet, and one can find many other nice, creative, and useful mathlets for diverse classroom purposes.

Last but not least, all the other web sites that consist mathlets are good sources to investigate and build your own pool of interactive diagrams to use in your classroom. I included here only a restricted set of examples that are reliable and easy to find on the Internet. The most common characteristic of these web sites is their duration of being a mathlet source over Internet is more than three years at least. Technology is a little time consuming especially when something unexpected happens or when a little problem with your network disables your access to one of your favorite mathlets. The best way to deal with that is to be flexible, and have many options at once to start with. Giving the

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mathlets as "mathematical experiments" to students for investigation of mathematics can be another way of stress reduction. Besides, as with all the other mathematics education tools, individual differences affect students' impression and engagement into mathematics. Hence, acknowledging these differences right away and being prepared to different situations that may arise while students' learning may save time and energy of both you and the student.

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APPENDIX B

Permissions to use mathlets and van Hiele Test

Subject: request for permission to use the quadratic functions applet
From: "Lori Ristoff" <Lori.Ristoff@gov.ab.ca>
Date: Fri, March 14, 2003 6:33 pm
To: "'oc7815@csc.albany.edu'" <oc7815@csc.albany.edu>
Priority: Normal
Options: View Full Header | View Printable Version

Dear Ozlem,

Thank you for your e-mail permission request of March 12, 2003, to Susan Schroeder. Your request has been forwarded to me, since LearnAlberta.ca is a Learning Technologies Branch initiative.

Permission is hereby granted to you to reproduce the Quadratic Functions in General Form (The Discriminant)applet in your dissertation. I'm afraid we cannot, however, provide you with a modified version of the applet. The applet must carry the following acknowledgement: "Reproduced by permission of Alberta Learning. No further reproduction is allowed." This is a one-time-use-only permission. For our records, we ask that you provide us with a copy of your dissertation and your research proposal.

Thank you for your interest in Alberta Learning's resources. We wish you the best of luck with your studies.

Sincerely,

> Lori Ristoff
> Acting Editing and Copyright Coordinator
> Learning Technologies Branch
> Alberta Learning
> Box 4000, Barrhead, Alberta T7N 1P4
> Tel: (780) 674-8719
> Fax: (780) 674-6561
> E-mail: lori.ristoff@gov.ab.ca

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Subject: Re: Van Hiele test
From: "Zalman Usiskin" <z-usiskin@uchicago.edu>
Date: Tue, October 1, 2002 10:09 pm
To: "CEZIKTURK OZLEM" <oc7815@csc.albany.edu>
Priority: Normal
Options: View Full Header | View Printable Version | View as HTML

Dear Ozlem:

The report "Van Hiele Levels and Achievement in Secondary School Geometry" has not been available from us for over a decade but you should try to obtain a copy. It is 231 pages in length and available as ERIC reproduction service document no. ED 220 288. It includes quite a bit of information that may be useful to you, including the various ways in which van Hiele levels can be derived from scores on the test. (Merely having the test would not be enough.)

We are happy to give permission to copy the van Hiele test for use in your research, but we need

(1) a description of the research

(2) assurance that you will write for copyright purposes on each copy: "© 1980 by the University of Chicago. Reprinted with permission of the University of Chicago."

Thank you for the assurance that we will receive a copy of any write-up of the results you find in your study.

Best wishes for success in your work.

Zalman Usiskin
The University of Chicago

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UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

TO: Ozlem Cezikturk

FROM: Institutional Review Board

DATE: April 21, 2003

SUBJECT: **REVIEW OF PROTOCOL NUMBER: 03-038**
ENTITLED: **"Effect of Interactive Diagrams on Sec. Students' Understandings of Selected Mathematical Representations Based on Van Hiele Theory and Representations"**

At its meeting on February 25, 2003, the University at Albany Institutional Review Board (IRB) reviewed your above-referenced protocol and approved it pending the receipt of requested information. You have provided that information.

The IRB has reviewed this information, by its expedited review procedures, and found that it satisfied the Board's concerns. This constitutes **final approval** effective **April 21, 2003, through February 25, 2004.**

This approval is valid for one year only. You must request a continuation of the approval if the activity lasts more than one year. If approval for the above protocol lapses, all research must stop immediately until IRB review and final approval has been obtained.

If you have any questions regarding the review of your protocol, please do not hesitate to contact me.

Cheryl Savini

Cheryl Savini. CIP
Research Coordinator
On behalf of the IRB

Att: 1
Cc: Vicky L. Kouba

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APPENDIX C

The van Hiele Test

EDRS
ERIC Document Reproduction Service
1 800 443 3742

Answer Sheet

- 156 -

Test Number _____

VAN HIELE GEOMETRY TEST*

Directions

Do not open this test booklet until you are told to do so. . .

This test contains 25 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheet.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.

Wait until your teacher says that you may begin.

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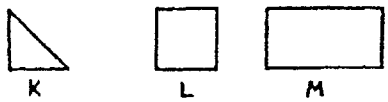
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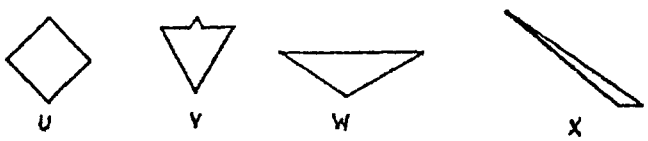
YAN HIELE GEOMETRY TEST

1. Which of these are squares?



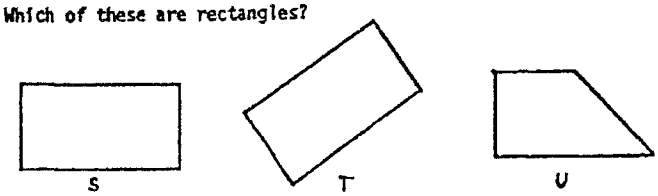
(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

2. Which of these are triangles?



(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3. Which of these are rectangles?



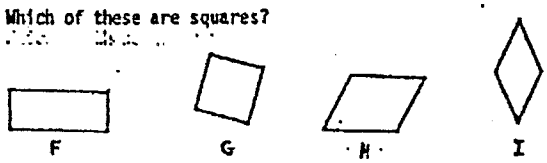
(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles.

uced by DynaEDRS

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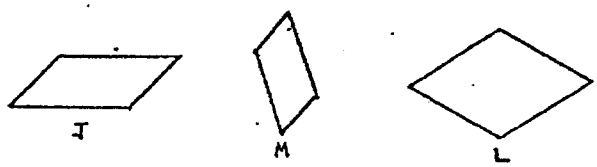
EDRS
ERIC Document Reproduction Service
1 800 443 3742

4. Which of these are squares?



(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares.

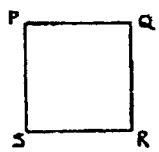
5. Which of these are parallelograms?



(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

6. PQRS is a square.

Which relationship is true in all squares?



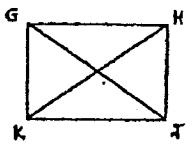
(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.

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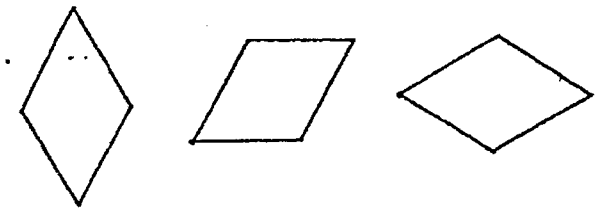
7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.



Which of (A)-(E) is not true in every rectangle?
(A) There are four right angles.
(B) There are four sides.
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.



Which of (A)-(D) is not true in every rhombus?
(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A)-(D) are true in every rhombus.

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9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.

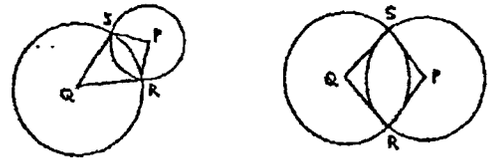
(B) One side must have twice the length of another side.

(C) There must be at least two angles with the same measure.

(D) The three angles must have the same measure.

(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is not always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will have at least two angles of equal measure.

(C) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

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Here are two statements.

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.

(B) If 1 is false, then 2 is true.

(C) 1 and 2 cannot both be true.

(D) 1 and 2 cannot both be false.

(E) None of (A)-(D) is correct.

12. Here are two statements.

Statement S: $\triangle ABC$ has three sides of the same length.
Statement T: In $\triangle ABC$, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.

(B) If S is true, then T is true.

(C) If T is true, then S is true.

(D) If S is false, then T is false.

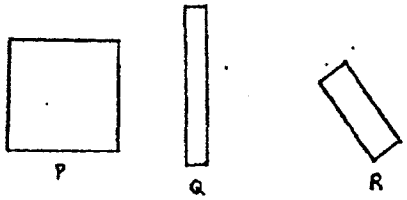
(E) None of (A)-(D) is correct.

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13. Which of these can be called rectangles?



(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

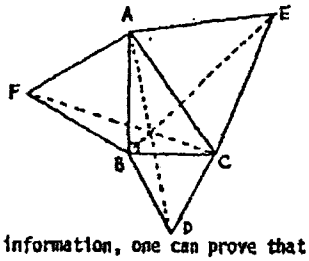
14. Which is true?
(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?
(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) none of (A)-(D)

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16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

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18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that II is true.

(B) To prove II is true, it is enough to prove that I is true.

(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.

(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

(E) None of (A)-(D) is correct.

19. In geometry:

(A) Every term can be defined and every true statement can be proved true.

(B) Every term can be defined but it is necessary to assume that certain statements are true.

(C) Some terms must be left undefined but every true statement can be proved true.

(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A)-(D) is correct.

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20. Examine these three sentences.

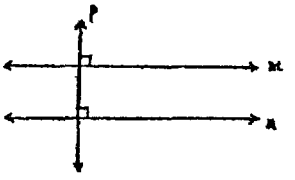
(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are

{P,Q} , {P,R} , {P,S} , {Q,R} , {Q,S} , and {R,S}



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

(A) {P,R} and {Q,S} intersect.
(B) {P,R} and {Q,S} are parallel.
(C) {Q,R} and {R,S} are parallel.
(D) {P,S} and {Q,R} intersect.
(E) None of (A)-(D) is correct.

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22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

(A) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

(B) In general, it is impossible to trisect angles using only a compass and a marked ruler.

(C) In general, it is impossible to trisect angles using any drawing instruments.

(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

(E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

(A) J made a mistake in measuring the angles of the triangle.

(B) J made a mistake in logical reasoning.

(C) J has a wrong idea of what is meant by "true."

(D) J started with different assumptions than those in the usual geometry.

(E) None of (A)-(D) is correct.

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24. Two geometry books define the word rectangle in different ways. Which is true?

(A) One of the books has an error.

(B) One of the definitions is wrong. There cannot be two different definitions for rectangle.

(C) The rectangles in one of the books must have different properties from those in the other book.

(D) The rectangles in one of the books must have the same properties as those in the other book.

(E) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I. If p, then q.
II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.

(B) If not p, then not q.

(C) If p or q, then s.

(D) If s, then not p.

(E) If not s, then p.

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Test Number _____

- 168 -

VAN HIELE GEOMETRY TEST

ANSWER SHEET

Project Use Only J

ID _____

Please print

Name 1 80 443 3742 Class period _____

Last First Middle

Math Teacher _____ School _____

Grade in School (circle): 8 9 10 11 12 Sex (circle): M F

Birth date _____ Test date _____

Month Day Year Month Day Year

Cross out the correct answer

1.	A	**B**	C	D	E
2.	A	B	C	**D**	E
3.	A	B	**C**	D	E
4.	A	**B**	C	D	E
5.	A	B	C	D	**E**
6.	A	**B**	C	D	E
7.	A	B	C	D	**E**
8.	**A**	B	C	D	E
9.	A	B	**C**	D	E
10.	A	B	C	**D**	E
11.	A	B	**C**	D	E
12.	A	**B**	C	D	E
13.	**A**	B	C	D	E
14.	**A**	B	C	D	E
15.	A	**B**	C	D	E
16.	A	B	**C**	D	E
17.	A	B	**C**	D	E
18.	A	B	C	**D**	E
19.	A	B	C	**D**	E
20.	**A**	B	C	D	E
21.	A	**B**	C	D	E
22.	A	B	C	D	**E**
23.	A	B	C	**D**	E
24.	A	B	C	D	**E**
25.	A	B	C	**D**	E

Space for drawing or figuring
(You may also use the other side)

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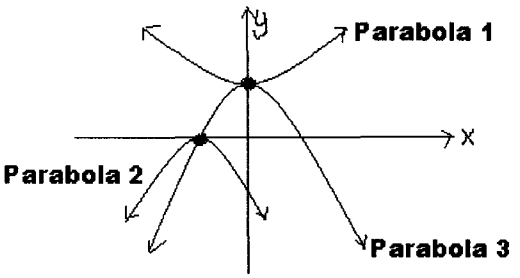
APPENDIX D

STD. ID#_____**Class**_____**Teacher**_____

PRETEST

Please answer all the questions in the appropriate spaces below.
DO NOT USE GRAPHING CALCULATORS.

Q1. Answer questions 1a and 1b using the graph on the right.



1A) What do parabolas 1 and 3 have in common? Use terms from the following list in your answer: vertex, maximum, minimum, x-intercept, y-intercept, root, and axis of symmetry.

1B) What do parabolas 2 and 3 have in common? Use terms from the following list in your description: vertex, maximum, minimum, x-intercept, y-intercept, root, and axis of symmetry.

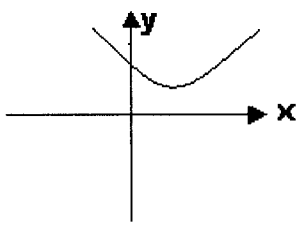
Q2. Find the x-intercepts of the graphs of the equations:

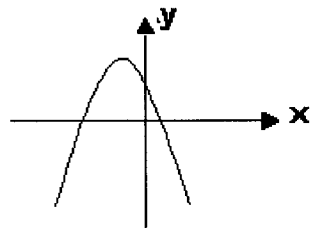
$Y= x^2 + 10x + 16$ _____ $Y= -3x^2 + 5x - 2$_____

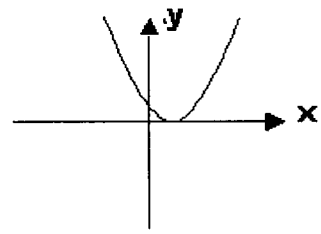
Q3. Match the discriminant (A, B and C) with its graph. Print the letter of each discriminant on the line below its matching graph.

A. $b^2-4ac = 2$
B. $b^2-4ac = 0$
C. $b^2-4ac = -3$

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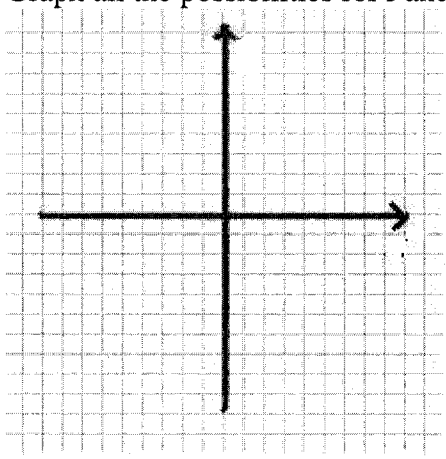
Q4. Consider the equation $(2x+1)(3x+7)=0$

A. What are the x-intercepts?_____
B. What is the y-intercept?_____
C. What do you predict discriminant (Δ) to be; $\Delta>0$, $\Delta<0$, $\Delta=0$?_____
D. In which quadrant is the vertex located?_____
E. The axis of symmetry lies on which side of the y-axis?_____

Q5. Find the range of the value(s) of k so that the graph of the following quadratic function $Y=3x^2 - 6x + k$ intersects the x-axis;

a. in only one point_____
b. in exactly two points_____

Q6. The area of a circle of radius r is $A = \pi r^2$.
Graph all the possibilities for r and A on a coordinate plane.



Q7. For each equation below, what new quadratic function will produce the described change?

A. Parabola: $y = x^2 - 1$
The change: Graph is shifted up two units
New quadratic function: _____

B. Parabola $y = - x^2 + 4$
The change: Graph is inverted w.r.t. x-axis

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New quadratic function: _____

C. Parabola $y = -x^2+2x-2$
The change: Graph is shifted one unit left
New quadratic function: _____

Q8. What is the vertex of the parabola of $y=3 + (x-2)^2$

Q9. The vertex of a parabola is at (1,3) and a>0. What if any, are the x-intercepts? (HINT: make a rough sketch)

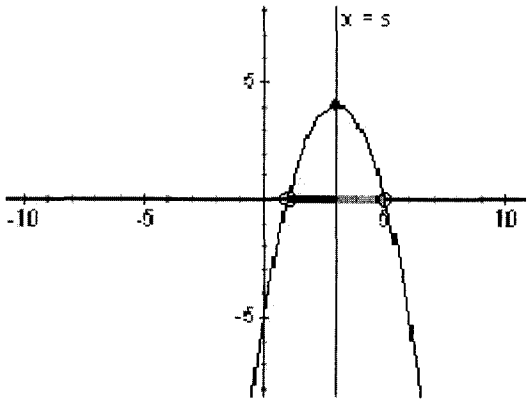
Q10. If the axis of symmetry is at x= -1/2, what is the x value of the vertex?

Q11. A number (n) is multiplied by (n + 2). The result is 168. State how you would find the two numbers.

Q12. Find the value of y for the point where the axis of symmetry intersects the quadratic $x^2-3x-4=0$.

Q13. Use the graph on the right to answer questions 13a and 13b.
A ball is thrown upward and falls back to its original position. The graph represents the position of the ball in one-second intervals.

13A.
What is the equation of the parabola?
Write it in the space below.



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13B.
What is the position of the ball (in coordinates) at t = 2.5 seconds.

Q14. Is it possible for a rectangle with a perimeter of 42 cm to have an area of 90 cm^2?
Circle your answer (Y / N)

YES NO

Explain your answer.

Q15. If the vertex of a parabola is at (0,0), what are y-intercept and axis of symmetry?

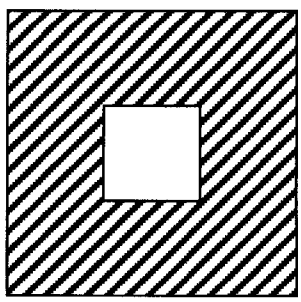
Q16. Using the rule for finding the x-intercepts of a quadratic function, find the points of intersection of the graphs of these equations.

$$\begin{cases} y_1 = -3x^2 + 4x + 1 \\ y_2 = 3x^2 - 3x + 2 \end{cases}$$

Q17.
17A. What is an example of quadratic equation that has a single solution?
Write the equation in the space below using the form $Y=a(x-h)^2 + k$ where (h, k) is the vertex.

17B. What is an example of quadratic equation that has a two solutions.
Write the equation in the space below using the form $Y=a(x-h)^2 + k$ where (h, k) is the vertex.

Q18. Calculate the shaded area as a function of x where one side of the inside square is 3 and one side of the outside square is x-5.



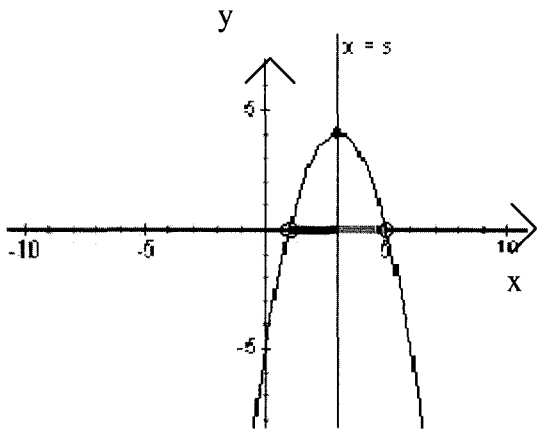
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Q19 A.
Which of the following statements are <u>always</u> true about quadratic functions?
Circle the letter of the statements that are always true.

a. A quadratic function is a second order equation.
b. The roots of the quadratic are at equal distance from the axis of symmetry.
c. The shape of the graph of a quadratic function is a parabola.
d. Any two linear function's multiplication ends up with a quadratic function.

19B.
For those statements that are <u>not</u> always true, give an example of a quadratic function that shows the statement is not true.

Q20.
Use the information in the graph to the right to answer questions 20a and 20b.



20A.
Write an equation for the graph in the form
$Y = ax^2+bx+c$

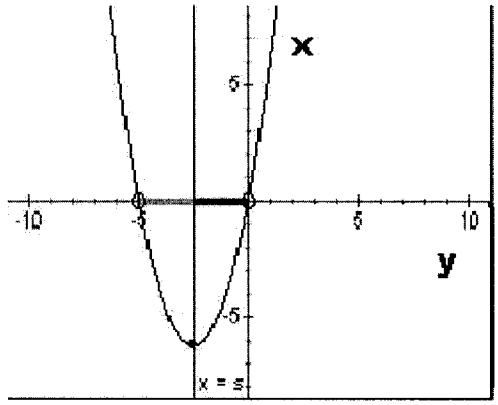
20B. Write and equation for the graph in the form
$Y = a(x-h)^2+k$.

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Q21. Use graph on the right to answer the following question.

Choose the equation of the parabola from the list below. (Circle the letter next to the equation you choose)

A. $y = x^2+5x-3$
B. $y = 2x^2+10x-1$
C. $y = x^2+5x$
D. $y = 2x^2-10x$
E. $y = 2x^2+5x-1$



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APPENDIX E

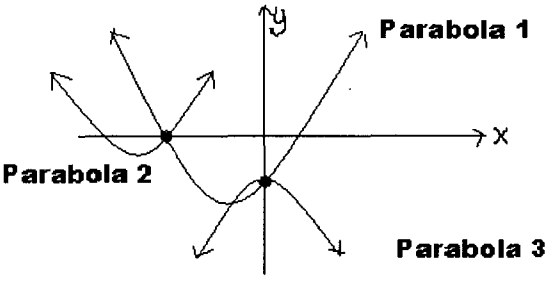
Posttest

STD. ID#_____**Class**_____**Teacher**_____

POSTTEST

Please answer all the questions in the appropriate spaces below.
DO NOT USE GRAPHING CALCULATORS.

Q1. Answer questions 1a and 1b using the graph on the right.



1A) What do parabolas 1 and 2 have in common? Use terms from the following list in your answer: vertex, maximum, minimum, x-intercept, y-intercept, root, and axis of symmetry.

1B) What do parabolas 1 and 3 have in common? Use terms from the following list in your description: vertex, maximum, minimum, x-intercept, y-intercept, root, and axis of symmetry.

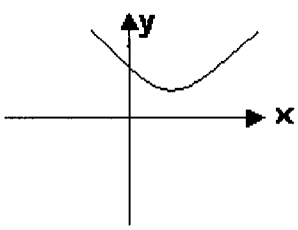
Q2. Find the x-intercepts of the graphs of the equations:

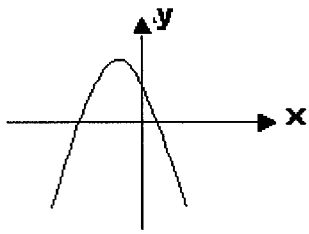
$Y= x^2 + x - 2$ _____ $Y= 2x^2 - x - 3$_____

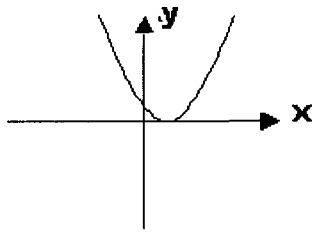
Q3. Match the discriminant (A, B and C) with its graph. Print the letter of each discriminant on the line below its matching graph.

A. $b^2-4ac = 10$
B. $b^2-4ac = 0$
C. $b^2-4ac = -1/2$

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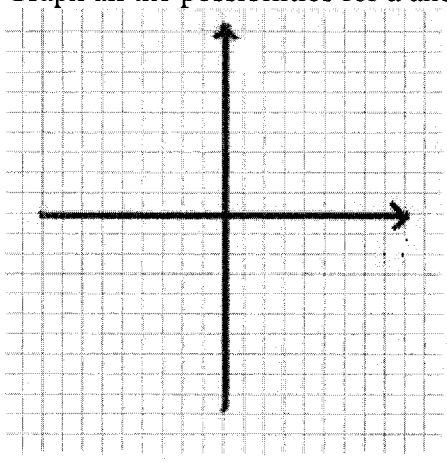
Q4. Consider the equation (-2x-1) (5x-10)=0

F. What are the x-intercepts?_____
G. What is the y-intercept?_____
H. What do you predict discriminant (Δ) to be; Δ>0, Δ <0, Δ=0?_____
I. In which quadrant is the vertex located?_____
J. The axis of symmetry lies on which side of the y-axis?_____

Q5. Find the range of the value(s) of k so that the graph of the following quadratic function $Y=2x^2 - 3x + k$ intersects the x-axis;

c. in only one point_____
d. in exactly two points_____

Q6. The area of a square of side a is $A = a^2$.
Graph all the possibilities for a and A on a coordinate plane.



Q7. For each equation below, what new quadratic function will produce the described change?

D. Parabola: $y = x^2 - 3$
The change: Graph is shifted up two units
New quadratic function: _____

E. Parabola $y = - x^2 + 3$
The change: Graph is inverted w.r.t. x-axis

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New quadratic function: _____

F. Parabola $y = -x^2 - 2x - 2$
The change: Graph is shifted one unit left
New quadratic function: _____

Q8. What is the vertex of the parabola of $y = -1 + (x+3)^2$

Q9. The vertex of a parabola is at (-1, -2) and $a<0$. What if any, are the x-intercepts? (HINT: make a rough sketch)

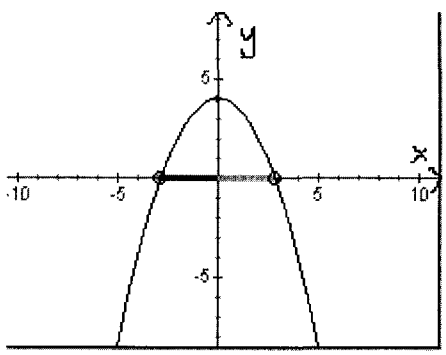
Q10. If the axis of symmetry is at $x = 3/2$, what is the x value of the vertex?

Q11. A number (n) is multiplied by (n + 2). The result is 63. State how you would find the two numbers.

Q12. Find the value of y for the point where the axis of symmetry intersects the quadratic $2x^2 - x + 3 = 0$.

Q13. Use the graph on the right to answer questions 13a and 13b.
A ball is thrown upward and falls back to its original position. The graph represents the position of the ball in one-second intervals.

13A.
What is the equation of the parabola? Write it in the space below.



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13B.
What is the position of the ball (in coordinates) at t = 1 seconds.

Q14. Is it possible for a rectangle with a perimeter of 50 cm to have an area of 136 cm^2?
Circle your answer (Y / N)

YES NO

Explain your answer.

Q15. If the vertex of a parabola is at (0,1), what are y-intercept and axis of symmetry?

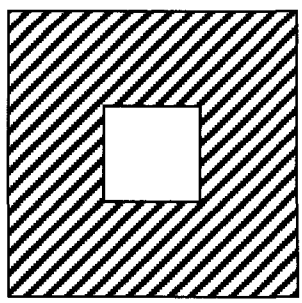
Q16. Using the rule for finding the x-intercepts of a quadratic function, find the points of intersection of the graphs of these equations.

$$\begin{cases} y_1 = 2x^2 + 3x - 4 \\ y_2 = -2x^2 - 4x + 1 \end{cases}$$

Q17.
17A. What is an example of quadratic equation that has a single solution?
Write the equation in the space below using the form $Y=a\,(x-h)^2 + k$ where (h, k) is the vertex.

17B. What is an example of quadratic equation that has two solutions.
Write the equation in the space below using the form $Y=a(x-h)^2 + k$ where (h, k) is the vertex.

Q18. Calculate the shaded area as a function of x where one side of the inside square is 3 and one side of the outside square is $-x+2$.



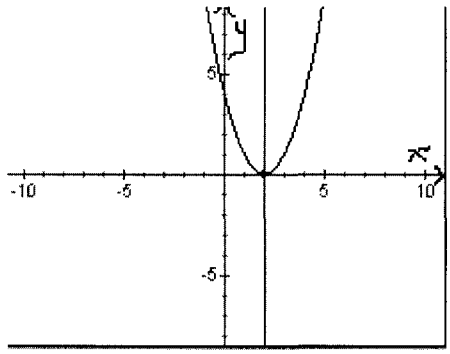
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Q19 A.
Which of the following statements are always true about quadratic functions?
Circle the letter of the statements that are always true.

a. The roots of the quadratic are at equal distance from the axis of symmetry.
b. A quadratic function is a second order equation.
e. Any two linear function's multiplication ends up with a quadratic function.
f. The shape of the graph of a quadratic function is a parabola.

19B.
For those statements that are not always true, give an example of a quadratic function that shows the statement is not true.

Q20.
Use the information in the graph to the right to answer questions 20a and 20b.



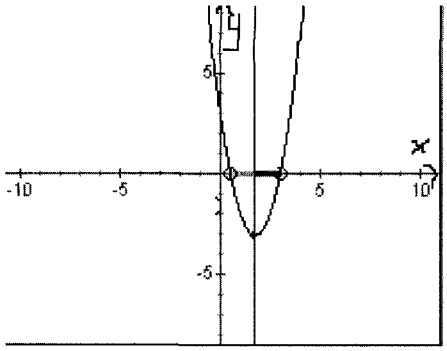
20A.
Write an equation for the graph in the form
$Y = ax^2+bx+c$

20B. Write and equation for the graph in the form
$Y = a(x-h)^2+k$.

Q21. Use graph on the right to answer the following question.

Choose the equation of the parabola from the list below. (Circle the letter next to the equation you choose)

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A. $y = x^2+5x-3$
B. $y = 2x^2+10x-1$
C. $y = x^2+5x$
D. $y = 2x^2-10x$
E. $y = 2x^2+5x-1$

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APPENDIX F

PRETEST ANSWERS

Q1)
1a. Parabola 1 has the same vertex, same y-intercept and same axis of symmetry with Parabola 3.
1b. Parabola 2's vertex and one of parabola 3's x-intercepts are same.

Q2) $y=x^2+10x+16=(x+8)(x+2)$

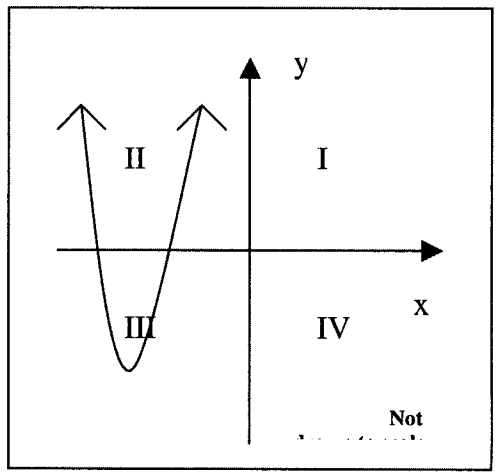
$x_1+x_2 = -10/1 \quad x_1*x_2=16/1$

$y= -3x^2+5x-2= (-1)(x-2/3)(x-1)$

$x_1+x_2 = 5/3 \quad x_1*x_2=2/3$

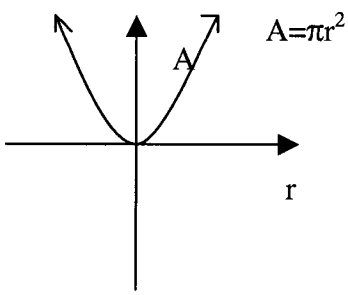
Q3) C, A, B

Q4) a. x-intercepts: $x_1= -1/2$, $x_2= -7/3$
b. y-intercept= 7 ; the only term without x
c. $\Delta > 0$, since there are two roots/x-intercepts .
Since there are two roots and a>0,
the parabola is pointing upward.
Hence the vertex would be at the third quadrant.
The axis of symmetry is on the left hand side of the y-axis.



Q5) $y= 3x^2 -6x+k$
$\Delta=0$ (for only one point) → $b^2-4ac=0$ → $36-12k=0$ → $k=3$
$\Delta>0$ (for two points) → $b^2-4ac>0$ → $36-12k>0$ → $k<3$

Q6. Find the range of the value (s) of k so that the graph of the following quadratic function

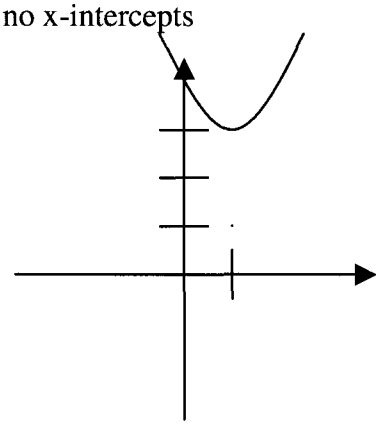


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Q7)
7a. $y = x^2+1$
7b. $y = x^2-4$
7c. $y = -x^2-1$

Q8) vertex= (2,3)
When $y=(x-h)^2+k$, vertex=(h,k)

Q9) no x-intercepts



Q10) x=-1/2, vertex is always at the axis of symmetry

Q11) $a(a+2) = 168$, $a^2+2a=168$, by solving the quadratic $a^2+2a-168=0$

Q12) The axis of symmetry intersects the parabola at the point of vertex.
Vertex = (-b/2a, f(-b/2a))
-b/2a= 3/2 , f(3/2)= 9/4-3(3/2)-4= 9/4-9/2-4/1= (9-18-16)/4= -25/4

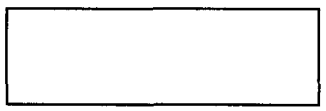
Q13) a. x-intercepts: 1, 5
Pointing downwards: a<0
Vertex = (3,4)

$-(x-1)(x-5)=0 \rightarrow -(x^2-5x-x+5)=0 \rightarrow -x^2+5x+x-5=0 \rightarrow y = -x^2 + 6x - 5$

b. We need to find f(2.5). t= 2.5 seconds = 5/2
$y= -(5/2)^2 +6(5/2) -5= 65/4$

Coordinates of the ball at t= 5/2 is (5/2, 65/4)

Q14) $2a +2b = 42 \rightarrow a + b= 21$
$a * b = 90 \rightarrow b= 90/a$
$\rightarrow a+90/a= 21 \rightarrow a^2 - 21a +90 = 0 \rightarrow$
$\Delta = (21)^2 -4\ (1)(90)= 441 - 360= 81$
$81>0 \rightarrow$ two real roots of the equation $\rightarrow$



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yes, there is a rectangle like that.
Q15) since vertex is on the y-axis, y- intercept= (0,0) and axis of symmetry: x=0

Q16) $-3x^2 +4x+1 = 3x^2 - 3x+2$ ➔ $0= 6x^2 - 7x +1$ ➔ $\Delta = 49- 4.6.1= 49-24= 25$

$X_{1,2}= (7 \pm \sqrt{25}) / 12$ (root formula)➔
$X_1= 1$
$X_2= 1/6$

Q17) There may be many answers for this question. Hence, as an example;

a. $2(x-2)^2 +0$
b. $3(x-1)^2+1$

Q18) $(x-5)^2 -3^2= x^2 - 10x+25 -9 = x^2-10x +16$

Q19) a, b, c is always true, but d is not
For example, y=3, y=x+5 does not

Q20) y-intercept= -5
x-intercepts= 1, 5

$(x-1)(x-5) = x^2 - 5x - x +5$
$= x^2 - 6x + 5$
a=1
vertex: (3,4)
➔ $y= x^2 - 6x + 5 = 1(x-3)^2 +4$

Q21) It passes through (0,0)➔ NOT A, B, E
Vertex= (-5/2, 0)
Only **C** satisfies the graph.

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APPENDIX G

POSTTEST ANSWERS

Q1)
1a. Parabola 1'and Parabola 2's one of the x-intercepts is same.
1b. Parabola 1's y-intercept is the same as y-intercept and axis of symmetry of Parabola 3.

Q2) $y=x^2 + x - 2=(x-1)(x+2)$

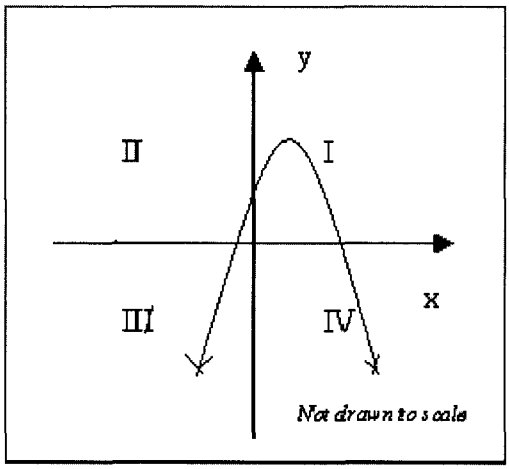
$x_1+x_2 = -1/2 \quad x_1*x_2= -2$

$y= 2x^2 - x -3= (x-3/2)(x-1)$

$x_1+x_2 = 1/2 \quad x_1*x_2=-3/2$

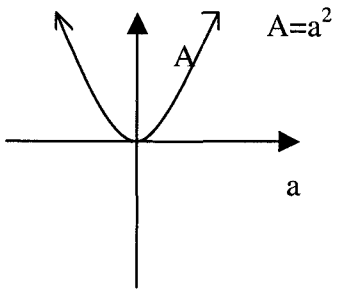
Q3) C, A, B

Q4) a. x-intercepts: $x_1= -1/2$, $x_2= 2$
b. y-intercept= 10 ; the only term without x
c. $\Delta > 0$, since there are two roots/x-intercepts .
Since there are two roots and $a<0$,
the parabola is pointing downwards.
Hence the vertex would be at the first quadrant.
The axis of symmetry is on the right hand side of the y-axis.



Q5) $y= 2x^2 -3x+k$
$\Delta=0$ (for only one point) → $b^2-4ac=0$ → $9-8k=0$ → $k=9/8$
$\Delta>0$ (for two points) → $b^2-4ac>0$ → $9-8k>0$ → $k<9/8$

Q6. Find the range of the value (s) of k so that the graph of the following quadratic function

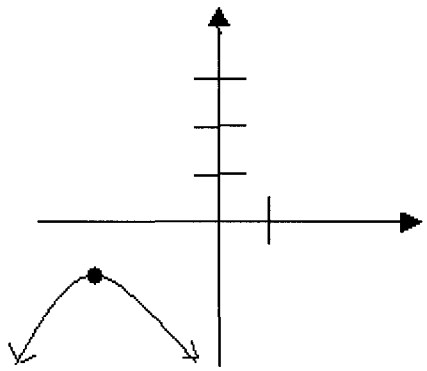


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Q7)
7a. $y = x^2+1$
7b. $y = x^2-3$
7c. $y = -x^2-1$

Q8) vertex= (3,-1)
When $y=(x-h)^2+k$, vertex=(h,k)

Q9) no x-intercepts



Q10) x=3/2, vertex is always at the axis of symmetry

Q11) $a(a+2) = 63$, $a^2+2a=63$, by solving the quadratic $a^2+2a-63=0$

Q12) The axis of symmetry intersects the parabola at the point of vertex.
Vertex = (-b/2a, f(-b/2a))
-b/2a= 1/2 , f(1/2)= 9/4-1/2+3= 9/4-1/2+3/1= (9-2+12)/4= 19/4

Q13) a. x-intercepts: -2, 2
Pointing downwards: a<0
Vertex = (0,4)

$-(x+2)(x-2)=0 \rightarrow -(x^2+2x-2x-4)=0 \rightarrow -x^2-4=0 \rightarrow y = -x^2 - 4$

b. We need to find f(1). t= 1 seconds
$y= -(1)^2 - 4 = -5$

Coordinates of the ball at t= 1 is (1, -5)

Q14) $2a+2b = 50 \rightarrow a + b= 25$
$a * b = 136 \rightarrow b= 136/a$
$\rightarrow a+136/a= 25 \rightarrow a^2 - 25a +136 = 0 \rightarrow$
$\Delta = (25)^2 - 4(1)(136)= 625 - 544 = 81$
$81>0 \rightarrow$ two real roots of the equation $\rightarrow$



b

a

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yes, there is a rectangle like that.
Q15) since vertex is on the y-axis, y- intercept= (0,1) and axis of symmetry: x=0

Q16) $2x^2 +3x+4 = -2x^2 - 4x+1 \rightarrow 0= 4x^2 + 7x +3 \rightarrow \Delta = 49- 4.4.3= 49-48= 1$

$X_{1,2}= (-7 \pm \sqrt{1}) / 8$ (root formula)$\rightarrow$
$X_1= - 6/8= - 3/4$
$X_2= -1$

Q17) There may be many answers for this question. Hence, as an example;

a. $2(x-2)^2 +0$
b. $3(x-1)^2+1$

Q18) $(-x+2)^2 -3^2= x^2 - 4x + 4 -9 = x^2-4x -5$

Q19) a, b, d is always true, but c is not
For example, y=3, y=x+5 does not

Q20) y-intercept= 4
x-intercepts= 2

$(x-2)^2 = x^2 - 4x + 4$
a=1
vertex: (2,0)

Q21) There is y-intercept, hence NOT C or E.
y-intercept is at 3, hence NOT B either.
Axis of symmetry is on the positive side. Hence the answer is A.

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APPENDIX H

WORKSHEET #4
Quadratics with SLIDER GRAPH applet
Student ID Number _____ Class _____ Teacher _____

Please use the "Slider Graph" Interactive Diagram, which is at http://math.kuwa.edu/jazmath/basic_applets/SliderGraph.html while filling empty boxes in the following table. For each equation/parabola, find if the discriminant (A) greater than zero, equal to zero, or smaller than zero. Find x-intercepts, y-intercept, maximum or minimum, axis of symmetry, and vertex. Note your observations as well.

Note that the values of the coefficients "a, b, c" from $y=ax^2+bx+c$ can be adjusted using the sliders at the bottom of the applet. The applet responds to the mouse actions click, shift-click, click-and-drag, and right-click-and-drag. Set the ranges of x and y-axes as following: xmin= -10, xmax= 10, ymin= -9, ymax= 9. Then push "set limits" button to save the changes.

$y= ax^2+bx+c$	(A)	x-intercepts	y-intercept	Min/ max	Vertex (h, k)	Other observations
$y=-x^2$						
$y=-3x^2$						
$y=.5x^2$						
$y=x^2+2$						
$y=(x+2)^2$						
$y=2x^2+3x+5$						
$y=2x^2-3x+5$						
$y=2x^2-3x-2$						
$y=3x^2-5$						

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WORKSHEET #2
Quadratics with QUADRATIC FUNCTIONS IN GENERAL FORM (THE DISCRIMINANT) applet
Student ID Number _____ Class _____ Teacher _____

Please use the "Quadratics" Interactive Diagram, which is at http://www.members.shaw.ca/ron.blond/QFA.GF.APPLET/index.html while filling empty boxes in the following table. For each equation/parabola, find if discriminant (Δ) greater than zero, equal to zero, or smaller than zero. Find x-intercepts, y-intercept, maximum or minimum, axis of symmetry, and vertex. Note your observations as well.

Note that the values of the coefficients "a, b, c" from $y=ax^2+bx+c$ can be adjusted using the sliders on the right side of the applet. The graph in the applet responds to the mouse actions click, click-and-drag (from each point of the graph). Here $x=s$ stands for the axis of symmetry and "d" stand for the distance of the axis of symmetry from each root of the equation respectively.

$y= ax^2+bx+c$	(Δ)	x-intercepts	y-intercept	Min/ max	Vertex (h, k)	Other observations
$y=x^2$						
$y=-3x^2$						
$y=.5x^2$						
$y=x^2+2$						
$y=(x+2)^2$						
$y=2x^2+3x+5$						
$y=2x^2-3x+5$						
$y=2x^2-3x-2$						
$y=3x^2-5$						

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APPENDIX I

Scoring Rubric

Each item will be scored according to the following rule..

Q1 (5 pts total)

1a. 1 pt. For each correct common feature (3 pts total)
1b. 1 pt. Of each common feature (2 pts total)

Q2 (4 pts total)

2 pts for each equation/ having 2 toots correct
1 pts for each correct root

Q3 (3 pts total)

1 pts for each correct match

Q4 (4 pts total)

2 pts for x-intercepts
1 pt. Y-intercept
0.5 pt. For discriminant
1 pt. For quadrant
.5 pt for axis of symmetry

Q5 (4 pts total)

2 pts for each 5a and 5b.

Q6 (5 pts total)

-graph (2)
-naming the axes (3)

Q7 (6 pts total)

2 pts for each

Q8 (1 pt.)

Q9 (3 pts total)

1 pt. For a correct sketch

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2 pts for No roots

Q10 (2 pts)

Q11 (6 pts total)
writing equation. 2
simplifying equation.2
stating how it would be solved.2

Q12 (4 pts total)

Stating y-value of vertex 2
Finding vertex 2

Q13 (8 pts total)

13a.
Finding equation (in any way) 5
Any conceptual miscalculation –1
13b.
f(y) 3 pts.

Q14 (6 pts total)

Identifying unknowns 2
Writing equations 2
Stating how to solve 2

Q15 (4 pts total)

y-intercept 2
axis of symmetry 2

Q16 (6 pts)

Simplifying equation 3
Solving equation 3

Q17 (6 pts total)

Each equation 3

Q18 (4 pts total)
Writing equation 2

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Simplifying it 2

Q19 (10 pts)

Each truth value 2 *4 = 8 pts
Counter example 2

Q20 (6 pts total)

Determining unknowns 2
Writing in standart form 2
Writing in vertex form 2

Q21 (3 pts total)

choosing correct equation 3

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APPENDIX J

Student Interview Descriptions

Student Number 107170
Van Hiele Level ***Inbetween***
Teacher 2

Student used words like "going down", "wider", "moved up and down", while describing the changes occurred for the set of equations given. In the case of a line equation, unplanned, the student chose to ignore it and the symbolic parts where s and d were indeterminate. Student could not figure it out the equation $y= (x+2)^2$ and what to do with it. For the overall changes, student stated that "parabola moved up y axis sometimes, and sometimes it would open up and wider." When possible parts were asked, student pointed to the graph and to special points like vertex. The relations between parts were a puzzle for the student. The movements were pretty planned, and not playful. The changes were imposed by little increments. Another line occurred while carrying out the remaining equations, but again it was ignored by the student. For graph movements, statements like "click on vertex and drag, it (graph) moves up" ' "as the parabola moves, the numbers (referring to the values of sliders) go up and down". Student used Internet for many purposes but especially for the projects, finding info, search engines, etc. Developed understanding was a feedback from the student.

2. Student Number 575110
Van Hiele Level Inbetween
Teacher 2

Had problems with $y= x^2+2$. Found in determined for lines. Overall, graphs got more complicated and smaller. Graphical part was this student's focus area. The reason student gave was "cause you can see the changes, positive, negative, smaller," . For axis of symmetry line, the student identified it as a pointer to indicate the bottom point (vertex). For positive and negative "b"s student identified the new graph as "pretty much the same

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graph but on the other side of this line (pointing y axis). "Vertex getting more of the page, they (graphs) are getting smaller. Most of them have "a", some of them have "b"& "c". Distance equation, these equations (discriminant and axis of symmetry) changes, too. Pointed to the changes occurred in the formulas above as a result of changes in the graph. Also, pointed to the changes in the tabloid slider part: "these all moving up and down". Student observed; when the graph is dragged from any arbitrary point, the d's become too large. Could not remember what b^2-4ac (discriminant) stands for. Then the numbers on the right table became too big. Student argued that in no time the focus of the student was on symbolic part. No much to say about how it makes sense but the changes that was made when some coefficients were same and when others were differing. Internet usage was for uses not other than research.

3. Student Number 915195
Van Hiele level Inbetween
Teacher 2

"Negative a opens down". Came up to indeterminate again, student did not further along for a while. "Something happening with "a" change, this is a straight line. There is no "a". After a little time of puzzlement, student realized a>0, there is no b or c instead with the specific equation at hand. "When "b" changes, it changes side of the y-axis.". Had problem with the equation $y=(x+2)^2$. Student stated that this equation is the same with $y= x^2+2$, it seemed like a misconception. Student identified "s" with the concept of slope. Graph changes with the slope change. Student explained the way to make sense as "need to figure out the equation, need to break the equation into a part of each number. It is hard. It is hard to understand". This student avoided using the "coordinate system" terminology. For the distance between the lines, student associated it with the coordinates of a point of which is indicated by blue lines on the applet when clicked on any arbitrary point of the parabola. Student pointed to the dragging of the vertex, for which the formulas were changing as well. For this student, formulas took the most of attention. Student stated that seeing the changing numbers helps you understand what is supposed

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to be. Did not learn anything (said by student), Internet usage is uses once-twice a day for chatting and search.

4. Student Number 636075
Van Hiele Level High
Teacher 1

"I noticed that negative numbers, parabola down otherwise up." "Parabola rises up above x-axis or drop down below it." For negative "b" the parabola goes to the right side of the y-axis, otherwise to the other side of the y-axis. For negative c, it goes under the x-axis. Student used above statements. For each equation, there was different parabola. Tabloid part and then graphical took the most attention. Student indicated that d and s looks like coordinates of a point. Student mentioned that if he knew more on s and d, his attention would be more over there. "Moving c adjusts graph on the y axis." Being able to specify the formula part would not make any difference for this student as he mentioned. In case of "homework, being able to change a,b,c whenever you need it would be really great." "by playing, I could figure it out somewhere." "helpful in understanding how the parabola change with fluctuating a,b,c. This student found intervention interesting but indicated the tests were a little bit hard for them. "I found the discriminant thing was the most difficult part of the test."

5. Student Number 139895
Van Hiele Level Low
Teacher 2

Student was hesitant to say anything. Changed b and c and for specific set of b and c's. Student used the word diminished for x^2 when "a" was brought to zero. Realized with lower c's the vertex goes down of the x-axis. Argued that graph changed as being wider, flip over, with a>0 or a<0. Used the word "adjust" to explain the corresponding changes of dragging a pt on parabola and seeing the set of values change in tabular representation, and formulas change above ("it adjusts into that"). Discovered to carry the parabola along

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the x-axis via dragging the axis of symmetry. They correspond each other ("That formula changes according to the change in the variables"). Student was color blind, so I needed to explain what part is what color. When vertex is moved, numbers changed in the formula of discriminant above according to the student. But also, when it is asked, how is it so, student gave the answer "I don't know, the program does it.". Did not recall what was discriminant. He found the applet too technical. "there is a lot of stuff going on; formulas, graph&s,d, all this ; a bunch of stuff tumbled together . This student was the only one who realized the pattern of the effect of changing b on parabola as being a path of parabola after being forced to see the pattern. He missed intervention, the tests were a little hard for what they are used to. Use Internet everyday for looking up things, but do not search for learning more. That is what he said.

6. Student Number 012810
Van Hiele Level High
Teacher 2

First a little bit puzzled but was able to point out the concepts well. Checked SET A button. Mostly added if the parabola opens up or down to the information asked. Moved careful. Mentioned that a makes it wider, c makes graph up or down, and b moves the graph from side to side. While seeing the dragging of the parabola, indicated that it was getting wider or stretching out. Was not able to see the possible other changes. By dragging vertex, still stated that the parabola was opening up. For explaining relationship between parts, used the wording as "when you change the settings, graph changes and causing this part to change (symbolic)." Mentioned that would use this applet since, no need to write anything. It is easier to visualize". Student uses Internet at least an hour a day, for doing research, homework notes, etc. Computer at home is relatively slow.

7. Student Number 789689
Van Hiele Level High
Teacher 1

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This student was pretty good with concepts. For the difference between positive and negative "a"'s, explained it with "it is just going down". Pointed to the concepts in the right place. Realized the change produced by b as moving the axis of symmetry back and forth around y-axis. For the effect of change in "c", hypothesized that it affects where the vertex is. For the change in "a", concluded that it makes wider, until it passes x-axis then it flips. Then added, " c changes how far the vertex is from x-axis". When made to click on the parabola and drag it, explained as making it wider, thinner. As it goes up, formulas changed, numbers got bigger, and as outside of the screen, they got smaller, according to this student. When the vertex was dragged only, student argued that only vertex changes. Was into playing with the applet to figure it out more, even after answered the questions. Mentioned that overall all numbers changed, but really do not know else. Indicated that, even though the applet may be helpful, still it is a little bit confusing, especially some parts of it. Not a frequent user of Internet, a few hours a week only. Used not for homework, but for more personal matters as of chatting, e-mailing. Argued that even if searches, could not find more of these kinds of applets.

8. Student Number 369423
Van Hiele Level High
Teacher 1

This student pointed and stated concepts in a correct way. Had problem with $y=x^2+2$ and $y=(x+2)^2$. For change in b, "graph changes quadrants from 4 to 1 and be like a reflection to axis of symmetry /y-axis. Saw equations as a separate part. Divided equations into two parts, hence saw 4 parts in the whole mathlet. "As you slide the tabular values up and down, the formulas change upwards, and then the graph changes accordingly." "Automatically changing all values so the equations are changing with congruence of the tabular values. Then formulas pretty much tells you where the equation to be (tabular part) when something changes on the graph". Was focusing on the vertex; a big red dot. "The graph takes most of my attention" "I just have to remember what all these means. Pointing to the formulas". Mentioned that outside of the frame for graph, only parabola is extending, hence, small part is better, since this portion specifies a parabola outside

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portion is not needed. Identified "c" as going below or up the screen. Stated that she/he does not know all the things on the screen. Played with applet to see how the upper formulas work. This made me think that little inaccuracies occur and unless they are corrected they may stay like that producing misconceptions. Confessed that does not like the topic for some reason, and it is a little bit confusing due to that. Argued that after that day, he/she would not remember anything. Claimed that the confusing formulas were making it difficult to understand. Stated that uses Internet for research, and talking to friends. Mentioned that she/he likes the topic "circle" better, and I showed another applet from the same web site indicating that there are more applets with this kind of applets for each topic. Was able to use the applet with whatever conventions he/ she got from the Quadratics applet. Thought it was helpful." I know how to move things around, etc."

9. Student Number 831980
Van Hiele Level Low
Teacher 1

Student found indeterminate results for formulas with a line, too but ignored it and continued with the applet. Identified the role of "a" as moving upwards or downwards the parabola. Claimed that it really doe not help to see the parabola and values on the screen. But also mentioned "As you can see the stuff, you can set numbers in and see the results". "Now parabola is up, it is a positive number now, instead of a negative. For s & d, she/he thought them to be coordinates. Pointing to the discriminant and that number increased so the other numbers here. " I use Internet every day to check weather, talking to friends, schoolwork, and projects" "Probably, would not search the Internet for finding applets like these".

10. Student Number 797585
Van Hiele Level Inbetween
Teacher 1

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" I really don't know anything about this". This student was hesitant to start with but kept ok after a while. Pointed on the screen while talking about each part. "When there is no number, it is just "x", it is always zero, when it is negative, it is downwards" "It gets bigger, wider I mean" Positive a open up, negative a open down, b makes it wider may be, this one gets skinner, etc. It gets less, when it is really skinny, the numbers are high." "Overall, I drag graph, and a,b,c moves, b does not actually move, only a moves." Played back and forth. Decided they all move. While click and drag on a point, identified that numbers get bigger. Claimed that it is confusing since, he/ she does not know how it works exactly. If he/she would work a lot, said she/he would know how to use it, and then it could be helpful.

11. Student Number 901840
Van Hiele Level Low
Teacher 2

Pointed to vertex, and intercepts without hesitation. "Vertex moved to up and left from previous." Tried to ignore equations above, even after many repeated questions. Answer always included –vertex- possibly the concept student knows. "Drag the graph parallel to the x-axis, basically b and c changes. Yes, they would help. Having a,b,c and seeing changes on the graph. This student uses Internet pretty often, for some schoolwork, most personal use. If would know about them, would use them but would not search Internet to use them.

12. Student Number 986027
Van Hiele Level Low
Teacher 2

Had problem with English, said that "I came from Germany 8 months ago and am still adjusting to it". "No wait wait, here here" much talkative while identifying a,b,c. "I don't really know where vertex is" Paused for axis of symmetry as well. Found a line, did not get caught there. I don't know how equations differ graphically. "Ok, I can try this one.".

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"The graphs move higher, positive size, and the line (axis of symmetry) can move to the left and right of y-axis. But I don't know how they work together". "Graphs change in this form, when I change equation, that a different graph cause it looks different." "I see nothing" "I see x line (axis of symmetry) goes left, the graph goes bigger and equation is slower." Move "c", the numbers going down, it is going to the negative of the. X-coefficients are touching y-line (y-axis). Used interesting terminology. "I don't know what d is. Oh, the distance between points." For dragging any arbitrary point, student's wording was "Going bigger, it is changing after the graph". It is getting wider and wider. Noted that movement due to the vertex dragging and movement due to the point dragging were different. Pointed to the entire applet; "a,b,c are moving with it because a,b,c changes". "Equations and graphing are something together." It helps a lot, when you see by yourself as she stated. Student uses internet everyday, at home, too. For search for classes, checking e-mails, and writing e-mails.

13. Student Number 420606
Van Hiele Level High
Teacher 1

Student had such a low pitch voice. It was hard to listen to, and take notes. $Y=(x+2)^2=$ x^2+4x+4; student gave right away. With different a, noted that the parabola is higher above the x-axis. With different b, stated that the parabola changed sides of the x-axis. "Equation defines the parabola". For some values graph went out of the frame. Student took time, did not talk that much, when talked voice was almost un heardable. She claimed that applet made things easier to understand. Uses internet searching, search engines, etc. Would prefer to use it collaboratively. Must have some sort of techno phobia. This is what I believe.

14. Student Number 707021
Van Hiele Level Low
Teacher 1

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When a changed to negative, "it is no longer a maximum, it is a minimum." Student noted that as a changes, the square root above was changing. "as higher it gets, higher the square root becomes (seeing the change for small increments and generalizing to the all problem). Thought b as 2 in the equation $y=x^2+2$ (deciding right away without thinking using its face value). Was focusing on the graph while dragging vertex. Had some problems with discriminant and other concepts. Did not have any problem with $y=(x+2)^2$. Has named the standart form of this equation $y= x^2+4x+4$ by himself/herself. "vertex stays on the axis of symmetry for this one, but it would be at different place" "it gets a lot smaller"same but the axis of symmetry is tilted left a little bit for this one. Just the opposite side". The graph took most of attention. "Formula part is just taking the problem and dissecting more." Smaller took many meanings with this student. Here, used it to point the portion of the parabola seeable from the graphical frame. "With b and c, it is always funky stuff. It is a big change, the screen is compartmentalized to portion bigger. Claimed that after it is done here, it will slip out of her/his head. But mentioned that the "form" may remain but the rest would probably slip out of head. While dragging vertex, was still focusing on the graphical representation. Ignored the questions on the symbolic and tabular representation parts. This may related to the idea of "familiarity".

15. Student Number 794963
Van Hiele Level Low
Teacher 1

Had no problem with concepts. Student was pointing out for each equation without hesitation. Pretty confident with terminology. For "d", " I don't know what that means! Is it the distance between roots?" For b, Axis of symmetry moves left and right. For c, vertex moves higher or lower. "a,b, are all parts of the equation that runs the parabola." "Pull vertex, goes up and down". "Pull any arbitrary point, wider, skinner, etc.". She was looking for patterns while dragging any arbitrary point. She was confident while playing with it. "I don't think it would be helpful" "yes, it would be useful for understanding the shape of the parabola, changes in a,b,c and how these changes produce changes in the parabola. Uses, internet could be helpful with homework.

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16. Student Number 424188
Van Hiele Level Inbetween
Teacher 1

Little hesitant with vertex. But x and y-intercept knowledge is better. "Sometimes they open up... due to c shift" "with numbers these take, a,b,c , the parabola is different" For a, vertex is going lower, b makes the parabola move around y depending on equation. C makes it move up and down. While dragging vertex, "c moves", the equations are changing, the set numbers are changing as well, except a" Dragging any point, c is not affected by moving the point to the left or right. She found symbolic part pretty helpful for learning quadratics. Equation and graph, it helped to understand. "Yes, it helped to see when a parabola changes with respect to the equations. "Visual is helpful. " Half an hour a day. Checks e-mail, chats, research comes next.

17. Student Number 022525
Van Hiele Level High
Teacher 1

"I understand the a part is what makes parabola above or below the x-axis. "a is the one that decides on going up or down" "C is how high it is on the y-axis." "b is how far it is from y axis, and x-axis. " Had little misconception with vertex and c relation. No problem with axis of symmetry concept. Focused on graphical representation. Saw structure within it. Axis of symmetry, etc. "When I point the parts, they function together.". "Graph and sets of a,b,c shows where you are and how these position" . Graph and equations shows, how you compute all, your way of formulas. "When I moved my slider, the graph shows how it is with these." Drag any arbitrary point, the numbers in the formula changes by decimals" "After experimenting with it, I got to understand the points, so it got easier". "it helped me learn b.. it wasn't time. Assuming as I do the equations myself, how it helped." Uses internet for research. "I think I would use if I was given a set of web sites related to mathlets." "Intervention helped me understand it better. Since it helped me to experiment with it at first."

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APPENDIX K

The inter-item correlations for posttests

Correlation matrix

	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq	zpostq
zpostq1	1.0000																				
zpostq2	0.1446	1.0000																			
zpostq3	0.1271	0.0634	1.0000																		
zpostq4	0.1685	0.1427	0.1062	1.0000																	
zpostq5	0.0804	0.2100	0.2624	0.1047	1.0000																
zpostq6	-.1899	0.2308	0.1526	0.3153	0.0715	1.0000															
zpostq7	0.1597	0.0306	0.0598	-.0850	0.1299	-.1071	1.0000														
zpostq8	0.0007	0.0197	-.0983	-.0120	0.0929	-.0585	-.0321	1.0000													
zpostq9	0.3064	0.1792	-.0013	0.1569	-.0121	-.0122	0.1280	-.1285	1.0000												
zpostq10	0.1810	0.1882	-.0324	0.0320	-.0120	0.1287	0.0768	0.0987	0.1914	1.0000											
zpostq11	0.4403	0.1458	0.1088	0.1941	0.1305	-.0182	0.1324	-.0070	0.0324	0.1668	1.0000										
zpostq12	-.0673	-.0080	-.0381	-.0442	-.0603	0.0309	-0.1556	-.0972	0.0772	0.0703	-.0480	1.0000									
zpostq13	0.1370	-.0358	0.0624	0.3081	0.0462	0.0876	0.1995	0.0608	0.8630	0.2783	0.1149	0.1975	1.0000								
zpostq14	0.2604	0.1421	-.1658	0.0879	0.0133	-.1050	0.1617	0.1771	0.1049	-.0446	0.1010	-.0628	0.0560	1.0000							
zpostq15	0.4486	0.2167	-.0753	0.0739	-.0304	0.0792	0.0219	-.0823	0.3012	0.3669	0.1014	0.1213	0.2587	0.1629	1.0000						
zpostq16	0.1536	0.0636	0.0433	0.0197	0.1731	0.0344	0.2491	-.0204	0.0573	0.0798	0.0691	0.2291	0.1506	-.1537	0.2127	1.0000					
zpostq17	0.0429	-.1684	-.0285	0.0013	0.1109	0.1053	0.1256	-.0255	0.1211	0.1070	0.1106	0.3942	0.3817	0.0307	0.0831	0.1966	1.0000				
zpostq18	0.3071	-.1063	-.1115	0.0922	-.1039	-.1287	-.0478	0.1050	0.1989	-.0378	0.2257	0.0606	0.1980	0.4874	0.3032	-.1791	0.1955	1.0000			
zpostq19	0.0797	-.1798	-.0964	0.0626	-.0225	-.0971	0.2884	0.0514	0.2424	0.0115	0.1412	0.3407	0.4578	0.2021	0.2088	0.2737	0.4814	0.3362	1.0000		
zpostq20	0.2217	0.0997	0.0919	0.2020	0.3823	0.0486	0.2121	0.0517	0.2606	0.2833	0.0360	0.0076	0.4775	-.0556	0.3287	0.3559	0.3254	0.0799	0.2932	1.0000	
zpostq21	0.1102	-.0649	0.1476	0.0972	0.0885	-.0822	-.2649	-.1115	0.0562	0.0009	-.0015	0.1159	-.0269	0.0470	0.2063	0.1666	-.0265	0.0520	0.1824	0.1950	1.0000

N of cases:	75				
Statistics for	Mean	Variance	Std Dev	N of variables	
scale	.000	62.0839	7.8793	21	
Reliability Coefficients					
Alpha: .6948	Standardized item alpha:	.6948			

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APPENDIX L

The Homogeneity of scores test required for ANCOVA

Before conducting ANCOVA, the homogeneity of slopes assumption was tested. This test evaluates the interaction between the covariate and the factor in the prediction of the dependent variable. An insignificant interaction suggested that the differences on the dependent variable among groups does not vary as a function of the covariate. In case of the interaction was significant, the ANCOVA results would not be meaningful. My

Tests of between subjects effects

Dependent variable : gain

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Intercept	449.996	1	449.996	3.380	0.181	0.575
TEACHER * V3	15.152	1	15.152	0.207	0.652	0.006
TEACHER * LEVELS	52.985	2	26.492	0.362	0.699	0.020
TEACHER * MATH_ACH	16.582	1	16.582	0.227	0.637	0.006
V3 * LEVELS	279.945	2	139.973	1.913	0.163	0.099
LEVELS * MATH_ACH	381.770	2	190.885	2.609	0.088	0.130
TEACHER	14.924	1	14.924	0.204	0.654	0.006
V3	55.902	1	55.902	0.424	0.574	0.156
LEVELS	393.420	3	131.140	1.701	0.185	0.127
MATH_ACH	488.939	1	488.939	6.682	0.014	0.160

Table 8. Selected results of the test of homogeneity of slopes

Results let me proceed to the ANCOVA assuming homogeneity of slopes, without hesitancy since the partial $\eta^2 = .18$ (effect size) being large in size (>.14).

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APPENDIX M

Table for raw Pre and Post test means by vH levels and modes of representation

Modes >	**2 mode**			**3 mode**		
VH Levels>	*high*	*inbtw*	*low*	high	inbtw	low
Pretest	28.39	24.16	23.58	19.32	15.19	16.11
Posttest	29.75 ↑	23.45 ↓	22.95 ↓	25.71 ↑	12.38 ↓	17.58 ↑

Each cell corresponds to the mean of the subgroup represented and the arrows indicate the direction of the change (↑ for increase and ↓ for decrease)

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