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Longevity risk—Its financial impact on pensions

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ABSTRACT

We study the financial impact of longevity risk on defined benefit (DB) pension plan liabilities using the 2010 Ghana population census, and the Lee-Carter model. We compare the usual Lee-Carter model to an extended version. While we observe that Ghana's mortality has decreased, our results show that longevity risk raises the cost of liability. We propose that defined benefit scheme trustees and fund managers, assess their funds and act on the assets available to them in a Liability-Driven Investment (LDI) Strategy tailored to the dynamics of their scheme - since there is no single best method in LDI strategy. This study may help policymakers or practitioners in their efforts to mitigate the effects of longevity risk.

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Introduction

Countries saw a significant decrease in mortality rates during the twentieth century, with progress emerging - albeit at different ages across time. There have been enormous improvements in some decades but no signs of improvement in others [1]. There have been major concerns about forecasting increased life expectancy because of its nature of uncertainty.

The magnitude of increase or decrease in mortality is a point of contention. There are basically two schools of thought. One argues that there is no limit to the expectancy of life, Oeppen and Vaupel [2]. The other contends that great strides in the field of medicine, epidemiological change and healthy lifestyle have led to changes in mortality only to generate a minimal increase in life expectancy. This implies that, in the long run, life expectancy will either increase or stall [3]. The increase in life expectancy is significant as a result of human population dynamics as well as ongoing scientific and medical advancements and technology. The global increase in human life expectancy has far-reaching implications for many aspects of our society, including healthcare systems, pension plans, and the insurance industry [4]. Conversely, it poses a great risk about the social security system's long-term viability, since longevity is clearly linked to funds reserves. The decline in mortality contributes directly to the density of a growing number of elderly people, leading to additional costs for life insurers and annuity providers [5].

The traditional or deterministic method of quantifying mortality provides point estimates which tend to underestimate mortality gains [6,7].

Abbreviations: SVD, Singular Value Decomposition; DB, Defined Benefit; LDI, Liability Driven Investment.

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The Lee-Carter model, a recent model works best in attempting to reveal or bring out trends over time as well as forecast into the future. The Lee-Carter model and its numerous extensions are the most widely used methods for estimating mortality rates. To forecast, the Lee-Carter model and its many extensions employ a latent process [8]. The Lee-Carter model employs a probabilistic or stochastic approach to estimate and predict mortality - it has a stochastic component called the mortality time trend index. This model is the most recent and widely used in estimating and forecasting mortality trends, and it has successfully explained mortality data from many countries (eg, G7 countries). However, stochastic models bind probabilities to distinct outcomes, allowing for an accurate assessment of the probability or likelihood of occurrence. Thus, stochastic models are far superior because they are more likely to capture or assess the likelihood of uncertainty surrounding improvements in life expectancy.

The study of human population demography necessitates an understanding of and quantification of mortality rates. Mortality rates play an important role in actuarial calculations such as cash flow analysis, premium pricing, reserves, benefits, and life insurance product pricing. Although mortality rates have been observed to change over time, forecasting future mortality rates is critical in demography and actuarial science [9]. Clearly, systematic underestimation of mortality declines would eventually lead to financial strain or insolvency, as well as the collapse of life insurance and pension providers. This could be due to higher than expected pay-out ratios as policyholders live longer than expected. A pension fund must be financially sound in order to meet its own liabilities and administrative costs. Thus the present value of the outflow (benefits) should equal the present value of the inflow (contributions). Ultimately, the financial goal is to strike a balance between benefits and contributions. When this is achieved, the fund is able to meet its administration and financial obligations. A shift, which leads to a situation in which benefit outflows exceed inflow of funds, results in unfunded actuarial liabilities with long-term financial implications for the scheme.

Longevity risk is the possibility that some members of the reference population will live longer than expected [10]. Longevity risk has an impact on defined benefit (DB) pension plans. In general, longevity risk affects the net liabilities of defined benefit pension plans through lifetime annuity payments, because unexpected improvements in mortality and life expectancy lengthen payment periods [11]. Paying a high turn out of ratios of unexpected liabilities (claims) results in a loss, and these payments tend to put a strain on the funds because they are not anticipated. This gradually depletes funds in the reserves in the long run.

Longevity risk is one of the most significant and widely overlooked risks that insurance companies, reinsurance companies, pension plan sponsors, and the government face [12]. Clearly, if this risk is not managed, both fund managers and policyholders will be left disappointed in the long run. As a result, there is a need to address the uncertainties surrounding mortality trends by capturing and quantifying these gains.

In Ghana, insurance and pension companies generally adjust and use an alternate South African mortality table. Although the South Africa table may be adequate for administrative, business, or insurance purposes, a more accurate table based on Ghanaian data will be preferred.

We extend the basic Lee-Carter model for this important issue by introducing a simple parameter which is the needed exposure. In place of a well-constructed life table, this parameter could be used as a simple basic life table tool.

Using data from Ghana, we show how the mortality rate has decreased significantly. Utilising Lee-Carter's stochastic model, we forecast mortality using a random walk.

Methods

Ghana's 2010 population census data was used as the reference or base year for subsequent estimated population in successive years. The following assumptions were made: first, a constant force of mortality across all age groups for each year; and second, a constant growth rate across all age groups for each year.

The assumptions were considered because it is difficult to obtain data on the crude death rate for each age group each year. As a result, the assumption is based on the available annual crude death rates for the entire population.

Lee-Carter model

The Lee-Carter model employs a probabilistic or stochastic approach to estimate and predict mortality - it has a stochastic component,the mortality time trend index. This model is the most recent and widely used in estimating and forecasting mortality trends, and it has successfully explained mortality data from many countries (eg, G7 countries).

The Lee-Carter model is given by

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

where

$m_{x,t}$: the central age-specific death rate in the year t at age x, i.e. the number of deaths in an age group over exposure.
a_x : average age-specific mortality (arithmetic mean of the natural log mortality rates over time), describing the average mortality pattern.
b_x : how fast or slow mortality is, as the form of each age varies as k_t changes.
k_t : a mortality time-trend index describing the general mortality level each year.

$\varepsilon_{x,t}$: error term reflecting age-specific influences at x age and t time, not captured by the model, $N(0, \sigma^2)$

Key: all parameters on the right side of the equation are not measurable, it is difficult to fit the model with ordinary minimum squares as used in linear regression. We use [13] two-stage estimation method to solve this situation, which offers detailed solutions.

$ln(m_{x,t})$; The natural log of the central death rates in the matrix form is given as

$$\begin{bmatrix} ln(m_{0,2010}) & ln(m_{0,2011}) & \dots & ln(m_{0,2019}) \\ ln(m_{1--4,2010}) & ln(m_{1--4,2011}) & \dots & ln(m_{1--4,2019}) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ ln(m_{95+,2010}) & ln(m_{95+,2011}) & \dots & ln(m_{95+,2019}) \end{bmatrix}$$

$$a_x = \frac{1}{T}\sum_t lnm_{x,t} \tag{2}$$

where T is the total number of observation periods.

Estimation using exact solutions

To estimate a_x we do a row summation of the matrix and we divide each row summed value by the total number of years of observation. As an example, the estimated value of average age-specific mortality for age group under one is

$$a_0 = \frac{1}{T}\sum_{t=1}^{10} lnm_{x,t} \tag{3}$$

$$a_0 = \frac{1}{10}[lnm_{0,2010} + lnm_{0,2011} + \ldots + lnm_{0,2019}] \tag{4}$$

To estimate parameters b_x and k_t, we create a matrix $Z_{x,t}$.

$$Z_{x,t} = \ln(m_{x,t}) - a_x = b_x.k_t \tag{5}$$

There is a clear identifiability problem - the matrix $Z_{x,t}$ is composed of both the parameters b_x and k_t together. We need to extract the parameters from this matrix by applying the single value decomposition to the Zx, t matrix and traditionally resolving it by securing the parameters to meet a pair of restraints (b_x and k_t are normalized to unity and zero, respectively) given by;

$$\sum_x b_x = 1, \sum_t k_t = 0 \tag{6}$$

Estimation using singular value decomposition

Singular value decomposition (SVD) is applied to the $Z_{x,t}$ matrix, which divides the matrix into three matrices

$$ULV = SVD(Z_{x,t}) = L_1U_{x1}V_{t1} + \ldots + L_XU_{xX}V_{xX} \tag{7}$$

where the age element is represented by U, the singular values are L and the time part is represented by V. The first vector of the time dimension matrix obtains k_t and the first singular value component ($k_t = L_1V_{T1}$), b_x from the first age component matrix vector ($b_x = U_{x1}$).

The best rank-1 approximation was used to derive k_t obtained from the first vector of the time variable matrix averaged by the first singular value, and b_x is obtained from the first vector of the matrix of the age component.

When b_x and k_t were determined, Lee and Carter found that the total number of deaths observed is not assured to be equal to the fitting number of deaths (slight difference). Therefore by seeking a value that makes the observed number of deaths equal to the expected number of deaths, they made a second stage estimate of k_t. Re-estimating k_t is at the researcher's discretion since the difference is small.

The time series model

The Lee-Carter approach is distinguished by the fact that, after reducing the time dimension of the mortality to a single index, k_t they model and forecast this index using statistical time series methods. The standard univariate time series model ARIMA (0,1,0) is used by Lee and Carter to estimate or forecast the mortality index. Other ARIMA models have been shown to be superior for certain data sets, but some studies have agreed with Lee and Cater overwhelmingly and shown that the

Random Walk with Drift Model (RWD) works best in practice. The RWD model is presented next.

$$k_t = k_{t-1} + \theta + \varepsilon_t \tag{8}$$

$$\varepsilon_t \sim N(0, \sigma_{rw}^2) \tag{9}$$

The drift parameter, θ which is usually negative, represents the linear trend component in the change of k_t and its maximum likelihood is simply given by

$$\theta = \frac{k_T - k_1}{T-1} \tag{10}$$

which is only dependent on the first and last k_t estimates. The standard error is estimated as

$$\sigma_{rw} = \sqrt{\frac{1}{T-1}\sum_{l=1}^{T}[k_t - k_{t-1} - \theta]^2} \tag{11}$$

The variance is estimated as

$$\sigma_{rw}^2 = \frac{1}{T-1}\sum_{l=1}^{T}[k_t - k_{t-1} - \theta]^2 \tag{12}$$

Then we plug in the drift parameter θ in equation, and also replace k_t pushed back in time one period to predict two periods ahead:

$$k_t = k_{t-1} + \theta + \varepsilon_t \tag{13}$$

$$k_t = (k_{t-2} + \theta + \varepsilon_{t-1}) + \theta + \varepsilon_t \tag{14}$$

$$k_t = k_{t-2} + 2\theta + (\varepsilon_{t-1} + \varepsilon_t) \tag{15}$$

We repeat the same procedure iteratively to forecast the mortality index k_t at time $T + (\Delta t)$, and obtain

$$k_{T+(\Delta t)} = k_T + (\Delta t)\theta + \sum_{l=1}^{\Delta t}\varepsilon_{T+1-1} \tag{16}$$

$$k_{T+(\Delta t)} = k_T + (\Delta t)\theta + \sqrt{(\Delta t)}.\varepsilon_t \tag{17}$$

where Eq. (17) is an expression enabled by the assumption that the random variables ε_t in this model are independent and have the same variance. The equation shows that as the square root of the distance to the forecast horizon $\Delta(t)$ increases, so does the forecast's conditional standard errors. Forecast point estimates that follow a straight line as a function of $\Delta(t)$, with slope, θ can be obtained as

$$k_{T+\Delta(t)} = k_T + (\Delta t)\theta \tag{18}$$

Now, we plug these expressions into the empirical and vectorized versions of Eq. (18) to generate a point estimate forecast for log-mortality as

$$\hat{m}_{T+(\Delta t)} = \hat{a} + \hat{b}.k_{T+(\Delta t)} \tag{19}$$

$$\hat{m}_{T+(\Delta t)} = \hat{a} + \hat{b}[\hat{k}_T + (\Delta t)\theta] \tag{20}$$

For example, given data from 2010 to 2019, the forecast for 2030 is as follows.

$$\hat{m}_{2030} = \hat{a} + \hat{b} \times [\hat{k}_{2019} + 11\hat{\theta}] \tag{21}$$

$$\hat{m}_{2030} = \hat{a} + \hat{b} \times \left[\hat{k}_{2019} + 11\frac{\hat{k}_{2019} - \hat{k}_{2010}}{9}\right] \tag{22}$$

Simulation of random walk

A bit of simulation may be the best way to understand the stochastic nature of the projection. We set time horizons and generate random trajectories. The strategy is to generate normally distributed errors, add the drift, and then add the previous value to each value, which is easiest done as a running sum; we simulate using this process to generate random trajectories.

Graphing

We use the ggplot2 package in R statistical software, which is for exploratory graphs, model diagnostics, and presentation of models. We store estimated parameters or data into data frame before we can execute using the ggplot2 package or commands to graph our data.

Update on the model

Our model, which is an extension of the Lee-Carter model, is provided by

$$ln(m_{x,t}) = a_x + b_x k_t + n_{x,t} + \varepsilon_{x,t} \tag{23}$$

The updated model is based on the exact solution, singular value decomposition, and time series component of the Lee-Carter model, but we include the needed exposure parameter in the equation, which is also estimated using the exact solution and time series method of the Lee-Carter model. Where $n_{x,t} = \frac{1}{m_{x,t}}$, is the needed exposure to produce a unit death case in each age group. $n_{x,t}$ is a time series variable which can be simulated or forecasted using random walk.

We add each needed exposure,$n_{x,t}$ from all age groups to get the sum total of the needed exposures for each year, N_t. This N_t is used as a weight. Then each $n_{x,t}$ is weighted over the sum total, N_t (i.e. $n_{x,t}/N_t$ will now become our new $n_{x,t}$ value). These weighted values are then used in the model.

The needed exposure parameter, is a simple tool that aids insurance companies and pension plan schemes in product engineering and pricing, benefit calculation, and future liabilities.

Second update

The new parameter, $n_{x,t}$ increases the error sum of squares. As a result, we were motivated to reduce the sum of squares of errors. This is solved by squaring the parameter, $n_{x,t}$ in Eq. (24) below

$$ln(m_{x,t}) = a_x + b_x k_t + n_{x,t}^2 + \varepsilon_{x,t} \tag{24}$$

Model performance

Explanation ratio, R

We assess the performance of the models by using the explanation ratio, R which is given by;

$$R = 1 - \frac{\sum_{t=0}^{T}\sum_{t=0}^{\infty}\varepsilon_{x,t}^2}{\sum_{t=0}^{T}\sum_{t=0}^{\infty}(lnm_{x,t} - a_x)^2} \tag{25}$$

R is defined as the proportion of variance explained (or linearly accounted for) by the first k_t principal components: When the provision with k_t components has no error, it returns a value of 1, implying that it explains 100 percent of the variance.

AIC and BIC

$$AIC = 2 * k - 2 * \ln(L) \tag{26}$$

where; $2 * k$ increases as we increase the number of parameters in the model this penalises us for building complex model and $-2 * \ln(L)$ decreases as the model gets better for explaining the data well, this rewards us for fitting the data well.

$$BIC = k * \ln(n) - 2 * \ln(L) \tag{27}$$

The likelihood of the model

$$W_i = \frac{exp(-1/2\Delta_i)}{\sum \exp(-1/2\Delta_i)} \tag{28}$$

where; W_i is the probability or odds that a model is the best of those listed.

Δ_i is the change in AIC.

Regression specification

The linear regression model is used to support the assertion or claim that longevity risk has a financial impact on pension liabilities. We transform the annuity certain formula into a linear regression. The annuity certain formula is given by

$$L = PB\left[\frac{1-(1+r)^{-n}}{r}\right] \tag{29}$$

where we define the variables as follows

P: number of pensioners or policyholders
B: benefit paid
r: interest rate
n: number of future payouts
L: liability to pay or offset

The annuity certain formula is the present value of all future benefits to be paid at the time of evaluation; we transform this formula by taking the logarithmic form of the equation.

$$\log(L) = \log(P) + \log(B) + \log\left[\frac{1-(1+r)^{-n}}{r}\right] \tag{30}$$

We can rewrite the last term of the equation as;

$$\log(L) = \log(P) + \log(B) + \log\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right] \tag{31}$$

$$\log(L) = \log(P) + \log(B) + \log\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right] \tag{32}$$

$$\log(L) = \log(P) + \log(B) + \log\left[\frac{(1+r)^n-1}{(1+r)^n}\right] \tag{33}$$

$$\log(L) = \log(P) + log(B) + log\left[\frac{\frac{(1+r)^n-1}{(1+r)^n}}{\frac{r}{1}}\right] \tag{34}$$

$$\log(L) = \log(P) + \log(B) + \log\left[\frac{(1+r)^n-1(1)}{(1+r)^n(r)}\right] \tag{35}$$

$$\log(L) = \log(P) + \log(B) - \log(r) + \log[(1+r)^n-1] - n\log(1+r) \tag{36}$$

Linearizing the two last two terms of Eq. (36), $\log[(1+r)^n-1] - n\log(1+r)$ we obtain

$$\log(L) = \alpha + \beta_1\log(P) + \beta_2\log(B) + \beta_3\log(r) + \beta_4 n + \beta_5 n\log(r) + \varepsilon \tag{37}$$

where parameters are defined as

α: Constant
β_1: Number of policyholders predictor variable
β_2: benefit predictor variable
β_3: Interest rate predictor variable
β_4: effect of one additional year predictor variable
β_5: effect of one additional year$*$interest rate predictor variable
ε: error term

Table 1
Estimated parameters.

Age	Age coding	a_x	b_x	k_t
0	1	−4.861079	0.04766996	1.6087340
1–4	2	−4.861076	0.04766346	1.2335557
5–9	3	−4.861074	0.04766671	0.8714742
10–14	4	−4.861071	0.04766570	0.5272888
15–19	5	−4.861072	0.04766473	0.1579730
20–24	6	−4.861076	0.04766243	−0.2284200
25–29	7	−4.861074	0.04767507	−0.5820044
30–34	8	−4.861062	0.04766654	−0.9197545
35–39	9	−4.861079	0.04766837	−1.2042400
40–44	10	−4.861049	0.04767321	−1.4646067
45–49	11	−4.861068	0.04767488	
50–54	12	−4.861087	0.04768055	
55–59	13	−4.861069	0.04765523	
60–64	14	−4.861084	0.04768866	
65–69	15	−4.861058	0.04769879	
70–74	16	−4.861046	0.04765278	
75–79	17	−4.861071	0.04771833	
80–84	18	−4.861127	0.04767034	
85–89	19	−4.860999	0.04761351	
90–94	20	−4.860777	0.04751030	
95+	21	−4.860856	0.04676044	

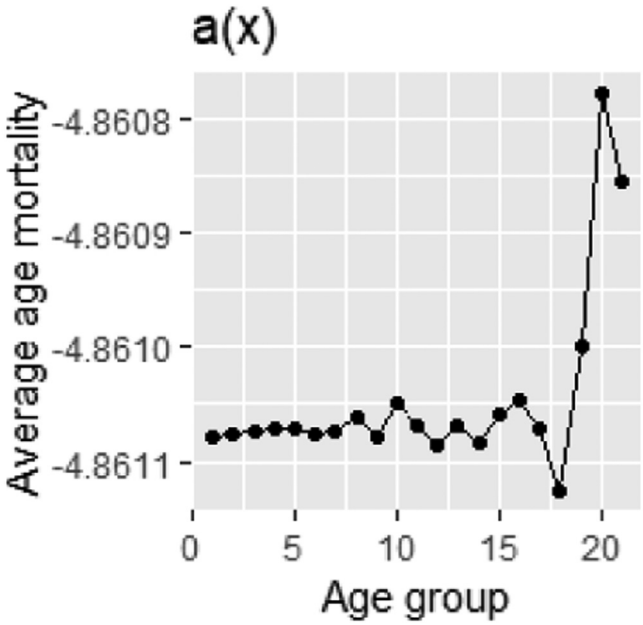


Fig. 1. A graph of average mortality for the period of observation (2010–2019).

The coefficient β_4, is of particular interest because it represents the effect of one additional year of life expectancy on the present value of pension liabilities.

Results and discussion

Estimated parameters

The estimated parameter values from the Lee-Carter model are shown in Table 1.

Average mortality, a_x

Fig. 1 depicts the average age mortality trends across all age groups.

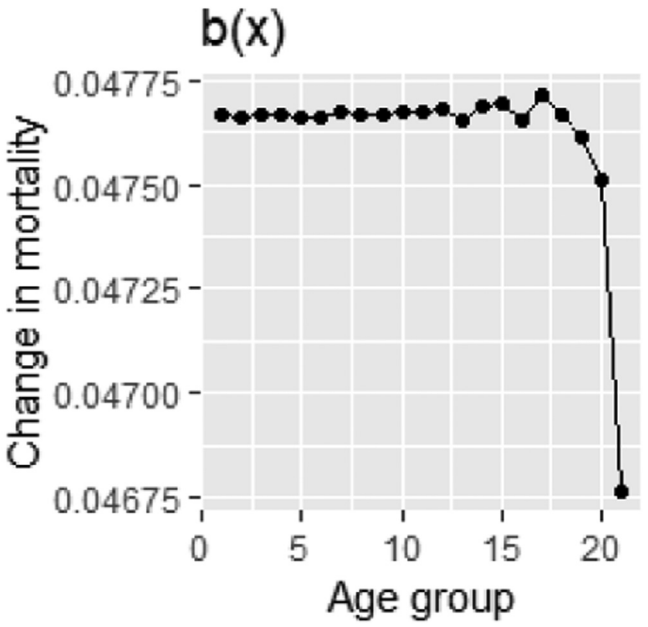


Fig. 2. A graph of change in mortality for the whole period of observation (2010–2019).

From Fig. 1, we ca see that the average mortality rate a_x is quite steady at the younger age group cohorts and then begins to fluctuate just after the age group 15–19 (which is coded as 5 on the age axis) to age group 80–84 (coded as 18) where average mortality is the highest, then it starts to peak to age group 90–94 (20) then drops.

Change in mortality, b_x

Fig. 2 depicts the change in mortality trends across all age groups.

From Fig. 2, we can see that the b_x deviations show how mortality changes quickly or slowly in each age group as the mortality index k_t, changes over time. When b_x for an age group is high, the death rate at that age group varies greatly, whereas when b_x is low, the death rate at that age varies minimally with changes in the overall mortality degree.

Change in pace of mortality is quite steady at the younger ages and begins to fluctuate just after the age group 50–54 (12) to 75–79 (17) where it peaks then it begins to drop steeply (change in mortality is high) at older ages as expected.

The comparative size of the correlated Lee-Carter shock is limited to the relative size of the mortality decline rate over time. For example, the ratio of the shock size in age group 80 to age group 60 is always equal to b_{80} /b_{60}. The premise is that, age groups that are expected to receive larger shocks are more likely to have a higher mortality rate than others. The observed shift or change in the mortality rate in a specific age group, is then easily reflective of its sensitivity to change. Potential mortality shocks, we believe, would have the greatest impact on age groups that have already increased the most rapidly.

The proportion of b_{70-74} to b_{95+} indicates how mortality rates are changing. Changes in mortality are only about 1.02 times faster at the age of 95 than at the age group 70–74.

Mortality time-trend index, k_t

Fig. 3 depicts the general mortality time trend index over the observation period.

From Fig. 3, it is clear that the mortality time-index k_t exhibits a linear trend over time (showing steady mortality decrease over the years). This means that over the last ten years, Ghana has seen a decrease in mortality (improvement in life expectancy). The mortality time trend index has a straight line trend for two reasons; as a result of the assumptions (constant force of mortality and constant growth rate) used in estimating or projecting population numbers, the graph displays a straight trend. These assumptions give all age group nearly equal weights, so age groups with small numbers get the same weight as age groups with large numbers. Second, in using singular value decomposition (SVD) and best rank of order-1 approximation in the estimation of Lee-Carter model parameters; the values of k_t were weighted over time using the first vector of the time component averaged by the first singular value component.

Random walk

Figs. 4 and 5 exhibit a range of simulated random walks.

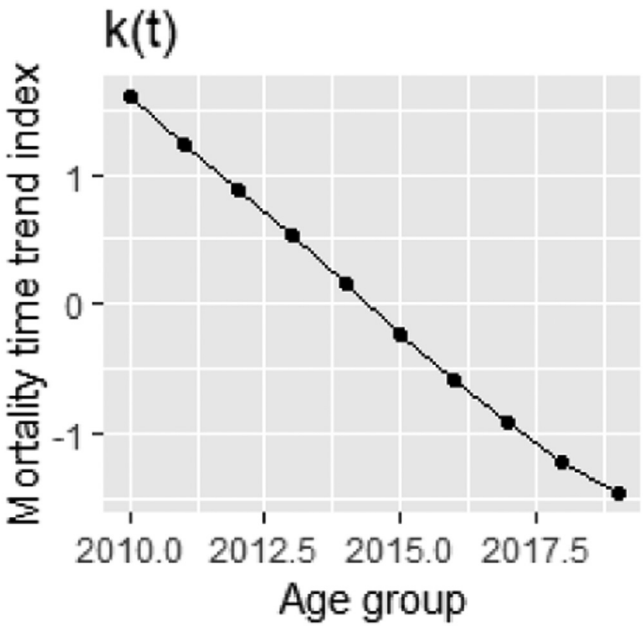


Fig. 3. A graph of mortality time trend index for the whole period of observation (2010–2019).

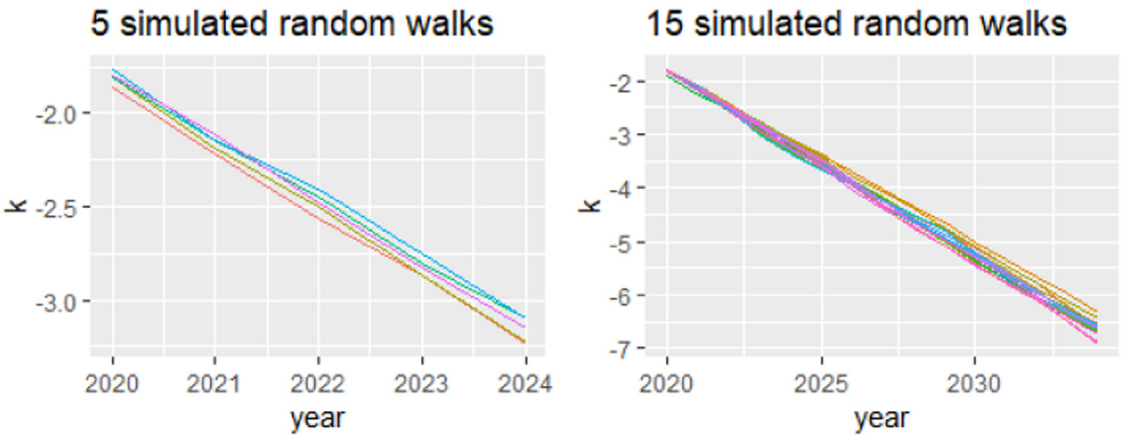


Fig. 4. Graphs of random walk for random trajectories.

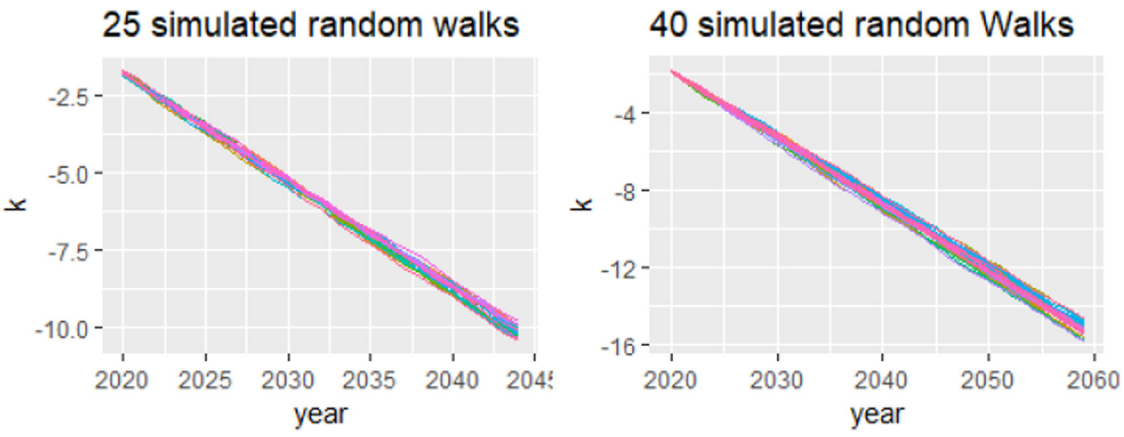


Fig. 5. Graphs of random walk for random trajectories.

Table 2
Needed exposures, 2010.

Age	2010($N_{x,t}$)
0	26
1–4	150
5–9	499
10–14	801
15–19	593
20–24	425
25–29	324
30–34	204
35–39	173
40–44	134
45–49	124
50–54	97
55–59	93
60–64	66
65–69	52
70–74	42
75–79	33
80+	19

We simulate a stochastic process driving the mortality time index k_t, a random walk with drift parameter, θ is −0.3441585 and innovations with standard deviation of 0.040318686. The drift parameter is computed from Eq. (10) and the deviation is computed from Eq. (11). The following is an example of a simulation of forty random walk trajectories in R.

```
S=matrix(0,40,40)
for(j in 1:40) S[,j]= -1.4646067 + cumsum(-0.3441585+ rnorm(40,0,0.040318686))
DF=data.frame(S);DF$year = 2020:2059
DF
forty<-ggplot(gather(DF,sim,k,-year),aes(year,k,color=sim))
+ geom_line()+ guides(color=F)+ggtitle('40 simulated random Walks')
```

Age-specific forecast

Once we have a forecast for k_t we can combine it with vectors a_x and b_x to produce a forecast of age-specific mortality. This can be used to create or revise a life table every year if necessary (due to the nature of uncertainty in life expectancy improvement), especially for pension and insurance companies.

Needed exposure

The needed exposure parameter, $n_{x,t}$ is shown in Table 2 below and indicates the number of exposures that are likely to result in a unit death case.

This parameter can be useful in present value of benefit evaluations and product or policy development.

Model performance

Lee-Carter model

For the Lee-Carter model, the explanation ratio, R is 99.9937 percent thus the model explains about 99.9937 percent of variability of the data. In other words, above 99 percent of the age-specific mortality change was accounted for by k_t (the general mortality index). This implies that the model explains the data well.

Update on the model

The addition of the needed exposure parameter, $n_{x,t}$ to the basic Lee-Carter model significantly increased the error sum of squares, indicating that this model is unable to explain the variability. It only explains 0.0044 percent of the variability.

Second update

The motivation was to minimize the error sum of squares because the parameter $n_{x,t}$ increases the error sum of squares. This is solved by squaring $n_{x,t}$. For this model, the explanation ratio R is 99.767 percent of the variability. As a result, the general mortality index adequately accounts for age-specific mortality changes.

AIC and BIC

The AIC and BIC values for the Lee-Carter model and the Second Updated Lee-Carter model are shown in Table 3 below.

Table 3
AIC and BIC.

Model	AIC	ΔAIC	BIC	W_i
Second Update (quadratic)	34.30085	0.00	52.91031	0.9315352
Lee-Carter	39.52188	5.22103	51.03639	0.06846475

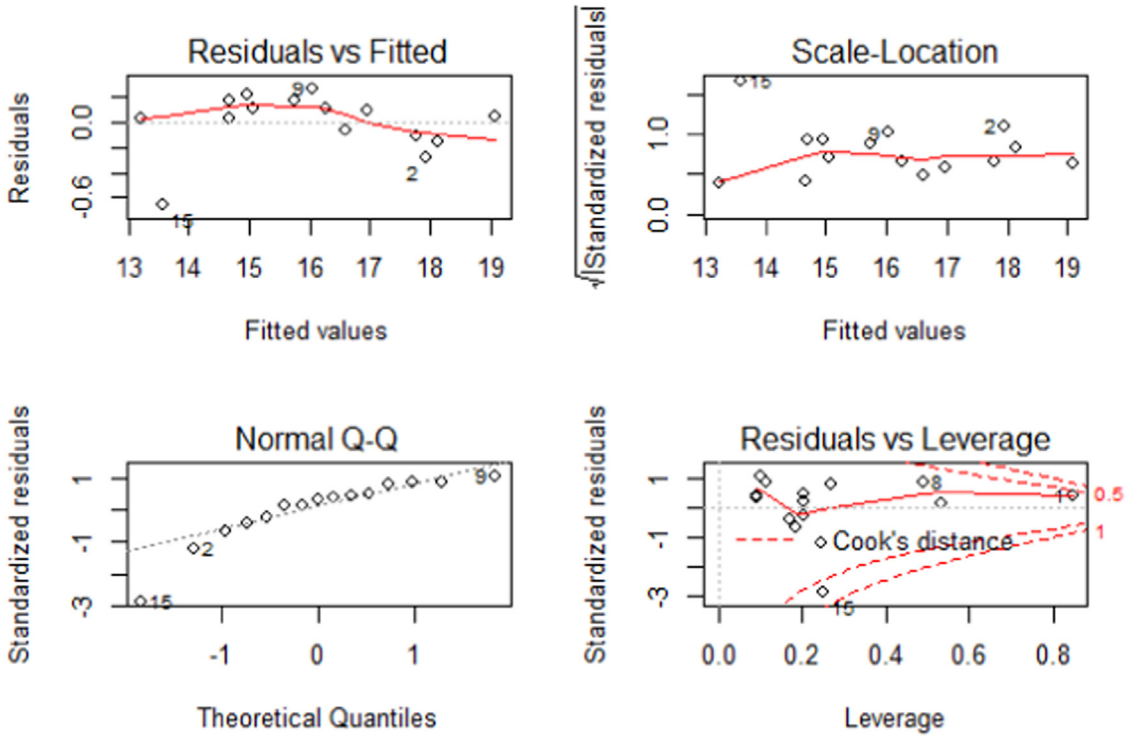


Fig. 6. diagnostic plots.

Going by the AIC numbers reported in Table 3, the second update (quadratic version of the model) is preferred. The greater the delta (change) in AIC, which is a measure of entropy, the less plausible the model. W_i denotes the likelihood of each model, or the odds that a model is the best of the ones listed. Where W_i is

$$W_i = \frac{\exp(-1/2\Delta_i)}{\sum \exp(-1/2\Delta_i)} \tag{38}$$

Although the additional parameter increases the model's complexity, it adequately explains the data in the second update. When it comes to AIC and BIC, the lower the number, the better.

Further analysis

Prior findings and analysis show that Ghana has experienced a decrease in mortality rate, which translates or implies an increase in life expectancy. We use a linear regression on the present value annuity-certain formula (log-transformed) to test our hypothesis that longevity risk has a financial impact on pensions. The assumptions were as follows: first, everyone in the sample lived to the terminal age of 75, and the second assumption was using an annuity-certain formula rather than a life annuity formula based on the first assumption.

Diagnostic plot

Fig. 6 depicts the various diagnostic plots, which provide us with additional information about the data set used in our regression.

The Residual vs. Fitted plot in Fig. 6 shows that the residuals are spread quite evenly around the horizontal line, indicating a linear relationship. This indicates that the regression specification model is a good fit and adequately explains the data. The Q-Q plot clearly shows that the residuals have a normally distributed distribution. The Spread-Location plot indicates whether residuals are distributed equally across predictor ranges. In this case, they actually show homoscedasticity. The Residuals vs. Leverage plot depicts influential cases that may alter results if they are excluded or included. There are

Table 4
Summary table.

Residuals: Min	1Q	Median	3Q	Max
−0.65601	−0.08030	0.04295	0.14295	0.26432
	Estimate	Std. Error	*t* value	Pr(> |*t*|)
(Intercept)	1.78201	1.59043	1.120	0.286385
LogP	1.38840	0.30407	4.566	0.000808
LogB	0.74461	0.17386	4.283	0.001293
n	0.14128	0.03352	4.215	0.001450

some extreme cases where the regression line may or may not be influenced. This plot shows that the eighth, tenth, and fifteenth observation cases could potentially change the analysis of the results.

Summary table

The model summary table, Table 4, provides information about the regression line's ability to account for the total variation in the dependent variable.

- Residual standard error is 0.2643 on 11 degrees of freedom.
- Multiple *R*-squared is 0.9818
- Adjusted *R*-squared is 0.9768
- *F*-statistic is 197.7 on 3 and 11 DF
- *p*-value: 7.566e−10

Although a single rate was used in calculating benefits regardless of time of valuation, it is clear from the summary Table 4 that the parameters Log(r) and nLog(r) are not defined. This tells us that the two variables are not linearly independent because the information they provide is contained in the other variables, rendering them redundant.

The Social Security and National Insurance Trust (SSNIT) calculates benefits at the current treasury bill rate (10 percent or lower) per evaluation date. But the treasury bill rate has always been higher than 10 percent and has never fallen below 10 percent, so they use 10 percent for valuations. Considering discounting, using high treasury bill rates to calculate benefits results in a much smaller benefit being paid. SSNIT settles for 10 percent to avoid small valued benefits.

The R^2 (coefficient of determination) is 98.18 per cent, and the adjusted R^2_{adj} is 97.68 percent, implying that the predictors explain about 98 percent of the variation. The overall *F*-statistic of 197.7 and the *P*-value of 7.566e−10 indicate that the combined effect of all variables is statistically significant - that is, the dependent variable Log(L) is well explained by the independent variables (the model's predictors), and thus the model is significant.

We notice that liability in this case is a function of the number of participants (policyholders), benefits (claims), and additional year lived by policyholders or pensioners, all of which play a significant role in the cost of liabilities. Our main interest is the model's coefficient β_4, which is the effect of one additional year of life expectancy on the present value of life expectancy. This means that an extra year of life expectancy at retirement increases pension liabilities by 14.13 percent, which is significant and has a long-term adverse influence on pension funds, potentially leading to insolvency.

Conclusion

The empirical result shows that Ghana's life expectancy has increased over the ten year period that the data for this study covers, implying the existence of longevity risk on life contingent annuities. We demonstrated empirically that longevity risk on defined benefit schemes has a significant impact on the cost of liabilities. The results of this study show that the models (both Lee-Carter and quadratic) fit the data well. The second model (quadratic) which is an update to the Lee-Carter model with the needed exposure parameter, is a simple tool that will assist insurance companies and pension plan schemes in product engineering and pricing, benefit calculation, and future liabilities.

The liability of pension funds is a function of the number of participants (policyholders), benefits (claims), and additional years lived by policyholders or pensioners, all of which account for a significant portion of the cost of liabilities.

Recommendation

In light of the evidence presented, it is the responsibility of DB scheme trustees and fund managers to assess their funds and act on the assets available to them in a Liability-Driven Investment (LDI) Strategy tailored to the dynamics of their scheme - there is no single best method in LDI strategy.

The essential rule in this regard is to hedge against interest rate risk in order to meet future obligations. Further research should be directed towards developing a high-quality human mortality database that will benefit everyone (researchers, students, insurance, reinsurance, and pension companies), thereby addressing the issue of insufficient or poor data quality. Future research can update this research by using 2021 population census data of Ghana as well as a finer administrative data.

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CRediT authorship contribution statement

Nana Poku Appiagyei Nantwi: Conceptualization, Visualization, Writing – original draft, Data curation. **Anani Lotsi:** Conceptualization, Supervision, Writing – review & editing. **Godwin Debrah:** Methodology, Supervision, Project administration.

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