JAST, 2018;11(2):35-47

DESIGN OF FRACTIONAL ORDER SLIDING MODE CONTROLLER FOR 2 DOF HELICOPTER SYSTEM

Gurkan KAVURAN*

Firat University, Department of Mechatronics Engineering, Turkey
gkavuran@firat.edu.tr

Celaleddin YEROGLU

Inonu University, Department of Computer Engineering, Turkey
c.yeroglu@inonu.edu.tr

Received: *05th April 2018* ***Accepted:*** *07th July 2018*

ABSTRACT

This paper presents a real time control application for a prototype helicopter system by using Sliding Mode Control strategy modified with small differential deviations. Experimental and simulation studies demonstrate the performanceof the proposed controller for a class of flight systems with unstable dynamics. It is shown that the proposed controller decreases the steady state errors of the vertical and horizontal motion of the system. Small differential deviations strengthen or weaken the effect of the differential to obtain better tracking response in case of external disturbances. Numerical and experimental results match each otherto demonstratethe tracking of desired rotor angles and suppression the disturbance effects.

Keywords: *Fractional Calculus, Sliding Mode Control, 2 DOF Helicopter System, TRMS.*

2 SERBESTLİK DERECELİ HELİKOPTER SİSTEMİ İÇİN KESİR DERECELİ KAYMA KİPLİ KONTROLÖR TASARIMI

ÖZET

Bu çalışma, küçük diferansiyel değişimler ile modifiye edilmiş Kayma Kipli Kontrol stratejisini kullanarak, prototip helicopter sistemi için gerçek zamanlı bir kontrol uygulaması sunmaktadır. Deneysel ve benzetim çalışmaları, kararsız dinamiklere sahip bir uçuş sistemi için önerilen kontrolcünün performansını ortaya koymaktadır. Önerilen kontrolcünün, sistemin dikey ve yatay hareketinin kalıcı durum hatalarını azalttığı gösterilmiştir. Küçük diferansiyel sapmalar, harici bozucu durumunda daha iyi takip cevabı elde etmek için diferansiyelin etkisini güçlendirir veya zayıflatır. Sayısal ve deneysel sonuçlar, istenen rotor açılarının takibi ve bozucu etkilerin bastırılmasını göstermek için birbirleriyle eşleşmektedir.

AnahtarKelimeler: *Kesirli Hesaplama, Kayma Kipli Kontrol, 2 Serbestlik Dereceli Helikopter Sistemi, TRMS.*

1. INTRODUCTION

Nowadays, researches on air vehicles are rapidly increasing owing to requirements and technological advancements. Acceptable new designs enable different types of aircrafts to have effective vertical take-off and landing. One of the well-known vehicles of this type is the conventional helicopter with hovering, high maneuverability and flying at very low speeds. Thus, autonomous or non-autonomous helicopters have attracted the attention of many researchers in military and civil fields. However, developing control algorithms for these vehicles have been considered as one of challenging problems of recent researches[1].

The flight control problem involves many complications. Altitude, payload, weather conditions, complex mechanisms, nonlinear aerodynamics and changes in flight conditions may disturb flight stability [2, 3]. The control algorithm for the aerial vehicles should overcome these difficulties. Thanks to Sliding Mode Controller (SMC) structure having strong ability to deal with high nonlinearity and

* Corresponding Author

parameter fluctuations of control systems and thus it may help to improve system stabilization. In variable structure control theory, the conventional sliding mode controller (CSMC) structure is known as an essential topic. The CSMC increases robustness of the system and compensate parameter fluctuations [4]. Many researches still have been conducting the SMC in recent studies [5-10]. Some researches proposes to combine the SMC structure with the Fractional Order Differentiation (FOD) for various applications [11,12].

This paper proposes a robust controller that is based on variable structure control technique and small deviations in differential terms of the discontinuous and continuous parts of the control law of SMC structure. The proposed control strategy is applied to control an experimental flight control set-up, so called Twin Rotor Multi-Input Multi-Output (MIMO) System (TRMS), which has two degrees of freedom related with pitch and yaw angles. Controlling the TRMS is one of the challenging control problems, due to the coupling effects between two axes and nonlinearities in motion mechanism. Unpredictable environmental factors and nonlinearities affect the system output that results in undesired control action. Several methods have been proposed to achieve better control performance for TRMS in the literature [13, 14]. Tao et. al, proposed parallel distributed fuzzy LQR to control the pitch and yaw angles of TRMS [15]. Martinez et. al, have used a nonlinear H∞ control strategy for disturbance rejections [16].

In the point of flight control practice, this study aims to use online SMC enhanced with Small Differential Deviation (SDD) to achieve precise and smooth control of TRMS. Experimental and simulation results have proved that the SDD strengthen or weaken the effect of differentiation in the SMC depending on the sign of deviation. In this case, Sliding Mode Controller with Small Differential Deviation (SMC-SDD) can successfully suppress the external disturbances and overcome the negative impacts. Thus, the main contribution of this paper is to use FOD to modify the derivative term of the SMC for control performance of multivariable system. The performance improvement is clearly illustrated with simulation studies and real time experiments.

Following sections provide problem statements, basic definitions, SMC-SDD design, simulation and experimental results, respectively.

2. PROBLEM STATEMENT AND PRELIMINARIES

The importance of real time application of the smooth and precise control of the prototype helicopter system is the main motivation of the paper. In the literature survey, authors encountered several SMC methods for TRMS. Thus, enhancement of the SMC for aerial vehicle will be appreciated. Some recent studies propose fractional order SMC design methods. However, this study employs SDD in the SMC that is simple to implement for multi-variable real time running system, yet effective for external disturbance suppression, capability of fast, precise and overshot-free control.

Recent improvements in implementation of FOD enable researchers to apply fractional order expressions to various research areas. We investigated that deviations from integer order differential terms of SMC structure effects the performance of control. Positive effect of the deviation motivates us to design more effective SMC structure for real time running prototype helicopter system.

2.1. Fractional Order Differentiation

Fractional-order derivative was discussed in 1690s with the letter between LeHopital and Leibniz [17]. Theoretical improvement of fractional calculus and related developments in computing, make the it easy to use in different science and engineering fields [18], [19]. Fractional calculus is the general form of integer order integro-differential expressions and it is represented by ${}_aD_t^{\alpha}$.While $\frac{d^{\alpha}}{dt^{\alpha}}$ represents fractional derivative, $\int_a^t (d\tau)^{-\alpha}$ shows fractional order integration, where α is the order of integro-differential expression [20], [21]. In fact, the order of integro-differential expression, α, defines an amount of deviation from integer order derivative or integration. So, one may say that the fractional order α is amount of deviation from the integer order one.

This paper discusses small deviations in derivative that make fine tuning on the effect of the derivation. Results are verified via simulation and real time applications. In other words, small deviations of the integer order derivative and integral make the effect of differentiation softer or harder depending on the sign of deviation α. In this study, authors aimed to make fine tuning on the effect of differentiation in the SMC by using small deviations in the integro-differential expressions.

2.2. Conventional SMC

Conventional SMC is widely used in nonlinear systems to satisfy the stability when parameter fluctuations exist [22]. The SMC structure has two steps. In the first step, the sliding surface $S(t)$ is selected. Secondly, the equivalent control law is determined for reaching and keeping on sliding axis. The $S(t)$ is given as [22],

$$S(t) = \left(\frac{d}{dt} + \lambda \right)^{n-1} e(t) \tag{1}$$

here, $e(t) = x_d(t) - x(t)$ is tracking error of $x(t)$, $x_d(t)$ is reference input, λ is slope of the sliding surface, n is system order. The purpose of the control is to preserve the state variables on the sliding axis in (1). Following Lyapunov function was defined to determine the control law,

$$V(S) = \frac{1}{2} S^2(t) \tag{2}$$

If the Lyapunov function in (2) satisfies (3) for positive constant η, the system reaches to $S(t) = 0$.

$$\frac{1}{2} \frac{d}{dt} S^2(t) \le -\eta \left| S(t) \right| \tag{3}$$

One can obtain (4) using (3), to enforce $e(t)$ to become zero all times.

$$S(t) D S(t) \le 0 \tag{4}$$

where D is integer order derivative operator. The total control law $u(t)$ consist of equivalent and discontinuous control terms as,

$$u(t) = u_{eq}(t) + u_d(t) \tag{5}$$

This paper uses small deviations in the differentiation of the discontinuous and continuous parts of the control law.

2.3. SMC with FOD

Authors have recently proposed to use fractional order differentiation to improve the control performance of SMC for unstable time delay systems. SMC with FOD can be summarized from [11] as following;

The $S(t)$ in (1) can be written as,

$$S(\bar{x};t) = \left[\sum_{k=0}^{i} \binom{i}{k} \left[D \right]^k \lambda^{i-k} \right] e(t) \tag{6}$$

here $i = n-1$, $k = 0,1,\mathrm{L}\ n-1$, $(d/dt) = D$. The fractional operator D^{α} widen the differentiation effects in (6) as [11],

$$S(\bar{x};t) = \left[\sum_{k=0}^{i} \binom{i}{k} \left[(D)^k D^{\pm \alpha_k} \right] \lambda^{i-k} \right] e(t) \tag{7}$$

where $0 < \alpha_k < 1$. Equations (1)-(7) can be re-arranged for multi-variable systems to strengthen or to weaken the effect of the differentiation at sliding surface of SMC to make precise and overshoot-free control of MIMO systems. The proposed structure will also be robust against external disturbances. Next section provides the proposed controller design.

3. SMC-SDD DESIGN FOR LTI SYSTEMS

Consider the integer-order controllable system as,

$$Dx(t) = \mathbf{A}x(t) + \mathbf{B}(u(t) + d\left(x(t),t\right) \tag{8}$$

here, $x(t)$, $u(t)$ represents state vector and control input respectively. $\mathbf{A}$ and $\mathbf{B}$ are system parameter matrix and $d(x(t),t)$ is an external disturbance.

Assumption 1: The disturbance with known upper bound is continuous derivable as, [22].

$$\left\| d(x(t),t) \right\| < d_0\left(x(t),t\right) \tag{9}$$

The sliding surface can be written for the LTI system as [22],

$$S(t) = \mathbf{c}^T \mathbf{x}(t) \tag{10}$$

where $c \in \mathbf{R}^{mxn}$, $m\,x\,n$ represents parameter matrix. For $DS(t) = 0$ one can write the equivalent control law as,

$$\mathbf{c}^T Dx = \mathbf{c}^T \left(\mathbf{A}x(t) + \mathbf{B}u(t) \right) \tag{11}$$

$$u_{eq}(t) = -(\mathbf{c}^T \mathbf{B})^{-1} \mathbf{c}^T \left(\mathbf{A}x(t) \right) \tag{12}$$

One can write the total control law by adding discontinuous term, $u_d(t) = -Ksgn\left(S(t)\right)$, as,

$$u(t) = -(\mathbf{c}^T\mathbf{B})^{-1}\mathbf{c}^T\left(\mathbf{A}x(t)\right) - Ksgn\left(S(t)\right) \tag{13}$$

Assumption 2: In accordance with (10) and (13), sliding mode exists if the matrix $(\mathbf{c}^T\mathbf{B})$ has an inverse and $rank(\mathbf{B}) = m$ [22].

Let Assumptions 1 and 2 are satisfied under the control law (13). The sliding mode exists and $e(t)$ will be zero at all times if (4) is fulfilled. One can derivate the Lyapunov function as,

$$\begin{aligned} DV &= S(t)DS(t) \\ &= S(t)\left[\mathbf{c}^T\mathbf{A}x(t) + \mathbf{c}^T\mathbf{B}u(t)\right] \\ &= S(t)\left[\mathbf{c}^T\mathbf{A}x(t) + \mathbf{c}^T\mathbf{B}\begin{pmatrix}-(\mathbf{c}^T\mathbf{B})^{-1}\mathbf{c}^T(\mathbf{A}x(t)) - \\ Ksgn(S(t))\end{pmatrix}\right] \\ &\le -K\left|S(t)\right| \end{aligned} \tag{14}$$

where, $sgn\left(S(t)\right) = \left|S(t)\right| / S(t)$. It can be seen that, the tracking error converges to sliding surface if the parameter of K is chosen as $K > 0$.
For the LTI system with $n = 2$, the control law can be defined as,

$$\begin{aligned} u(t) = -\left(\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}b_{11}\\ b_{21}\end{bmatrix}\right)^{-1}\begin{bmatrix}c_1 & c_2\end{bmatrix}\left(\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}X_1(t)\\ X_2(t)\end{bmatrix}\right) \\ -K\,sgn\left(\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}X_1(t)\\ X_2(t)\end{bmatrix}\right) \end{aligned} \tag{15}$$

Following equation defines the dynamic relations of the system in (15).

$$X_2(t) = DX_1(t) \tag{16}$$

Then, one can define following Remarks,

Remark 1: $D^{\pm\beta}$ weaken or strengthen the effect of differentiation in the continuity term, which forces $DS(t)$ to be zero and keeps it to stay on the sliding surface, with $X_2(t) = D^{1\pm\beta}X_1(t)$, where $0 < \beta < 1$.

Remark 2: $D^{\pm\alpha}$ weaken or strengthen the effect of differentiation in the discontinuity term, which compensates the fluctuations while reaching to the sliding surface, with $X_2(t) = D^{1\pm\alpha}X_1(t)$, where $0 < \alpha < 1$.

3.1. Stability Analysis and Reachability Condition of SMC-SDD

Stability of dynamical systems is an essential property to design a necessary and satisfactory controller. There are several methods in the literature to investigate the stability of the integer systems. An integer-order LTI system is stable if all roots of the characteristic polynomial are in the left half of the *s* plane. However, the numerical solutions such as Routh-Hurwitz criteria cannot be used directly for a fractional order systems [23]. Oustaloup et. al was used Linear Matrix Inequality methods to check if matrix of Eigen values belong to a definite region in the complex plane [24]. This technique has been developed for ordinary and interval fractional order LTI systems. In addition, Matignon was investigated internal and external stabilities with the closed angular segment for linear fractional systems [25].

Fractional order LTI system in (17) can be considered in state-space form as [26],

$$\begin{aligned} {}_0D_t^{\boldsymbol{\alpha}}x(t) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) \end{aligned} \tag{17}$$

where $y \in R^p$, $u \in R^r$ are $x \in R^n$ are output, input and state vectors respectively. $\mathbf{C} \in R^{p\times n}$, $\mathbf{B} \in R^{n\times r}$, $\mathbf{A} \in R^{n\times n}$ and $\boldsymbol{\alpha} = [\alpha_1, \alpha_2 \text{K}\ \alpha_n]^T$ are the fractional orders.

Theorem 3.1: If the condition in (18) is satisfied for $\alpha_1 = \alpha_2 = \text{K} = \alpha_n \equiv \boldsymbol{\alpha}$, one can say that the commensurate-order system in (17) is stable [26].

$$\left|Arg(eig(\mathbf{A}))\right| > \alpha\frac{\pi}{2} \tag{18}$$

where $eig(\mathbf{A})$ is eigen values and $0 < \alpha < 2$.

However, Theorem 3.1 cannot be used if the system in (17) has various derivative orders. The system in (17) is called as an incommensurate-order order system if $\alpha_1 \neq \alpha_2 \neq \text{K} \neq \alpha_n$. Let us define Fractional Order Differential Equation (FODE) with *n*-term as,

$$a_nD_t^{\alpha_n}y(t) + \text{L} + a_1D_t^{\alpha_1}y(t) + a_0D_t^{\alpha_0}y(t) = 0 \tag{19}$$

here $\alpha_k (k = 0,1,\text{K},n)$ are real numbers ($\alpha_n > \alpha_{n-1} > \text{K} > \alpha_0 \ge 0$), $a_k (k = 0,1,\text{K},n)$ are constant coefficients. The analytical solution of the

equation in (19) is assumed to be written as [27],

$$y(t)=\frac{1}{a_n}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\sum_{\substack{k_0+k_1+\mathrm{L}+k_{n-2}=m\\ k_0\geq 0;\mathrm{K};k_{n-2}\geq 0}}(m;k_0,k_1,\mathrm{K},k_{n-2})\times \prod_{i=0}^{n-2}\left(\frac{a_i}{a_n}\right)\varepsilon_m\left(t,-\frac{a_{n-1}}{a_n};a_n-a_{n-1},a_n+\sum_{j=0}^{n-2}(a_{n-1}-a_j)k_j+1\right) \tag{20}$$

where $(m;k_0,k_1,\mathrm{K},k_{n-2})$ are coefficients and $\varepsilon_k(t,\lambda;\mu,v)$ is Mittag-Leffler function [27]. So the analytical solution of the FODE in (20) for $\alpha_1\neq\alpha_2\neq\mathrm{K}\neq\alpha_n$ and $u(t)=0$ is stable if $\lim_{t\to\infty}y(t)=0$. Also, the stability test techniques for general fractional order LTI system can be found in [28]. Let the desired sliding mode surface is given by the fractional derivative form for LTI system in (17) as,

$$S(t)=c_1X_1(t)+c_2D^{1\pm\alpha}X_1(t) \tag{21}$$

Remark 3: The operator $D^{1\pm\alpha}$ is the $(\pm\alpha-1)^{th}$ order integration of $X_2(t)$ that behave as a low-pass filter. Also, it may compensate the high-frequency fluctuations of disturbances. In this regard, the fractional-order sliding surface defined in (21) is smoother compared with conventional one.

Equation 15 can be rewritten using Remarks 1 and 2 as,

$$u(t)=-\left(\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}b_{11}\\ b_{21}\end{bmatrix}\right)^{-1}\begin{bmatrix}c_1 & c_2\end{bmatrix}\left(\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}X_1(t)\\ D^{1\pm\beta}X_1(t)\end{bmatrix}\right) -K\,sgn\left(\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}X_1(t)\\ D^{1\pm\alpha}X_1(t)\end{bmatrix}\right) \tag{22}$$

If (21) is rearranged,

$$\begin{aligned}c_1X_1(t)+c_2D^{1\pm\alpha}X_1(t)&=0\\ D^{1\pm\alpha}X_1(t)&=-c_1/c_2\,X_1(t)\end{aligned} \tag{23}$$

where $0<1\pm\alpha<2$. It is shown according Theorem 3.1 that the sliding surface dynamics are asymptotically stable if the sliding plane constant, c_1/c_2, is selected to be positive.

Let us define the sliding surface according to (7) for higher order system ($n=3$) to show the same procedure as,

$$\begin{aligned}S(t)&=\left(\binom{2}{2}\left(D^2D^{\pm\alpha_2}\right)\lambda^0\right)e(t)+\left(\binom{2}{1}\left(D^1D^{\pm\alpha_1}\right)\lambda^1\right)e(t)\\ &\quad+\left(\binom{2}{0}\left(D^0D^{\pm\alpha_0}\right)\lambda^2\right)e(t)\\ &=D^{2\pm\alpha_2}e(t)+2D^{1\pm\alpha_1}\lambda e(t)+D^{\pm\alpha_0}\lambda^2e(t)\end{aligned} \tag{24}$$

The curve of tracking error attains to $S(t)=0$, to satisfy the reaching law as,

$$D^{2\pm\alpha_2}e(t)+2D^{1\pm\alpha_1}\lambda e(t)+D^{\pm\alpha_0}\lambda^2e(t)=0 \tag{25}$$

It can be seen from (25) that the expression of the sliding surface corresponds with the FODE in (19). If fractional orders are $2\pm\alpha_2\neq1\pm\alpha_1\neq\pm\alpha_0$ and $2\pm\alpha_2>1\pm\alpha_1>\pm\alpha_0$ then, the $S(t)$ becomes incommensurate-order system. The analytical solution of this equation can be computed by using (20) satisfying the convergence of error function $\lim_{t\to\infty}e(t)=0$. Otherwise, the least common multiple (LCM) and Riemann sheets will be helpful to determine the stability of the incommensurate-order systems.

Theorem 3.2 : Reaching mode can be shown by direct switching function approach that the system trajectory is under the following reaching condition [29].

$$\lim_{S_i\to0^+}S_i(t)<0\ \text{ and }\ \lim_{S_i\to0^-}DS_i(t)>0 \tag{26}$$

$$S_i(t)DS_i(t)<0\qquad i=1,\mathrm{L}\ ,m \tag{27}$$

One can add small differentiation deviations to the time derivative of sliding surface in (11) as,

$$DS(t)=\begin{bmatrix}c_1 & c_2 & c_3\end{bmatrix}\left(\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}X_1(t)\\ D^{1\pm\beta}X_1(t)\\ X_3(t)\end{bmatrix}\right) \tag{28}$$

where, $X_2(t)=D^{1\pm\beta}X_1(t)$. The dynamics of the switching function can be specified by using Gao's constant rate reaching law approach as in [30],

$$DS(t)=-K_d\,\mathrm{sgn}\left(S(t)\right) \tag{29}$$

where $K_d>0$. By using (28) and (29),

$$DS(t) = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1(t) \\ D^{1\pm\beta} X_1(t) \\ X_3(t) \end{bmatrix} \right) = -K_d \operatorname{sgn}(S(t)) \quad (30)$$

Following equation can be obtained from (30) by adding $D^{1\pm\alpha}$ to discontinuity term as,

$$DS(t) = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1(t) \\ D^{1\pm\beta} X_1(t) \\ X_3(t) \end{bmatrix} \right) = -K_d \operatorname{sgn} \left(\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} X_1(t) \\ D^{1\pm\alpha} X_1(t) \\ X_3(t) \end{bmatrix} \right) \quad (31)$$

If the time derivative of saturation function is used instead of signum function in (31), following equation can be written,

$$DS(t) = -K_d \frac{\left(\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} X_1(t) \\ D^{1\pm\alpha} X_1(t) \\ X_3(t) \end{bmatrix} \right)}{\left| \left(\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} X_1(t) \\ D^{1\pm\alpha} X_1(t) \\ X_3(t) \end{bmatrix} \right) \right| + \delta} \quad (32)$$

where $\delta > 0$ is a small value. Sign of switching functions vary under the following conditions.

$$\frac{S(t)}{|S(t)| + \delta} = \begin{cases} < 0, & c_1 X_1(t) + c_2 D^{1\pm\alpha} X_1(t) + c_3 X_3(t) < 0 \\ > 0, & c_1 X_1(t) + c_2 D^{1\pm\alpha} X_1(t) + c_3 X_3(t) > 0 \end{cases} \quad (33)$$

Thus the time derivative of sliding surface can be obtained as,

$$DS(t) = \begin{cases} > 0, & when \ \dfrac{S(t)}{|S(t)| + \delta} < 0 \\ < 0, & when \ \dfrac{S(t)}{|S(t)| + \delta} > 0 \end{cases} \quad (34)$$

It is clear that;

- $DS(t) > 0$ for $K_d > 0$ and $S(t) < 0$
- $DS(t) < 0$ for $K_d > 0$ and $S(t) > 0$

According to Theorem 3.2, $S(t)DS(t) \le 0$ and $e(t)$ is zero at all times and the system is stable in the presence of the SMC-SDD controller with the time derivative of saturation function.

4. EXPERIMENTAL SETUP

The flight control simulator, TRMS, is composed of two propellers joined by a shaft pivoted on its base. The shaft can rotate freely on both vertical and horizontal planes. TRMS is widely used in laboratories in the literature to mimic the flight controls of a helicopter.

TRMS control involves some difficulties, due to the mutual interaction between the two axes, and the nonlinearity in motion of mechanism [31]. The control purpose of TRMS is to achieve a quick and accurate position. A mathematical model of the TRMS should be correctly derived to design SMC-SDD accurately. Approximate mathematical models of the TRMS are obtained by using Newtonian and Lagrangian methods by considering effects of various forces existing in the system[32].

4.1. Dynamical Model of TRMS

Basic definitions in (35)-(42) represent vertical and horizontal model of the system. These equations are obtained by considering the system dynamics. Dynamic model of the vertical subsystem is derived as follows [5], [6], [30],

$$\frac{d\psi(t)}{dt} = D\psi(t) \quad (35)$$

$$D^2\psi(t) = \frac{a_1}{I_1}\tau_1(t)^2 + \frac{b_1}{I_1}\tau_1(t) - \frac{M_g}{I_1}\sin\left(\psi(t)\right) + \frac{0.0326}{2I_1}\sin\left(2\psi(t)\right)\left(D\varphi(t)\right)^2 - \frac{B_{1\psi}}{I_1}D\psi(t) - \frac{k_{gy}}{I_1}\cos\left(\psi(t)\right)\left(a_1\tau_1(t)^2 + b_1\tau_1(t)\right)D\varphi(t) \quad (36)$$

$$D\tau_1(t) = -\frac{T_{10}}{T_{11}}\tau_1(t) + \frac{k_1}{T_{11}}u_\psi(t) \quad (37)$$

Dynamic model of the horizontal subsystem can be given as,

$$\frac{d\varphi(t)}{dt} = D\varphi(t) \quad (38)$$

$$D^2\varphi(t)=\frac{a_2}{I_2}\tau_2(t)^2+\frac{b_2}{I_2}\tau_2(t) -\frac{B_{1\varphi}}{I_2}D\varphi(t)-\frac{1.75}{I_2}k_c\left(a_1\tau_1(t)^2+b_1\tau_1(t)\right) \quad (39)$$

$$D\tau_2(t)=-\frac{T_{20}}{T_{21}}\tau_2(t)+\frac{k_2}{T_{21}}u_\varphi(t) \quad (40)$$

where $\psi(t)$ and $\varphi(t)$ are pitch and yaw angles, $D\psi(t)$ and $D\varphi(t)$ are angular velocity around the vertical and horizontal axis, $\tau_1(t)$ and $\tau_2(t)$ are output torque of main and tail rotors modeled as a first order linear system, $u_\psi(t)$ and $u_\varphi(t)$ are voltage applied to main and tail rotorrespectively. Remaining parameters of the TRMS are provided in [30]. The mathematical model in (35)-(40) can be represented in state space form as,

$$D\begin{bmatrix}\psi(t)\\ D\psi(t)\\ \tau_1(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -B_{1\psi}/I_1 & b_1/I_1\\ 0 & 0 & -T_{10}/T_{11}\end{bmatrix}\begin{bmatrix}\psi(t)\\ D\psi(t)\\ \tau_1(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ k_1/T_{11}\end{bmatrix}u_\psi(t)+f_\psi(t) \quad (41)$$

$$D\begin{bmatrix}\varphi(t)\\ D\varphi(t)\\ \tau_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -B_{1\varphi}/I_2 & b_2/I_2\\ 0 & 0 & -T_{20}/T_{21}\end{bmatrix}\begin{bmatrix}\varphi(t)\\ D\varphi(t)\\ \tau_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ k_2/T_{21}\end{bmatrix}u_\varphi(t)+f_\varphi(t) \quad (42)$$

where $f_\psi(t)$ and $f_\varphi(t)$ are assumed to be unknown nonlinearity.

5. SMC-SDD DESIGN FOR TRMS

In this section, the proposed SMC-SDD structure given in Section 3 is adapted to the TRMS to eliminate the undesirable effect of external disturbance and to obtain fast and overshoot-free step response. Approximate mathematical modelof the system can be rewritten in the following form as a convenience,

$$\begin{aligned}DX_1(t)&=X_2(t)\\ DX_2(t)&=\gamma_1X_2(t)+\gamma_2X_3(t)\\ DX_3(t)&=\gamma_3X_3(t)+Bu(t)\end{aligned} \quad (43)$$

$[X_1(t)\ \ X_2(t)\ \ X_3(t)]^T=[\psi(t)\ \ D\psi(t)\ \ \tau_1(t)]^T$ state vector and input $u(t)=u_\psi(t)$ are defined in the case of pitch angle response, while $[X_1(t)\ \ X_2(t)\ \ X_3(t)]^T=[\varphi(t)\ \ D\varphi(t)\ \ \tau_2(t)]^T$ and input $u(t)=u_\varphi(t)$ are defined for yaw angle. The γ parameters are defined as $\gamma_{1\psi}=-B_{1\psi}/I_1$, $\gamma_{2\psi}=b_1/I_1$ $\gamma_{3\psi}=-T_{10}/T_{11}$ and $\gamma_{1\varphi}=-B_{1\varphi}/I_2$, $\gamma_{2\varphi}=b_2/I_2$, $\gamma_{3\varphi}=-T_{20}/T_{21}$ for the vertical and horizontal subsystem respectively. The error dynamics is given as,

$$e(t)=X(t)-X_d(t) \quad (44)$$

where $X(t)=\left[X_1(t)\ \ X_2(t)\ \ X_3(t)\right]^T$ is state vector, $e(t)$ is tracking error of state variable $X(t)$ and $X_d(t)$ is desired input. The sliding surface can be written as follows,

$$S(t)=c^Te(t) \quad (45)$$

where $c^T=\left[c_1\ \ c_2\ \ c_3\right]$ is coefficient vector of the error and $e(t)=\left[e_1(t)\ \ e_2(t)\ \ e_3(t)\right]^T$ is the error state vector. The sliding surface given in (44) can be rewritten as,

$$S(t)=c_1e_1(t)+c_2e_2(t)+c_3e_3(t) \quad (46)$$

where $e_1(t)=X_1(t)-X_d(t)$, $e_2(t)=X_2(t)$ and $e_3(t)=X_3(t)$. Thus the sliding surface becomes as,

$$S(t)=c_1\left(X_1(t)-X_d(t)\right)+c_2DX_1(t)+c_3X_3(t) \quad (47)$$

The derivative of $S(t)=c^Te(t)$ is given by,

$$DS(t)=c_1\left(X_2(t)-DX_d(t)\right)+c_2DX_2(t)+c_3DX_3(t) \quad (48)$$

Table 1. Parameters of the TRMS

Symbols	Definitions	Values
$B_{1\psi}$	Friction momentum function parameter of vertical axis	6x10^{-3}Nms/rad
$B_{1\varphi}$	Friction momentum function parameter of horizontal axis	1 x10^{-1}Nms/rad
I_1	Moment of inertia of vertical rotor	6.8x10^{-2} kg.m^2
I_2	Moment of inertia of horizontal rotor	2x10^{-2} kg.m^2
b_1	Static characteristic parameter	0.0924
b_2	Static characteristic parameter	0.09
a_1	Static characteristic parameter	0.0135
a_2	Static characteristic parameter	0.02
τ_1	Momentum of main rotor	$\tau_1 = \frac{k_1}{T_{11}s + T_{10}} u_\psi$
τ_2	Momentum of tail rotor	$\tau_2 = \frac{k_2}{T_{21}s + T_{20}} u_\varphi$
k_1	Gain of main motor	1.1
k_2	Gain of tail motor	0.8
T_{10}	Denominator of main motor	1.2
T_{11}	Denominator of main motor	1.1
T_{21}	Denominator of tail motor	1
T_{20}	Denominator of tail motor	1
k_{gy}	Gyroscobic momentum parameter	0.05
k_c	Gain of cross reaction momentum	-0.2 s/rad
M_g	Gravity momentum	0.32 .m

Equation (49) is obtained by substituting (43) into (48) as,

$$DS(t) = c_1\left(X_2(t) - DX_d(t)\right) + c_2(\gamma_1 X_2(t) + \gamma_2 X_3(t)) + c_3(\gamma_3 X_3(t) + Bu(t)) \quad (49)$$

The derivative of the sliding surface can be written by using Gao's reaching law as in [30],

$$DS(t) = -K_d sgn(S(t)) \quad (50)$$

The total control law $u(t)$ is obtained by using (49) and (50) as follows,

$$u(t) = \frac{1}{c_3 B}\begin{pmatrix} -c_1\left(X_2(t) - DX_d(t)\right) - c_2(\gamma_1 X_2(t) + \\ \gamma_2 X_3(t)) - c_3\gamma_3 X_3(t) - K_d sgn(S(t)) \end{pmatrix} \quad (51)$$

One can write the control law relating to the pitch and yaw angle from (51), respectively as,

$$u_\psi(t) = \frac{1}{c_3 B}\begin{pmatrix} -c_1 D\psi(t) - c_2\left(-\frac{B_{1\psi}}{I_1} D\psi(t) + \frac{b_1}{I_1}\tau_1(t)\right) \\ +c_3\frac{T_{10}}{T_{11}}\tau_1(t) - u_{\psi d}(t) \end{pmatrix} \quad (52)$$

where, $u_{\psi d}(t) = K_d sgn\begin{pmatrix} c_1\left(X_1(t) - X_d(t)\right) \\ +c_2 X_2(t) + c_3 X_3(t) \end{pmatrix}$

$$u_\varphi(t) = \frac{1}{c_3 B}\begin{pmatrix} -c_1 D\varphi(t) - c_2\left(-\frac{B_{1\varphi}}{I_2} D\varphi(t) + \frac{b_2}{I_2}\tau_2(t)\right) \\ +c_3\frac{T_{20}}{T_{21}}\tau_2(t) - u_{\varphi d}(t) \end{pmatrix} \quad (53)$$

where, $u_{\varphi d}(t) = K_d sgn\begin{pmatrix} c_1\left(X_1(t) - X_d(t)\right) \\ +c_2 X_2(t) + c_3 X_3(t) \end{pmatrix}$

The proposed SMC-SDD method can be derived in a state space form using Remark 1 and 2 for pitch and yaw angles respectively as,

$$u_\psi(t) = -\left(\begin{bmatrix} c_{\psi 1} & c_{\psi 2} & c_{\psi 3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ k_1/T_{11} \end{bmatrix} \right)^{-1} \begin{bmatrix} c_{\psi 1} & c_{\psi 2} & c_{\psi 3} \end{bmatrix} \left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & -B_{1\psi}/I_1 & b_1/I_1 \\ 0 & 0 & -T_{10}/T_{11} \end{bmatrix} \begin{bmatrix} \psi(t) \\ D^{1\pm\beta}\psi(t) \\ \tau_1(t) \end{bmatrix} \right) - K_{d\psi} sgn \left(\begin{bmatrix} c_{\psi 1} & c_{\psi 2} & c_{\psi 3} \end{bmatrix} \begin{bmatrix} \psi(t) - \psi_d(t) \\ D^{1\pm\alpha}\psi(t) \\ \tau_1(t) \end{bmatrix} \right) \quad (54)$$

$$u_\varphi(t) = -\left(\begin{bmatrix} c_{\varphi 1} & c_{\varphi 2} & c_{\varphi 3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ k_2/T_{21} \end{bmatrix} \right)^{-1} \begin{bmatrix} c_{\varphi 1} & c_{\varphi 2} & c_{\varphi 3} \end{bmatrix} \left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & -B_{1\varphi}/I_2 & b_2/I_2 \\ 0 & 0 & -T_{20}/T_{21} \end{bmatrix} \begin{bmatrix} \varphi(t) \\ D^{1\pm\beta}\varphi(t) \\ \tau_2(t) \end{bmatrix} \right) - K_{d\varphi} sgn \left(\begin{bmatrix} c_{\varphi 1} & c_{\varphi 2} & c_{\varphi 3} \end{bmatrix} \begin{bmatrix} \varphi(t) - \varphi_d(t) \\ D^{1\pm\alpha}\varphi(t) \\ \tau_2(t) \end{bmatrix} \right) \quad (55)$$

A block diagram of SMC-SDD control structure is given in Fig. 1, using (54) and (55). The *sgn* function is changed with the derivative form of saturation function in continuous time to overcome the chattering problem. This function can be written as follows,

$$\frac{S(t)}{|S(t)| + \delta} \quad (56)$$

where $\delta > 0$ is provided as a small value.

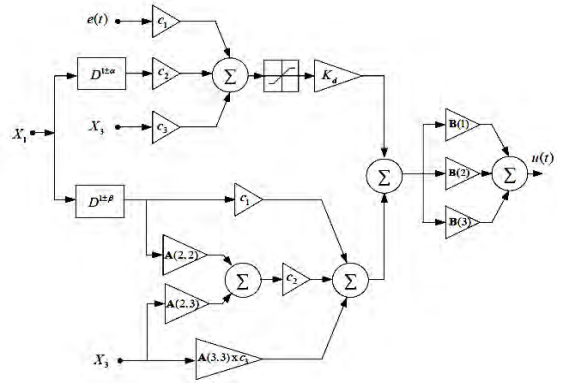


Figure 1. Block diagram of proposed SMC-SDD.

6. SIMULATION AND EXPERIMENTAL RESULTS

This section provides simulation and experimental results for vertical and horizontal axes of the TRMS. The SMC-SDD structure, proposed in this paper is implemented on the TRMS to demonstrate the performance of control effort. An external disturbance is applied to the system to show the robustness of the proposed control strategy. The hardware setup of the TRMS has two encoders for position feedback of the propellers and I/O cards for data acquisitions. The Matlab/Simulink modules are used to obtain the model of the TRMS and SMC-SDD controller. For simulation and experimental studies, sampling frequency was selected as 1 *KHz*. The continuous fraction expansion method was used to obtain Forth order rational approximations of the FOD operator to simulate SDD [33]. Block diagram of the closed-loop system and experimental setup of TRMS are given in Figs. 2 and 3 respectively.

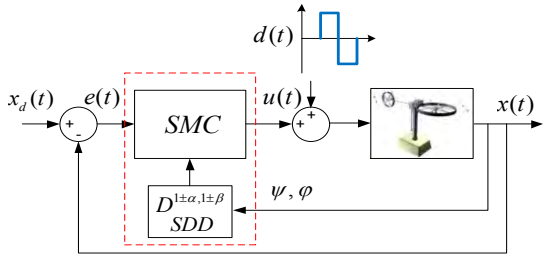


Figure 2. Block diagram of the controlled system.

The initial conditions of the TRMS were considered as $\psi(0) = 0$, $\varphi(0) = 0$ and the desired values of the pitch and yaw angles were chosen as $\psi_d(t) = 0.5$, $\varphi_d(t) = 0.5$. The SMC-SDD parameters for the vertical and horizontal subsystem were given as $K_{d\psi} = 3$, $K_{d\varphi} = 2.4$, $\delta_\psi = 0.57$, $\delta_\varphi = 0.8$, $c_\psi{}^T = [5 \;\; 0.5 \;\; 0.01]$, $c_\varphi{}^T = [4 \;\; 0.1 \;\; 0.2]$.

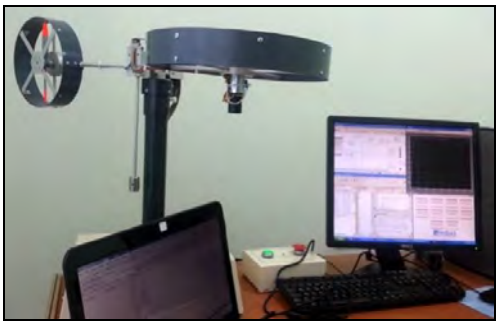


Figure 3. Experimental setup of TRMS.

A low pass filter was used at the feedback line to reject noise of the encoder. The parameters of CSMC and SMC-SDD control structures are the same except the fractional orders α and β in SMC-SDD. In other words, One can obtain CSMC from SMC-SDD if $\alpha = 0$ and $\beta = 0$.

Different values of differentiation orders α and β in (54) and (55) are used in block diagram of SMC-SDD in Fig. 1 to see the impacts of the SDD on the system performance. Figs. 4 and 5 present the simulation and experimental results of the pitch angle response of TRMS respectively, to illustrate the effect of parameter α while the β parameter was zero. One can see from inset figures that when an external disturbance is applied to the system, the amplitude of the disturbance by the SMC-SDD with $\alpha = 0.2$ parameter is smaller than the CSMC. The disturbance $d(t) = 4V$ has continued for time interval 20-21 sec. The numerical simulation in Fig. 4 confirms the experimental results in Fig. 5 in tracking desired angles and suppression the disturbance. Fig. 6 and 7 show the simulation and experimental results of the performance of the CSMC and SMC-SDD, with the $\alpha = 0$ and $\beta = (0.1,\ 0.2, 0.3)$. The disturbance remains same as in the previous case. One can see from inset figures that when an external disturbance is applied to the system, the amplitude of the disturbance by the SMC-SDD method is smaller than that of the CSMC. The β is more effective to suppress the disturbances than the α.

The higher values of β parameter usually compensate disturbance but causes higher overshoots. Simulation and experimental results in Figs. 8 and 9 were given to demonstrate the effect of α and β parameters together on SMC-SDD. The disturbance was added to the system as in previous two cases. The desired response to SMC-SDD was achieved at values $\alpha = -0.4$ and $\beta = 0.25$. There is no doubt that the variation of α and β parameters together is sufficient to improve the response of the system in the presence of disturbance.

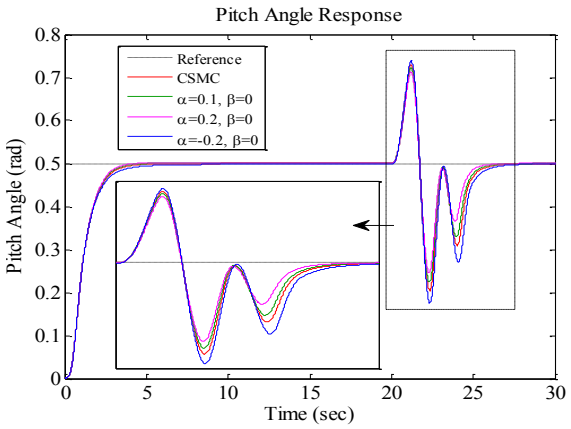


Figure 4. Simulation results of pitch angel response for deviation of α.

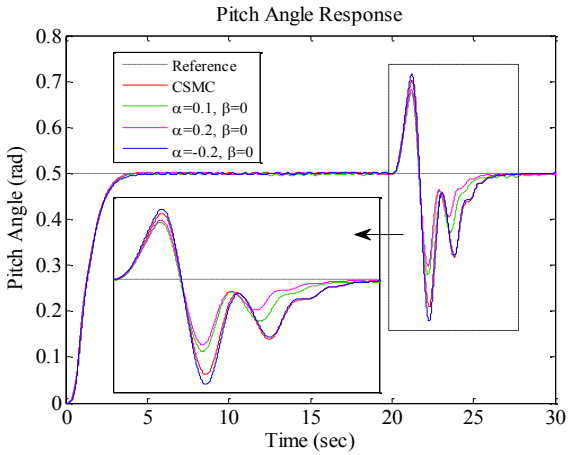


Figure 5. Experimental results of pitch angel response for deviation of α.

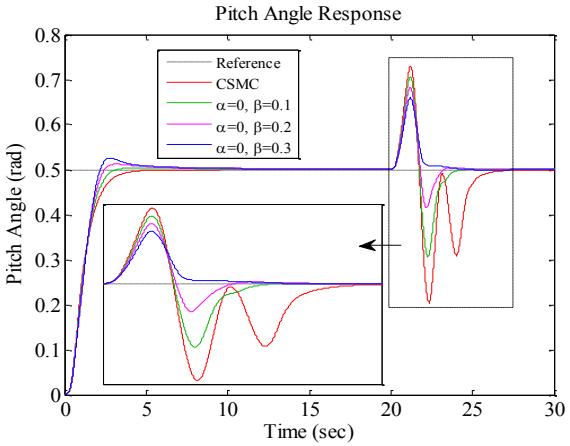


Figure 6. Simulation results of pitch angel response for deviation of β.

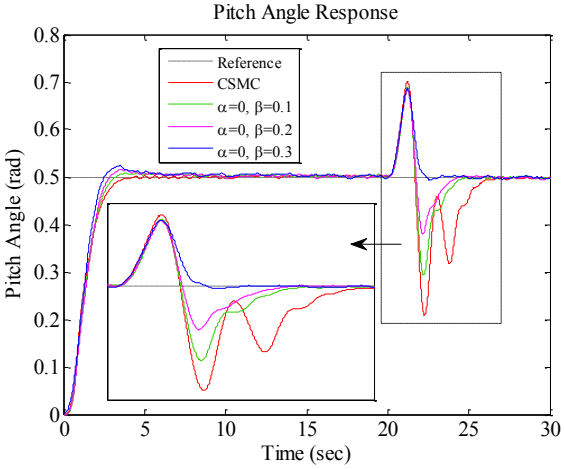


Figure 7. Experimental results of pitch angel response for deviation of β.

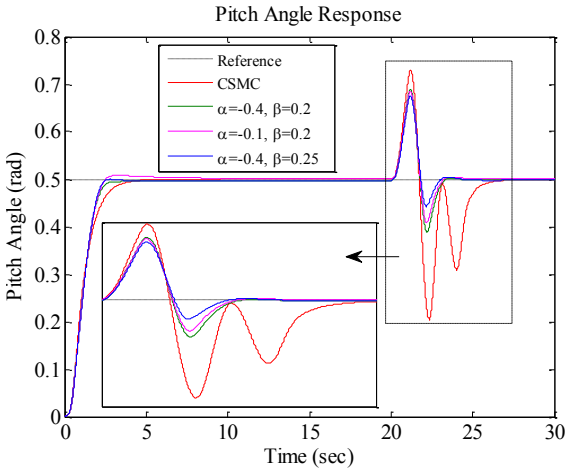


Figure 8. Simulation results of pitch angel response for deviations of α and β .

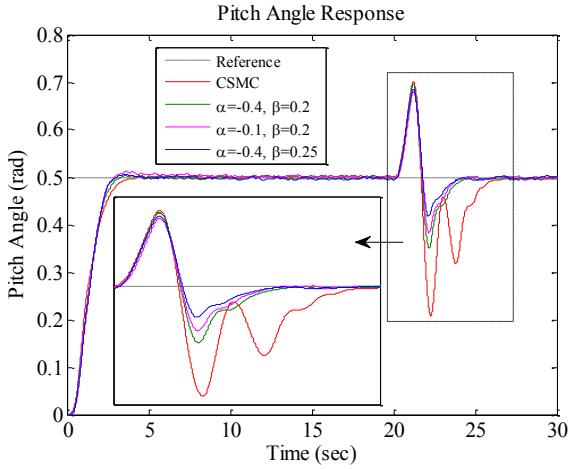


Figure 9. Experimental results of pitch angel response for deviation of α and β .

Despite the high disturbance suppression ability, the parameter β causes some overshoot. However this overshoot is eliminated adjusting α and β together. Also, β enhances the disturbance suppression effect of α . It was proved that the proposed controller method with $\alpha = -0.4$ and $\beta = 0.25$ parameters is more robust than the CSMC method.

The same SMC-SDD design procedure is applied to the yaw angle response of TRMS to illustrate the effect of the parameter α and β . In order to show the robustness of the SMC-SDD method following disturbance $d(t)$ is applied to the system for the simulation and experimental setup,

$$d(t) = \begin{cases} 2V, & 20 < t < 21 \\ -2V, & 21 < t < 22 \end{cases} \qquad (57)$$

The yaw angle response with external disturbance is depicted in Figs. 10 and 11 for simulation and experimental study respectively, to illustrate the effect of α and β parameters on SMC-SDD together. The best response of SMC-SDD was obtained for values $\alpha = -0.3$ and $\beta = 0.1$ in our tests. Despite the high disturbance suppression ability, the parameter β causes some overshoot. But this overshoot can be eliminated by using α and β together. Also, β enhances the disturbance suppression effect of α as in pitch angle response of TRMS. It is shown that the proposed controller method with $\alpha = -0.3$ and $\beta = 0.1$ parameters is more robust than the CSMC method for yaw angle response.

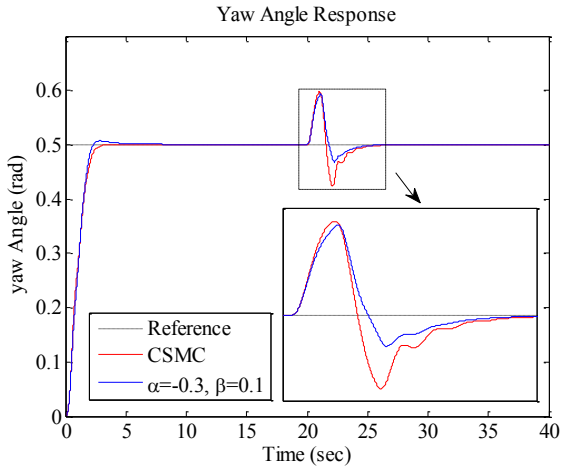


Figure 10. Simulation results of yaw angel response for deviation of α and β .

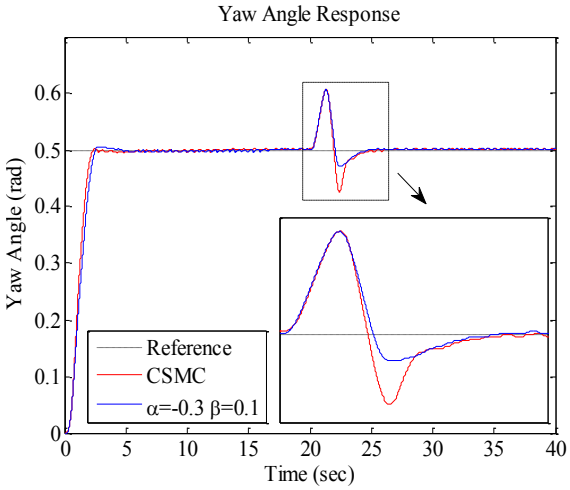


Figure 11. Experimental results of yaw angel response for deviation of α and β .

7. CONCLUSIONS

In this paper, CSMC structure was modified to enhance the control success of the multivariable TRMS by adding SDD. The SMC-SDD and CSMC method were used for the vertical and horizontal rotation of TRMS to decrease the steady state error. Numerical simulations confirm the experimental results in tracking desired angles and suppression of the disturbance. The SDD at the differential term of SMC strengthen or weaken the effect of differentiation according to the sign of deviation. The proposed method enables user to make fine-tuning for better output response. SMC-SDD successfully suppresses the external disturbances and exhibits fast and overshoot-free step response. The robustness of the system for the proposed method is better than the CSMC with the small tracking errors.

8. REFERENCES

[1] J.-G. Juang, W.-K. Liu, and R.-W. Lin, "A hybrid intelligent controller for a twin rotor MIMO system and its hardware implementation," *ISA Trans.*, vol. 50, no. 4, pp. 609–619, 2011.

[2] S. Mondal and C. Mahanta, "Chattering free adaptive multivariable sliding mode controller for systems with matched and mismatched uncertainty," *ISA Trans.*, vol. 52, no. 3, pp. 335–341, 2013.

[3] Q. Hu, B. Li, and Y. Zhang, "Robust attitude control design for spacecraft under assigned velocity and control constraints," *ISA Trans.*, vol. 52, no. 4, pp. 480–493, 2013.

[4] J. J. Slotine and W. Li, *Applied Nonlinear Control*. Prentice Hall, 1991.

[5] B. Pratap, "Neuro sliding mode controller for twin rotor control system," *2012 Students Conf. Eng. Syst.*, pp. 1–5, Mar. 2012.

[6] C. W. Tao, J. S. Taur, Y. H. Chang, and C. W. Chang, "A novel fuzzy-sliding and fuzzy-integral-sliding controller for the twin-rotor multi-inputmulti-output system," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 5, pp. 893–905, 2010.

[7] L. Huang, "An approach for robust control of a twin-rotor multiple input multiple output system," *2011 IEEE Int. Conf. Robot. Autom.*, pp. 4423–4428, May 2011.

[8] Y. Liu, T. Jia, Y. Niu, and Y. Zou, "Design of sliding mode control for a class of uncertain switched systems," *Int. J. Syst. Sci.*, vol. 46, no. 6, pp. 993–1002, 2015.

[9] E.-H. Zheng, J.-J. Xiong, and J.-L. Luo, "Second order sliding mode control for a quadrotor UAV," *ISA Trans.*, vol. 53, no. 4, pp. 1350–1356, 2014.

[10] S. Poddar, R. Kottath, V. Kumar, and A. Kumar, "Adaptive Sliding Kalman Filter using Nonparametric Change Point Detection," *Measurement*, vol. 82, pp. 410–420, 2016.

[11] B. M. Vinagre and A. J. Calderon, "On Fractional Sliding Mode Control," in *The 7th Portuguese Conference on Automatic Control*, 2006.

[12] G. Kavuran and C. Yeroglu, "Sliding mode control for fractional order plants via limit cycle prediction," in *2014 International Conference on Fractional Differentiation and Its Applications, ICFDA 2014*, 2014.

[13] A. P. S. Ramalakshmi and P. S. Manoharan, "Non-linear modeling and PID control of twin rotor MIMO system," *2012 IEEE Int. Conf. Adv. Commun. Control Comput. Technol.*, pp. 366–369, Aug. 2012.

[14] J. G. Juang, M. Te Huang, and W. K. Liu, "PID control using presearched genetic algorithms for a MIMO system," *IEEE Trans. Syst. Man Cybern. Part C Appl. Rev.*, vol. 38, no. 5, pp. 716–727, 2008.

[15] C. W. Tao, J. S. Taur, and Y. C. Chen, "Design of a parallel distributed fuzzy {LQR} controller for the twin rotor multi-input multi-output system," *Fuzzy Sets Syst.*, vol. 161, no. 15, pp. 2081–2103, 2010.

[16] M. Lopez-Martinez, C. Vivas, and M. G. Ortega, "A Multivariable Nonlinear H_inf Controller for a Laboratory Helicopter," *Proc. 44th IEEE Conf. Decis. Control*, pp. 4065–4070, 2005.

[17] G. W. Leibnitz, "Letter from Hanover, Germany, September 30, 1695 to GA L'hospital, Leibnizen Mathematische Schriften Olms Verlag," 1962.

[18] M. Zamani, M. Karimi-Ghartemani, N. Sadati, and M. Parniani, "Design of a fractional order PID controller for an AVR using particle swarm optimization," *Control Eng. Pract.*, vol. 17, no. 12, pp. 1380–1387, 2009.

[19] M. da Graça Marcos, F. B. M. Duarte, and J. A. Tenreiro Machado, "Fractional dynamics in the trajectory control of redundant manipulators," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 13, no.

9, pp. 1836–1844, 2008.

[20] R. Caponetto, G. Dongola, L. Fortuna, and I. Petras, *Fractional Order Systems: Modeling and Control Applications*. Singapore: World Scientific, 2010.

[21] K. B. Oldham and J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*. New York: Academic Press, 1974.

[22] V. I. Utkin, "Sliding mode control design principles and applications to electric drives," *Ind. Electron. IEEE Trans.*, vol. 40, no. 1, pp. 23–36, 1993.

[23] I. Petras, *Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation*. Springer Press, 2011.

[24] A. Oustaloup *et al.*, "An overview of the CRONE approach in system analysis, modeling and identification, observation and control," *IFAC Proc. Vol.*, vol. 41, no. 2, pp. 14254–14265, 2008.

[25] D. Matignon, "Stability results for fractional differential equations with applications to control processing," *Comput. Eng. Syst. Appl.*, pp. 963–968, 1996.

[26] D. Matignon, "Stability properties for generalized fractional differential systems," *ESAIM Proc.*, vol. 5, pp. 145–158, 1998.

[27] I. Podlubny, *Fractional Differential Equations*. San Diego: Academic Press, 1999.

[28] A. G. Radwan, A. M. Soliman, A. S. Elwakil, and A. Sedeek, "On the stability of linear systems with fractional-order elements," *Chaos, Solitons & Fractals*, vol. 40, no. 5, pp. 2317–2328, 2009.

[29] V. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Automat. Contr.*, vol. 22, no. 2, pp. 212–222, 1977.

[30] J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: a survey," *Ind. Electron. IEEE Trans.*, vol. 40, no. 1, pp. 2–22, 1993.

[31] "Twin Rotor MIMO System Control Experiments Manuel 33-949S." Feedback Instruments, 2006.

[32] A. Rahideh and M. H. Shaheed, "Mathematical dynamic modelling of a twin-rotor multiple input-multiple output system," *Proc. Inst. Mech. Eng. Part I J. Syst. Control Eng.*, vol. 221, no. 1, pp. 89–101, Feb. 2007.

[33] B. T. Krishna and K. V. V. S. Reddy, "Active and Passive Realization of Fractance Device of Order 1/2," *Act. Passiv. Electron. Components*, pp. 1–5, 2008.

VITAE

GURKAN KAVURAN received his B.Sc degree in Electrical and Electronics Engineering from Firat University in 2008. He received his master degree in Electrical and Electronics Engineering from Firat University in 2011 and also PhD degree in Computer Engineering from Inonu University in 2017. From 2008 to 2010, he was a Research Assistant with the Electrical and Electronics Engineering Department, Bozok University. Since 2010, he has been a Research Assistant with the Mechatronics Engineering Department, Firat University. His research interests include robotics, fractional calculus, control theory and applications, modeling and simulation, signal processing.

CELALEDDIN YEROGLU received his B.Sc. degree in Electrical and Electronics Engineering from Hacettepe University in 1990. He received his first Ph.D. degree in Computer Engineering from Trakya University in 2000 and his second Ph.D. degree in Electrical and Electronics Engineering from Inonu University in 2011. His research interests include fractional order control systems, robust control, nonlinear control, modeling and simulation, optimization algorithm, flight control.