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Developing mathematical competence: From the intended to the enacted curriculum

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ABSTRACT

This study investigates the impact of a national reform in Sweden introducing mathematical competency goals. Data were gathered through interviews, classroom observations, and online surveys with nearly 200 teachers. Contrasting to most studies of this size, qualitative analyses were conducted. The results show that teachers are positive to the message, but the combination of using national curriculum documents and national tests to convey the reform message has not been sufficient for teachers to identify the meaning of the message. Thus, the teachers have not acquired the functional knowledge of the competence message required to modify their teaching in alignment with the reform. The results indicate that for complex reform messages, such as the competency message, to have intended impact on classroom practice, special attention needs to be put on the clarity of the message. To have high-stakes tests, for example, does not alone seem to be sufficient.

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1. Introduction

Mathematics teaching is often regulated by some form of governing text through a curriculum or a set of standards. Because ideas about mathematics education are affected by educational, philosophical, and political positions, standards and curricula are changed over time (Ernest, 1991; Niss, 1981). Naturally, stakeholders in this process have great interest in seeing the intentions behind the standards or curricula influence the educational outcome (Ellsworth, 2000). The U.S. is implementing the Common Core State Standards (www.corestandards.org). Australia is in a similar position, and recently, India implemented their National Curriculum Framework in mathematics (Australian Curriculum Assessment and Reporting Authority, 2011; National Council of Educational Research and Training, 2005). Additionally, many countries with longer traditions of national standards, including Norway and Sweden, have recently implemented new national standards.

Our research investigates the impact of goals roughly similar to the NCTM process standards (NCTM, 2000) mediated through Swedish national curricula documents and national mathematics tests since 1994. Our studies were conducted in 2009 and 2010 meaning that teachers from a pure time perspective had ample possibility to absorb and implement the goals. Classroom activities, including each single task performed by any student, were analyzed in terms of their relation to the goals. Further, we conducted interviews with each teacher regarding their goals for student learning and how they viewed the goals expressed in the national curriculum documents and in national tests. The teachers also completed a survey. The

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data collection was conducted in cooperation with the Swedish School Inspectorate.[1] The schools chosen for inspection represented large and small schools, rural and city areas, and large and small municipalities. We believe that our results accurately represent Sweden as a whole concerning how a reform message, 15 years after the initiation of the reform, can be said to permeate teachers' goals and practices. Hence, this study presents an important example of using national curriculum documents and national tests as carriers for a reform message. The present article is largely based on two 60-page technical reports published in Swedish (Bergqvist et al., 2010a, 2010b).

1.1. Reforms for change?

Questions of how policy changes affect classroom practices have been frequently addressed, either in the form of qualitative classroom studies (see e.g., Ball, 1990), in system-wide research based on teacher attitude surveys coupled with assessment outcomes (see e.g., Cohen & Hill, 2000, 2001), from a theoretical perspective (see e.g., Gregoire, 2003), and with empirical support (see e.g., Charalambous & Philippou, 2010). Despite approaching the problem in different ways, all four of these examples paint a complicated but fairly coherent picture of the relation between policy and practice. Whenever the reform message is of a more complicated nature, the effects on teacher beliefs and classroom practice generally deviates from the intended. In general, it seems that if a curriculum includes content goals, such as arithmetic, then arithmetic is indeed taught, but if the curriculum includes competency goals, such as problem solving ability, then the effect on teaching may vary significantly. A commonly described mechanism behind this is that teachers only pick up surface aspects of the reform message and interprets these in terms of their own beliefs instead of actually changing their beliefs and their teaching (Ball, 1990; Charalambous & Philippou, 2010; Cohen, 1990; Frykholm, 1996; Grant, Peterson, & Shojgreen-Downer, 1996). Later in this article we describe how a theoretical and psychology-based model by Gregoire (2003) is applied to analyze how such a situation can occur and to identify the psychological motives (and subsequent actions) needed for a teacher to truly change conceptually.

Cases in which curriculum change did in fact lead to changes in teaching seem sparse, but Cohen and Hill (2000) note that when the state and several professional organizations, such as the NCTM and its California affiliate, home-office curriculum developers, university schools and departments, and others, all worked toward a similar goal, the results were positive. However, even then, only 15–20% of the teachers achieved a level of "coherent relationships among teachers' opportunities to learn, their practice, school curriculum and assessments, and student achievement" (Cohen & Hill, 2000, p. 331). However, conversely, it is not enough for a system to be more homogeneous. Cyprus can be regarded as a highly centralized system in which mandatory curriculum and textbooks are used. Nonetheless, a reform in Cyprus did not achieve the intended impact, and in fact, some teachers even claimed to be more comfortable teaching problem solving using pre-reform approaches rather than the recommended reform techniques (Charalambous & Philippou, 2010).

As Cohen and Hill (2000) point out, there are many aspects of a system that affect whether a message in a curriculum text affects the teachers or not. One tool to consider is national or state mandated tests. The underlying assumption is that by designing the tests so that they reflect important curricula goals the teachers will revise their teaching to make it more aligned with the tests. This assumption is however not thoroughly investigated in large-scale empirical studies (Cimbricz, 2002; Mehrens, 2002). In particular, it is not clear under what conditions such tests have a formatting function concerning a complicated message such as one associated with mathematical competency.

1.2. What reform?

A common feature of all the standards or curricula mentioned in Section 1 is that they present an enriched view of what it means to know mathematics. However, in what sense is this feature new and can be considered a reform?

Mathematics can be described as an autopoietic system in the sense that it produces the objects it discusses (Maturana & Varela, 1980; Sfard, 2008; Varela, Maturana, & Uribe, 1974), which means that the act of being mathematical (Mason, Burton, & Stacey, 1982) or doing mathematics (Schoenfeld, 1994) is scarcely separable from the act of studying or using mathematical structures. Traditional curriculum messages mainly specified the mathematical structures (notions, concepts, theories, methods, results) that should be studied, with little reference to the practice of mathematics (Hoffmann, 1989). During the 1990s, however, a clear trend emerged: the curricula now aimed to clarify the relationship between mathematical content and practice to make the development of a sound mathematical practice an explicit curricular goal. In Freudenthal's words,

> Every researcher, every producer of mathematics will readily admit that mathematics is an activity. (Freudenthal, 1991, p. 14)

[1] The Swedish School Inspectorate (SSI) is an agency of the government that has three main tasks: asserting the right of each individual to knowledge and personal development in a safe environment in school, asserting national equivalence, and contributing to higher national educational standards. The Inspectorate works in different ways: regular inspections of all schools, thematic quality evaluation within certain areas, investigations of complaints concerning the situation for a specific child/student, and approval of independent schools. The collaboration between SSI and the researchers came about due to SSI having the need for specific mathematics education expertise in this thematic quality evaluation.

> [. . .] the learner should reinvent mathematising rather than mathematics, abstracting rather than abstractions, schematising rather then schemes, formalising rather then formulas, algorithmising rather then algorithms, verbalising rather than language – let us stop here, now that it is obvious what is meant. (Freudenthal, 1991, p. 49)

The influential work Curriculum and Evaluation Standards for School Mathematics, hinges on the principle that "[. . .] knowing mathematics is 'doing' mathematics" (NCTM, 1989, p. 9). The Strands of Mathematical Proficiency and the Process Standards in the two U.S. publications Adding it Up (Kilpatrick, Swafford, & Findell, 2001) and Principles and Standards for School Mathematics (NCTM, 2000), respectively, are two examples of constructs that build on such ideas. Another example is the Danish Mathematical Competency Framework (Niss & Jensen, 2002). Frameworks for international comparative studies, such as TIMSS and PISA, incorporate similar constructs (Mullis et al., 2003; Neubrand et al., 2001; OECD, 1999), with the latter being a particularly distinct example. The national curricula of many countries have also picked up on these ideas, with Norway, China, and Singapore as a few of many examples (Kunskapsdepartementet, 2010; Ministry of Education of the People's Republic of China, 2004; Singapore Ministry of Education, 2007).

Retrospectively, these ideas have a certain top-down flavor, with inspiration perhaps coming from developments in the areas of the ontology and epistemology of mathematics (Hoffmann, 1989; Lakatos, 1976, 1978; Pólya, 1954; Schoenfeld, 1994; Steen, 1988) and from social constructivist ideas of teaching and learning mathematics (Bishop, 1985; Ernest, 1991, 1998; NCTM, 1989; Restivo, 1988), rather than from a grass roots teacher movement. Moreover, while the motives behind the above referenced frameworks are often complex, it is clear that they intend to break with a traditional teaching of mathematics associated with mainly practicing procedures and instead communicate a richer view of what doing mathematics means. Hence, it makes sense to assume that for most teachers, the concept of formulating specific goals about the process of doing mathematics represents a major shift of attention concerning the intended focus of the teaching of mathematics in school: a reform. We will denote these types of goals as *competence goals*.

The relevant Swedish national curriculum documents for primary and secondary school were published in 1994, with the upper secondary school documents revised in 2000 (Swedish National Agency for Education, 2001a, 2001b). The writing of the mathematical parts of these documents was influenced by the NCTM Standards (Emanuelsson, Johansson, & Lingefjärd, 1992), and references to competence goals can be found throughout the national curriculum documents (Palm, Eriksson, Bergqvist, Hellström, & Häggström, 2004). Although previous curriculum documents also mentioned problem solving, the documents from 1994 still represented a reform in terms of broader competence goals. However, the curriculum documents are intentionally designed to leave scope for local school and professional interpretation of the aims and content of the subject. For example, the central notions of the competencies (problem, reasoning, communication and so on) are included but not defined, explained, or exemplified. The intention is that the participants working in the education system should thoroughly analyze the curriculum documents by interpreting and discussing them and that this process should increase the quality of the teaching and learning in the schools. The national tests measure all the competencies, and each competence is required in several tasks (Boesen, 2006). Based on their results from the national tests, students receive a test grade. The teachers assign the final course grade, which is based on both the national test result and other achievements made during the course, such as the performance on teacher-made classroom tests. However, the national tests are often considered as the main foundation for teachers' grading since it covers the whole course. The course grades from lower-secondary school are used for admission to upper secondary-school programs, and course grades from upper-secondary school are used for admission to the university. There are no direct sanctions or rewards for the teachers or schools as a direct consequence of the students' national test results, but teachers' salaries may be based on their general performance and the schools may benefit from good test results when recruiting new students.

Fifteen years after the introduction of the competence goals in Sweden we will still conceptualize it as an ongoing reform not only in national curricula documents and tests, but also in other aspects such as textbooks, assessment, teaching, teacher education and policy. Our results will also show that it makes sense to think about the reform in terms of a continuous struggle rather than something that happens right when the ideas are introduced.

2. Mathematical competencies: a framework

We begin by defining what we mean by mathematical competence and specific mathematical competencies.

> To possess a competence (to be competent) in some domain of personal, professional or social life is to master (to a fair degree, modulo the conditions and circumstances) essential aspects of life in that domain. Mathematical competence then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. [. . .] A mathematical competency is a clearly recognizable and distinct, major constituent of mathematical competence (Niss, 2003, p. 6).

The main purpose of publications such as the KOM-project (Niss & Jensen, 2002), Adding it Up (Kilpatrick et al., 2001) or the NCTM Standards (NCTM, 2000) is to communicate goals and means for educational development. These frameworks are, however, not ideally suited for the analysis of empirical data because the descriptions of the competencies are often overlapping and sometimes not precisely defined. To this end, the Mathematical Competency Research Framework (MCRF) was developed with inspiration from these frameworks (Lithner et al., 2010). On a general level, the competencies coincide with the process standards in NCTM's Standards (2000) except for the addition of procedural competency, but in the MCRF,

the individual competencies are defined to be more separated and less overlapping. To enable the researcher to clearly identify competencies in different practical incarnations, the MCRF also include the *competency-related activities* (CRAs), namely:

(I) *Interpret*. Taking in information in relation to the competencies.
(II) *Do and use*. Using one's knowledge to solve tasks. The distinction between *do* and *use* is that do concerns developing knowledge within mathematics as a scientific discipline, and use concerns applying this within and outside mathematics. We also consider two main versions of this activity: (a) imitate and (b) construct (see Lithner, 2008).
(III) *Judge*. Meta-level considerations such as evaluating, reflecting, and forming opinions and conclusions on mathematics and on the activities related to learning, understanding, doing and using mathematics.

The descriptions of the CRA's for each of our six competencies comprises a key element of the MCRF. Here however, we will only describe each competency briefly and refer to Lithner et al. (2010) for the full description of corresponding CRA's.

2.1. Problem solving competency

Problem solving is "engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 51). This definition implies that in this perspective, there are only two types of mathematical tasks: problems and non-problems. The latter are often denoted "routine tasks" or "exercises" (Schoenfeld, 1985; Weber, 2005). Note that some aspects often included in similar definitions of problem solving are not included in this one, for example that the task must be a challenge (Schoenfeld, 1985) or that the task requires exploration (Niss & Jensen, 2002).

2.2. Reasoning competency

Reasoning is the explicit act of justifying choices and conclusions by mathematical arguments. This definition is based on a selected part of the definition of reasoning in the NCTM (2000): "to develop and evaluate mathematical arguments and proofs" (p. 55). In addition, this definition particularly focuses on the reasoning being explicit, in line with an idea from Niss and Jensen (2002) in which the reasoning competency is intimately connected with problem solving and modeling as the so-called juridical counterpart. We further define arguments to be mathematically founded if they, in the terms used by Lithner (2008), motivate why the conclusions are true or plausible and are anchored in intrinsic properties of the mathematical components (objects, transformations, and concepts) involved in the reasoning. Reasoning does not have to be logically strict but, for example, may be plausible in line with Pólya (1954). A proof is a sequence of reasoning in which the mathematical arguments are logically strict.

2.3. Procedural competency

A *mathematical procedure* is a sequence of mathematical actions that is an accepted way of solving a task. There are no definitions of the concept of procedure in either of the NCTM Standards (NCTM, 2000), the KOM project (Niss & Jensen, 2002), or Adding it Up (Kilpatrick et al., 2001). The above definition is inspired by ordinary dictionaries (e.g., the Cambridge Dictionary). An additional component in the definition is that a procedure is or can be reformulated as an algorithm, which is "a finite sequence of executable instructions which allows one to find a definite result for a given class of problems" (Brousseau, 1997, p. 129).

2.4. Representation competency

Mathematics is built on abstract mathematical entities, for example, numbers, functions, geometrical objects, tasks, methods, principles, concepts, phenomena, and ideas, and their properties. When we do mathematics, we must think about these entities and their relations or about some aspects of them. It is often impossible or at least very difficult to think about such an entity in abstract and/or fully general terms, and it is usually advantageous to think about something more concrete (mental or real) that replaces the abstract entity but still carries the aspects of the entity that are useful and relevant for the particular situation. A *representation* is therefore defined to be a concrete mental or real replacement (substitute) of an abstract mathematical entity.

2.5. Connection competency

The concept of connection is defined similarly in different dictionaries: to place or establish a relationship (Merriam-Webster online dictionary); when something joins or is joined to something else, or the part or process that enables this (Cambridge online dictionary). Here, *connection* is to use something that makes a link between two things, for example, a relationship in fact or a causal or logical relation. Following the discussion about the representation of entities, the "something" that is connected is a representation or an entity (including a part or a sub aspect of a representation or entity).

2.6. Communication competency

To *communicate* is to engage in a process in which information is exchanged between individuals through a common system of symbols, signs, or behavior. Therefore, communication includes a sender and a receiver, and a medium within which both can understand the communicated information. Within mathematics education, the sender usually is the teacher, the textbook author, or a student, while the receiver is most often a student or a teacher, and the medium is usually auditory (e.g., speaking, listening) or physical (e.g., writing, gestures).

3. A model for the relationship between teachers' interpretations of reform messages and their teaching

A model supposed to account for teachers' reactions to an educational reform must acknowledge that a teacher might well claim (and believe) to be teaching in accordance with the reform, whereas observations indicate the contrary (Ball, 1990; Cohen, 1990). Gregoire (2003) reviews research on this phenomenon and builds on other models of conceptual belief change to create her cognitive-affective model for conceptual change (CAMCC) with this very insight at its core. A central distinction is made between two ways of processing information: heuristic and systematic processing. The idea stems from Piaget and the distinction between thinking based on experience and thinking based on reflective abstraction (Piaget, 1980). The model builds on the following assumptions grounded in psychology research (see Gregoire, 2003 for a more elaborate discussion):

1. The systematic processing of information requires greater cognitive effort than does heuristic processing.
2. The systematic processing of information requires motivation and ability (Eagly and Chaiken (1993) present a review of psychology research supporting these claims).
3. What gets noticed (in the environment in general or in this case, in a reform message) is influenced by the individual's prior goals and beliefs (Fazio, 1986).
4. A threat to an individual's (professional) identity can motivate change to occur (Schlenker, 1982).
5. Implementing a reform message, such as the NCTM Standards, is a difficult process for teachers (Brosnan, Edwards, & Erickson, 1996) and requires systematic processing if a true belief change is to occur (Gregoire, 2003).
6. Initial benign or positive appraisals of a reform serve as a cue to promote predominantly heuristic processing (Mackie & Worth, 1991; Schwarz, Bless, & Bohner, 1991).

Our simplified version of Gregoire's model is illustrated by the flow chart below.

The starting point in the model is the presentation of a reform message. If the teacher interprets the message and concludes that her identity (her self) is not affected by the message, this conclusion automatically associates with a positive or benign (neutral) appraisal and a lack of motivation for systematic processing of the message. With only heuristic processing, the only possible outcomes are either a superficial assimilation of the message or no belief change at all, depending on prior beliefs and experiences (Chaiken, Liberman, & Eagly, 1989). The route on the right side of the flow chart represents this chain of events.

When the individual perceives the message as affecting the individual's identity, the result is psychological stress (Lazarus & Folkman, 1984). Depending on motivation and ability factors, the stress can either be a feeling of threat or a feeling of challenge. Challenge is a cue for an intention to approach the message and process it systematically (by different means). Based on systematic processing of the reform message, a deliberate choice of either yielding toward or away from the message is possible. However, threat is a cue for avoiding systematic processing of the message. The end result is heuristic processing followed by no belief change or superficial assimilation, as in the case in which no initial stress is present (Eagly & Chaiken, 1993; Gregoire, 2003). In the following sections, we refer to the model above as the CAMCC framework.

4. Research questions

The aim of the present study is to investigate the relation between the intentions of the competence reform and the implementation of the competence reform in Swedish classrooms. We operationalize this in terms of two research questions:

What impact has the competence reform had on the mathematics teaching practice in Swedish classrooms?

Since the reform concern goals for learning, the effect should ultimately be measured in terms of learning outcomes. However, the relationship between what students learn (the learned curriculum) and what is enacted in the classroom (the implemented curriculum) is very complex (Robitaille & Garden, 1989). Due to this, we focus on students' opportunities to learn (McDonnell, 1995) and investigate this by means of classroom observations of activities that through the MCRF can be related to the six mathematical competencies.

The second research question concerns possible mechanisms behind the impact:

How can the impact of the competence reform be explained?

The mediation between the competence reform intentions and enactment of it in the classrooms largely goes through the teacher. The principal communication of the reform message is through the national curriculum documents and the national tests. We therefore investigate the teachers' reactions to the competence goal message in the national curriculum

documents and the national tests. This is done by means of teacher interviews and questionnaires that are designed and analyzed with help of the CAMCC framework.

5. Method

Data were collected through observing lessons, writing structured field notes, collecting textbooks and other texts used in the classroom, performing interviews with teachers, and an online survey to the same teachers. The development of clusters of questions for the interviews and questionnaires and the procedures for classroom observations and text analyses are all based on the research questions and the frameworks presented above. Seven researchers (the authors) and twelve additional experts were involved in collecting the data 2009–2010. These experts were either researchers or Ph.D. students who received special training in the data collection procedures.

5.1. Sample selection

The selections of schools were based on stratified random sampling and carried out by the Swedish Schools Inspectorate (SSI). The principle was to include schools from all of Sweden, rural and urban districts, small and large schools as well as high-performing and low-performing schools. The selection comprised 82 teachers at 23 schools from the compulsory level and 145 teachers at 53 schools from the upper secondary level. One researcher did not have the possibility to complete the procedure, and 26 teachers (interviews and observations) were therefore excluded from the study. Due to technical problems 7 other interviews and 4 observations were left out, so in total the data consists of 197 observations, 194 interviews and in addition 178 completed questionnaires. Each school's principal made the selection of teachers. The instruction was not to choose teachers based on performances, but to arrange a schedule during the days which the researcher was present at the school.

5.2. Lesson observations

A researcher observed each lesson using an observation protocol that included taking notes on the teacher's actions and statements and the students' responses, taking photos of blackboards and of student notebooks, and recording which tasks each individual student was working on during the lesson. Copies of teacher handouts, teacher-made tests and lesson plans were also collected. The analysis included dividing the lesson into distinct episodes comprising coherent mathematical activities. The episodes were sorted in types A–D according to student involvement:

(A) The teacher provides some type of information (to the whole class or a large group) that is not directly related to tasks that the students work on during the observed lesson.

(B) The teacher provides information (to the whole class or a large group) that is supposed to be used later in tasks that the students individually (including small group) work on during the observed lesson. This can also include a few short questions to the students.

(C+/−) The teacher leads a task-solving session in a large group or with the whole class (C− if the teacher takes the majority of initiatives, C+ if the students do).

(D) The students engage in tasks individually or in small groups.

For each of the six competencies, it was decided whether the three competency-related activities of interpret (I), do and use (II) and judge (III) were present or not in each coherent mathematical episode. Each task performed by any student was analyzed in the same way. Because the episodes could be of different length and involve different numbers of students, each episode received a weight proportional to the length of time that the situation lasted and to the number of students involved. The total sum of all the episodes' weights equals the lesson's length. Each competency-related activity type that was present in a given episode was rewarded the same weight as that episode. A total weight (time) for each CRA-type over the lesson as a whole was calculated by summing over all episodes in the lesson.

It is important to note that for each episode, a particular competency-related activity needed only to be present one single, possibly very short, time to receive a weight score equal to the whole time of the episode. Thus the amount of time within each episode that was devoted to each competency does not influence the results. Moreover, the analysis was done in an inclusive rather than exclusive way. For example, any talking about a mathematical matter meant the episode scored a communication CRA. If a task in the textbook was of a kind that there were no similar examples or other templates provided by the teacher or the book, the task was categorized as a problem solving activity. Similarly, all tasks in the textbook where the students had to interpret a text were categorized as addressing the communication competence. To categorize a task as needing the reasoning ability, the main criterion was that the students were prompted to justify the answer. In the same way the representation ability is addressed every time a student must take the meaning of a representation into consideration, for example interpret a table or a diagram. A task where the students had to connect more than one representation was categorized as addressing the connections ability, for example a task including both a graph and a table. These examples show that it is in fact difficult for a task not to address any of the non-procedural competencies. The method is hence designed to include also elementary competency-related activities, rather than only advanced or high-level teaching.

A final note concerns our decision to give the same weight to each task that a student work with, despite that in real life some tasks might take longer time than others. Obviously, one reason for this is the practical difficulty of measuring the

amount of time that each student devotes to each individual task. A risk might be that our decision skews our measurement, by for example underestimating the time spent on more complex problems that might contain more competencies. We suggest that this risk is relatively low, since simpler tasks tend to have several subtasks in the format a–b, a–d or a–f, meaning that that the whole task still often take more time. To further indicate how the analysis is conducted, we will present two examples of lessons.

Example 1. The 64-min long lesson was divided into four separate episodes.

1. *The teacher leads a discussion on mean values and medians, 6 min.* This part involves all students, carrying a weight of 6 min. The teacher reasons about the concepts of mean and median and presents the procedures for calculating means and medians. The teacher presents this verbally and by writing on the blackboard, so there is communication from the teacher to the students present. The students also answer some questions from the teacher. Analyses using the MCRF reveal elements of reasoning, procedure and communication activities that all receive a score of 6 min. No other competency-related activities are present. The episode is categorized as type C−.
2. *All students work with a worksheet on means and medians, 23 min.* This part also involves all students and carries a 23-min weight. The tasks mainly involve procedures, but some involve the conceptual aspects of mean and median, and the solution requires reasoning. Some tasks are open, and it is a new field to the students, so not all procedures are previously known to them. Thus, the episode involves problem solving. In this episode, reasoning, procedure and problem-solving competencies receive a 23-min score. The episode represents type D.
3. *The teacher and the students discuss some tasks, 5 min.* This part involves all students and carries a 5-min weight. Mainly, students present ideas and reason about the problems, but with weak connections to the concepts and procedures involved and with little guidance from the teacher. In this episode, reasoning and communication competencies receive a 5-min score. The episode represents type C+.
4. *Individual student work in textbooks, 30 min.* In the last 30-min session the 14 students in the class work on a total of 62 tasks spread over 10 distinct tasks. Each single task is handled as one episode and to simplify the analysis, it is assumed that each task takes the same amount of time. A single task performed by a single student thus corresponds to 30/62 min. The tasks were divided in three categories (all of type D). Five tasks required only procedural competency. In total, the students performed 53 such tasks with a score of 53*30/62 = 25.6. Three tasks performed by a total of 6 students required reasoning and procedural competency, leading to a 6*30/62 = 2.9 min score for those competencies. One task performed by 3 students required both problem solving and procedural competencies, leading to a 3*30/62 = 1.5 min score for those competencies. Overall, this means that procedural competency is present in episodes corresponding to 6 + 23 + 25.6 + 2.9 + 1.5 = 59 min (92% of the lesson). Reasoning competency is present in episodes corresponding to 6 + 23 + 5 + 1.5 = 35.5 min (55%). Problem-solving competency is present in episodes corresponding to 23 + 2.9 = 25.9 min (40%). Communication competency is present in episodes corresponding to 6 + 5 = 11 min (17%). The connection and representation competencies are not present in this lesson.

Example 2. The 58-min long lesson was divided into three separate episodes.

1. The teacher discusses organizational issues, related to how to work in groups and presents a real world style problem that contains no mathematical information. No competency-related activities are involved. The episode is 6 min long and is classified as type C−.
2. The students work in groups for 41 min on a task whose solution method was not given or known to them, that is, a problem. The problem involved placing bushes in line with tiles around them where the number of tiles depends on the number of bushes, as illustrated by a picture. The problem leads the students to constructing equations that they can solve routinely; hence they also get opportunity to practice procedures. Connections need to be made between the picture of the tile arrangement and to different mathematical representations, hence both connections and representations are involved. Reasoning is involved since the task stated that reasons for how the problem is solved are to be clearly stated. Since the students work in groups, there is communication about mathematical matters going on. Overall, the episode provides opportunities to develop all six of the MCRF competencies, that all get a 41-min score. The episode is of type D.
3. The 11 min long final episode comprises the teacher leading a session where the students present and discuss their work on the problem. The session revolves around communication. Procedures, connections, reasoning and problem solving are discussed by presenting groups and the teacher while the rest get opportunities to interpret. Similarly as in Episode 2, students get opportunities to develop all six MCRF competencies.

Overall, this means that each of the six competencies are present in episodes corresponding to 41 + 11 = 52 min (90% of the lesson).

5.3. Interviews

After each observed lesson an approximately 90-min long structured interview was conducted with the teacher. The first of three parts treated the teachers' own goals and the influences on their teaching, without guiding the interview toward

competency goals. This part included questions such as: What do you want your students to learn? How do you work to achieve that? Have your goals for students learning and ways of achieving them changed over the years?

In the second part, the teachers were asked to interpret six excerpts from the national curriculum documents.[2] For example, the following was included to probe how teachers interpreted an excerpt related to the reasoning competency.

> The school in its teaching of mathematics should aim to ensure that pupils develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalize, as well as orally and in writing explain and provide the arguments for their thinking. (Skolverket (The Swedish National Agency for Education), 2008, p. 23)

Teachers were asked to give their interpretation of the quote, and subsequently how they interpreted the word reasoning.

Teachers were also shown four tasks from the national tests and were asked what a student needed to know to be able to solve the task. The task below is an example from the national test in mathematics year 5, 2003. The task was chosen to illustrate representation, connection and in applicable cases problem-solving competencies.



English translation: *Several thousand years ago in Egypt, different signs were used to show numbers. Find out what the different signs mean and write the missing number under the last picture.*

In part three, the interviewer presented the MCRF definitions of the six competencies and exemplified them by using the same curriculum document excerpts and tasks used in part two. The teachers were then asked about their views and beliefs concerning how the competency goals, now explicitly defined and exemplified, related to their own teaching. By using this method to create a common understanding we lessened the risk of talking about different phenomena while believing we were talking about the same (Speer, 2005).

To locate and characterize the teachers' statements concerning mathematical competence in the interview transcripts, a system using *competency indicators* was developed. Every utterance was examined to see if it contained something related to the competency goal reform. We searched not only for the special terminology associated with the used competency framework but also for any synonyms and other indications that the teacher was considering competency-type goals (perhaps without referring to a specific competency). Because the MCRF involves one among many possible choices of terminology, a teacher's statement using the word "communication" was not valued higher than a corresponding statement using related words, for example "talk mathematics" or "discuss mathematics." Each statement containing a competency indicator was classified according to the following criteria: *spontaneity* (was the reference to a competency a result of a direct question or not?), *clarity* (was the utterance *clear* or *vague*?), and *value* (was the competency mentioned in *positive*, *neutral*, or *negative* terms). A general assessment of the teachers' disposition toward the competency goals as positive, neutral, or negative was based on the whole interview, but mainly on the value classification of the competency indicators together with the answer to a specific question: "Is it important that the students develop the six competencies?" Further, it was determined whether the competency indicators concerned a competency in connection to some particular mathematical content (*local*), whether it illustrated that competencies have meaning for any mathematical content (*global*) or both. It was also noted whether the utterance concerned a lesson *activity*, a *goal* for student learning or both.

The teachers' displayed knowledge and awareness concerning competency goals during the interview was categorized as one of three qualitative categories: *non-existent*; *limited*; and *functional*. The assessment was in essence qualitative in nature, but heavily assisted quantitatively by the competency indicator and characterization system.

Teachers that had few competency indicators of any kind were placed in the category *non-existent knowledge*. They did not spontaneously mention the competencies, they mostly avoided them even on direct questions, and they did not mention the roles of the competency (or similar) goals. Teachers with an extensive amount of competency indicators that also displayed important insights in the meaning of the competency goals were placed in the category *functional knowledge.* They expressed utterances pertaining to the global character of the competencies, but also exemplified some of them in

[2] For both compulsory and upper secondary school, the Swedish national curriculum consists of two parts. The first includes fundamental values, goals, and guidelines for the school in general. The second states the goals for each individual subject. In this study we focus on the second part for both school levels. These documents are very compact and short, approximately 5 pages each. They are organized using the following headlines: Aim of the subject and its role in education, Goals to aim for, Structure and nature of the subject, Goals to attain, and Grading Criteria. There is nothing explicit on how teaching should be carried out, only what goals the student should reach at the end of the education.

relation to some particular (local) mathematical content. They also mentioned the competencies' nature as goals as well as exemplified some of them in terms of a concrete activity. The middle category, *limited knowledge*, included teachers who expressed some of the competencies quite clearly, but throughout the interview failed to treat key aspects like local-global and goal-activity. For example, it includes teachers that displayed knowledge of the meaning of the competencies but did not regard the competencies as goals for learning. Another example is teachers that talked about the competencies on a general level but failed to give a single example of any concrete activities where some competency could be expressed, was required, or could be developed. In addition to the competency indicators, the teachers' spontaneous goals, their valuing of the goals, their interpretation of the role of the national curriculum document as well as national tests, their processing of the curricular message, and their beliefs about teaching were also assessed.

The procedure is exemplified by a few answers from two teachers. When shown the excerpt from the national curriculum document above, Christine, a secondary teacher, answered.

> There are a lot of things to be interpreted in that. But if we take it backwards, this is what we did earlier today in class, to orally argue for their solution. Explain why, in written form . . . what they did as a preparation for this task. Draw conclusions . . . in situations where they should reach . . . something to argue . . . from their calculations for example.

When questioned about how she interpreted the word reasoning she answered.

> To think towards something . . . reason . . . discuss towards something, not necessarily towards an answer.

When discussing the question on what students needed to know to work out the task from the national test presented above Christine responded:

> Students have to have number sense, being able to distinguish between ones, tens and hundreds, seeing and recognizing patterns and to know our position system.

Christine sees a competency as a goal (*they should be able to explain*. . .) and she can also relate it to a concrete activity (referring to what they did earlier in class). In her interpretation of the word reasoning, she indicates that reasoning can be applied to any mathematics, not just something particular (global). In the reference to the lesson (discussed earlier in the interview) she refers to a concrete piece of mathematics where the student should use written arguments (local). Christine's answers throughout the interview were similar to these. She was overall *clear* in her comments and her responses were both a result of direct questions, as in the meaning of the word reasoning, and contained spontaneous reflections about competency-type goals and activities. Christine is one example of a teacher that was assessed as having functional knowledge of competency goals.

Turning to another secondary teacher, Anna, the response to the same excerpt from the national curriculum text was:

> Well, this is what it is all about, what they need to get by in life, kind of obvious I think.

On how she interpreted the word reasoning she answered.

> Own lines of thought, not necessarily talk. This makes sense.

When discussing the question on what students needed to know to work out the task from the national test presented above she answered.

> To know the position system and the meaning of each position.

Anna's response to the curriculum excerpt is *vague* and does not include any explanations pertinent to competency goals. When asked about the word reasoning, she responded in a short but reasonable way. The comment on the national test task relates only to content type goals. This pattern was representative for Anna's answers in the interview. She gave occasional interpretations of some competency goals when asked directly about them but she referred neither to *local* nor *global* aspects of the competency goals, and made no clear distinctions between *goals* or *activities*. She seemed to be *positive* toward the goals expressed in the quote. Anna was assessed as having limited knowledge of competency goals.

5.4. Questionnaire

The questionnaire contained 85 questions, which mainly dealt with teachers' interpretations, perceptions, and opinions. Most of the questions were written in the form, "To what extent. . .," and the options were numbered 1 (*very little* or equivalent) to 6 (*very much* or equivalent). After every ten questions, the teachers had the opportunity to comment on the earlier questions. The questions were linked to a number of constructs developed from the CAMCC framework (Gregoire, 2003). Some questions concerned the extent to which teachers were confident on how different parts of the national curriculum were intended to be used and the extent to which they felt they understood the formulations in the curriculum document. Groups of questions also addressed the teachers' beliefs in their ability to help their students develop the six competencies, teachers' feelings about the national curriculum document and national tests, and their epistemological beliefs concerning mathematics.

Table 1
The proportion (%) of minutes for each CRA (I: interpret, II: do and use, III: judge) for all competencies in each episode type A–D. The bottom line shows the proportion of the total number of minutes where at least one CRA type of each competency is present.

Minutes	%	Type	Problem solving competency			Reasoning competency			Representation competency			Connection competency			Communication competency			Procedural competency		
			I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
433	4	A	2	0	0	9	0	0	9	2	0	2	0	0	11	2	0	8	4	0
1386	13	B	15	11	10	22	15	4	41	26	2	43	24	3	44	26	2	51	36	5
1594	15	C−	21	32	11	34	36	9	50	36	9	48	48	10	38	53	4	56	59	7
714	7	C+	53	53	18	48	63	20	48	40	3	47	39	9	61	68	3	41	56	12
6496	61	D	26	28	2	13	22	4	28	29	2	26	32	2	24	24	2	44	84	3
10,623	100	All	25	27	4	19	25	5	34	29	3	32	33	4	31	30	2	45	69	5
				29			32			41			44			42			79	

6. Results

6.1. What impact has the reform had on the mathematics teaching practice in Swedish classrooms?

The first research question investigates in what ways the teaching provides students with opportunities to develop the mathematical competencies, especially the five non-procedural competencies in relation to the procedural competency. Students' competency-related activities during totally 10.623 min in 197 mathematics lessons are presented in Table 1.

The clearest result from the classroom observations is that opportunities to develop procedural competency is present in episodes corresponding to 79% of the total time observed (the bottom line, to the right) and hence the dominant activity. Opportunities to develop the non-procedural competencies are present in 29–44% of the time (left). Looking more closely, the tables show that the most common competency-related activity is *do and use procedures*. This result is visible in the second last row where CRA II for procedural competency occurs in episodes that cover approximately 69% of the total observed time. The corresponding figures for CRA I and II for all other competencies are 19–34%, and 45% for CRA I for procedural competency. The competency-related activity III (*judge*) is for each competency found in episodes corresponding to 2–5% of the total time. This is in itself a notable result, but we will not delve deeper into it in this article. In addition, and not visible in the table, during 35% of the total time there were no trace of the non-procedural competencies at all. The dominating type of episode (approximately 60% of the total time) is type D, where students solve tasks individually or in small groups. This is also the type of episode where procedural competency is most dominating. In addition, and not visible in the table, during almost half of the D episode time there is no trace of the other five competencies. The tasks that the students work with during type D episodes were mainly from their textbooks (70% of the D type episode time) but also from handouts provided by the teachers.

6.1.1. Conclusions regarding the impact of the reform on the teaching

Since we have no data from before the presentation of the reform message in 1994 we cannot compare and see if practice have changed, so our reasoning has to build on other arguments. Obviously, it is a challenge to answer such a general question based on large-scale classroom data on the level of learning opportunities given to individual students. This is however also the strength of our method: we do not rely on simple classroom observations of how things "look," nor on what teachers say about their practice. Every episode encountered by any student is analyzed in detail in terms of competency-related activities. So our task is to explain, in terms of how much competency-related activities of different sorts we have identified, if this represents a break with a tradition of mainly teaching procedures or not. And in simple terms, this could be answered by noting that procedural competency is present in episodes responding to almost twice as much time as each of the other competencies. Hence we conclude that the tradition has not been broken. In this line of reasoning, it is important to remember that, in terms of "time on task" the individual competency-related activities are overestimated in the sense that even relatively trivial instances of for example representation activities in an episode will make the representation-CRA get a score equal to the time of the whole episode. Even so, episodes corresponding to 35% of the time contain only competency-related activities related to practicing procedures. This in itself is a strong indication that practicing procedures has a dominant role in the classroom.

But there are also other ways of reasoning, more fundamental in relation to the intentions of the reform as well as to the nature of doing mathematics that the competence-goal system is intended to reflect. Let us start by discussing why teaching focused on practicing procedures will in many cases also contain elements of the five non-procedural competency-related activities. Practicing procedures is a wider notion than just doing calculations. A teaching focused on mainly practicing procedures includes the well-known, and almost defining, situation of traditional mathematics pedagogy where a teacher first demonstrates or leads a discussion on a template task and the students later practice tasks similar to the template task (Stigler & Hiebert, 1997). Obviously, such a task will often include mathematical representations and connections. The presentation and discussion will contain elements of reasoning and communication. Any non-trivial variation of the template task encountered by the students will show up as problem solving in our analysis. Typical textbooks often include template tasks and solutions followed by similar tasks for students to work on, so also in extended sessions where students

work in a textbook, elements of non-procedures will show up in a similar way. In fact very few tasks apart from carrying out a bare, and well-known, calculation can avoid containing also some non-procedural competency activities with the method of analysis used here. Still, as stated above, episodes corresponding to more than one third of the time did not include any competency-related activities other than those related to practicing procedures.

Example 1 in Section 5.2 could be a typical such lesson. The teacher presents some procedures for calculating the mean and median values of a sequence of numbers, and the tasks that the students work with in subsequent episodes focus on the same type of calculations in relatively standard situations. Yet, the analysis finds competency-related activities related to problem solving in episodes corresponding to 40% of the time, to reasoning in 55% of the time, and to communication in 17% of the time. Activities related to practicing procedures were present in episodes corresponding to 97% of the time. On a philosophical level, this is unsurprising given the autopoietic character of mathematics. Whenever mathematics is engaged, even if it is the process of practicing procedures, it is hard to fully avoid the spectrum of competency-related activities.

In Example 2 in Section 5.2, activities related to each of the six MCRF competencies appear in episodes corresponding to 90% of the lesson. This lesson is a classic problem solving session, similar to what is described by Stein, Engle, Smith, and Hughes (2008). There seems to be no particular intention to "teach" any of the competencies apart from perhaps problem solving but yet opportunities to develop each of the six MCRF competencies appear throughout the lesson. Naturally, there are many different ways of teaching a lesson that would be aligned with the intentions of the competence reform, and we cannot account for them all. But as described in Section 1, the competency reform is grounded in an idea of working mathematically, as for example expressed by Freudenthal (1991). In competency frameworks (Kilpatrick et al., 2001; NCTM, 1989, 2000; Niss & Jensen, 2002) the individual competencies are described as parts of a whole, as exemplified by "the intertwined strands of proficiency" (Kilpatrick et al., 2001, p. 117). It hence makes sense to assume that teaching episodes that honor these intentions would score high on activities related to all or most competencies. The result for the 10.623 min of observed lessons presented in Table 1 does not add up to that. It is reasonable to conclude that the examined teaching in general is consistent with a teaching defined by a very strong dominance of practicing procedures rather than teaching that is in line with the intentions of the competence-reform message we are examining.

6.2. How can the impact of the competence reform be explained?

Our conclusion above is that the reform has not had the intended impact. We formulate three sub-questions of research question 2 reflecting basic ideas in the CAMCC framework: In order to implement the reform message the teacher has to (A) see the message as important, (B) systematically process and identify the meaning of the message, and (C) change her beliefs about goals and means of teaching in line with the message. After the three sub-questions are examined, we answer research question 2.

(A) What are the teachers' attitudes toward the reform message?

Teachers who are confronted with the message make either a systematic or a heuristic processing, before deciding whether to yield to the message or not (Gregoire, 2003). The outcome of a heuristic processing is determined by the teachers' prior beliefs and experiences. The teacher will yield if these are in line with the message (Gregoire, 2003, p. 166). Similarly, for teachers to yield when performing systematic processing, "the message should be perceived as intelligible, plausible, and fruitful for promoting students' learning" (Gregoire, 2003, p. 168). Therefore, yielding teachers have views that are in line with the message, at least according to their own evaluation, and are not negative toward it. They will therefore see it as important that the students get the opportunity to develop the competencies. Teachers who have not yielded will be negative toward the message.

After the presentation of the six competencies, 87% of the 194 teachers responded to a direct question that it is important that the students develop the six competency goals. Although 13% of the teachers seemed indifferent and 1% skeptical toward the competencies, no one expressed that the goals are unimportant. This question might be biased due to the collaboration with the Swedish Schools Inspectorate, but when also considering the general assessment of the teachers' disposition toward the goals, the result remains. Less than 1% of the teachers were placed in the category negative, almost 57% in the category positive, and approximately 43% were assessed to be neutral. Our conclusion is that most of the teachers have yielded to the reform message in some way.

(B) Have the teachers systematically processed and identified the meaning of the reform message?

Systematic processing is "a comprehensive, analytic orientation in which perceivers access and scrutinize all informational input for its relevance and importance to their judgmental task, and integrate all useful information in forming their judgments" (Chaiken et al., 1989, p. 212; see also Gregoire, 2003, p. 159). A central feature of the CAMCC framework is that systematic processing is regarded as necessary for lasting belief change (Gregoire, 2003, p. 150). This study therefore maps indications of systematic processing in the teachers' own descriptions of their work with the goals. Indications of teachers' systematic processing: their view of competencies and/or curriculum documents have changed over time; they view curriculum documents as factors that affect their teaching; and they have been working with trying to understand the competency goals.

For 62% of the 194 teachers, there exists none of these indications that they have systematically processed the competency message. For 24% of the teachers, there are one or a few indications. A few teachers, 14%, showed many and clear indications

of systematic processing. Some of these teachers described that they worked with the national curriculum documents by reading them many times and also discussed them with colleagues, for example when constructing local curriculum documents, during in-service training, or during regular collegial cooperation. Still, reading the curriculum documents frequently does not guarantee systematic processing. For example, one teacher with few indications had trouble interpreting an excerpt from the national curriculum documents: "I haven't looked at this so carefully before. You read it many times but you don't think so closely about what it actually says."

If the reform message is to be implemented in schools, it is necessary that the teachers also manage to identify the meaning of the message. We therefore examine whether the teachers make reasonable, clear, and nuanced interpretations of excerpts from the national curriculum documents, and whether they produce reasonable, clear, and nuanced descriptions of the type of knowledge students need to solve national test tasks. For more than 60% of the teachers, their interpretations were reasonable, that is, their descriptions concerned some type of competency-type goal or activity, not only mathematical content, and to some extent coincided with interpretations based on the MCRF. However, their interpretations were often vague and many teachers had problems formulating their interpretations using abstract terms and rather gave examples of tasks or situations connected to the excerpt. Many also partly repeated the wording in the excerpt when trying to explain a concept, for example, one teacher interpreted 'reasoning' as "to be able to reason (. . .)." Most interpretations were also quite limited and not very nuanced, that is, they focused on narrow aspects of the competencies. One example is a teacher who interpreted "communicate mathematics" as to "be able to use words like *multiplication*." Many teachers seemed unused to interpreting the curriculum documents, expressed doubt concerning how to interpret the text, and a few asked the interviewer for confirmation of their interpretation.

When asked what the students needed to know in order to solve the four selected national test tasks, it varied greatly for how many tasks the teachers made reasonable interpretations, that is, not only mentioned mathematical content but also touched upon some kind of competency-type description. A quarter of the teachers do not mention competencies at all, while 15% makes a reasonable interpretation of one of the tasks. Two tasks are reasonably interpreted by 16% of the teachers, and 3 by 25%. The last group, 19% of the teachers, makes reasonable interpretations in connection to all tasks. In total, 56% of the teachers make reasonable interpretation for at most half of the tasks. One conclusion is that the teachers do not focus on the same type of aspects when discussing required knowledge for task solving. Also, only a handful of the teachers present clear descriptions of the students' needed knowledge in abstract terms, such as "they have to have some kind of language to use when explaining" (communication). It was more common that the knowledge was described as activities during task solving, for example that in order to solve this specific task they have to "find a strategy to be able to carry on" (reasoning). Just as for the excerpts, the teachers' descriptions in general concerned very limited aspects of the competencies. For example, "they have to know the words for these geometrical figures" (communication), and "patterns in general, they have to have worked with patterns" (connections). Some teachers say that tasks in the national tests differ from textbook tasks, for example by requiring more "original thinking" or by testing several goals simultaneously, but most find it problematic to express the differences in clear terms.

Some of the teachers interpret competency goals through a particular form of assimilation that we call filtering. This means that the competency goals are interpreted in ways that are in line with a preferred goal of the teacher, sometimes leading to interpretations that are far from the intentions of the national curriculum. The teacher's preferred goal could be an affective goal (e.g., that the students should be confident), a particular competency goal, or any other type of goal (e.g., everyday math). One of the most common ways of filtering is through the competency goal communication. One example of filtering is a teacher whose two main teaching goals seem to be "abstract thinking" and the "ability to use mathematics." He repeatedly returns to these goals when interpreting excerpts and tasks. Also when asked to clarify other goals that he mentions, he often refers to his two main goals. For example, when asked to clarify the word "understand," he replies after some hesitation "to use in an everyday context." In addition, his explanation of what he means by "abstract thinking" is also rather vague: "there is not so much information given, you have to make your own assumption, and formulate it."

Our conclusion is that most of the teachers have not made a systematic processing of the reform message and also, based on the teachers' varying, vague, and limited interpretations of both the excerpts and the tasks, that few teachers have identified the meaning of the reform message.

(C) Have the teachers made a belief change in line with the message?

The goal of the reform is that teachers break with the tradition of teaching mathematics as mainly practicing procedures, and formulate goals for the students' learning that concern the process of doing mathematics. According to Gregoire (2003), such a change of the teachers' practice can only take place if they make a true belief change. It might obviously be possible for a teacher to teach in accordance with the competency goals due to inspiration from other sources than the national curriculum documents. However, it is highly unlikely that a teacher *changes* her or his teaching in accordance with the competency goals without functional knowledge of the goals, since the competency goals are novel and intended to break both with traditions concerning how goals are formulated and with existing teaching traditions. We therefore see functional knowledge as a prerequisite for cognitive change. Furthermore, a teacher who has made a belief change has yielded to the message after systematic processing, which means that s/he has decided that it is important that the students get the opportunity to develop the competencies. The competencies, in particular the non-procedural ones, should therefore be part of the teacher's own goals for teaching mathematics. Based on this rationale, we have particularly examined the following two aspects: whether the teachers have functional knowledge of the competency goals, and intentions, whether the competency goals are a part of the their own goals for teaching mathematics.

Of the 194 teachers 18% displayed functional knowledge, 55%, displayed limited knowledge, and 28% displayed non-existent knowledge of the competency goals. One of the clearest conclusions is therefore that few teachers have functional knowledge of the competency goals, which is required for belief change.

When asked for their own goals for students' learning, 44% of the teachers spontaneously mentioned at least one non-procedural goal of competency type (not necessarily the competencies in the MCRF), often using vocabulary like "talk mathematics," "logical thinking," or other general formulations. However, approximately half of these teachers mention only one such competency type goal, which means that only around 20% of the teachers mention more than one (non-procedural) competency type goal. Still, when the teachers later were presented definitions of the six competencies, many teachers expressed that the competency goals are aligned with their own goals, for example by stating that there are "competencies in all mathematics." These interpretations are reasonable but often seem superficial, for example, one teacher said: "Now that we have talked here, I believe that I follow them [the MCRF competencies]. This is automatically included. Without seeing it, it is there." This type of vague statement is a sign of assimilation of the message.

In summary, only 18% of the teachers have functional knowledge of the competency goals and around 20% have intentions for teaching that include more than one competency goal. We therefore conclude that less than 18% of the teachers have made a true belief change.

6.2.1. Conclusions

So how can the impact of the competence reform be explained? Although there is some variation in the aspects analyzed above there are clearly predominating answers to the sub-questions A–C that might on the surface seem somewhat contradictory. On the one hand many teachers are positive to the reform, believe the message is important, and think that their teaching is in line with the message. On the other hand they have problems identifying the meaning of the message, have not processed the message systematically, have not functional knowledge, and do not teach in line with the reform. However, all this is consistent with the path of benign-positive appraisal, heuristic processing, and assimilation of message in the CAMCC framework (the path ending to the right in Fig. 1), that is, the model explains why teachers can be positive and believe they are implementing the message without actually doing so.

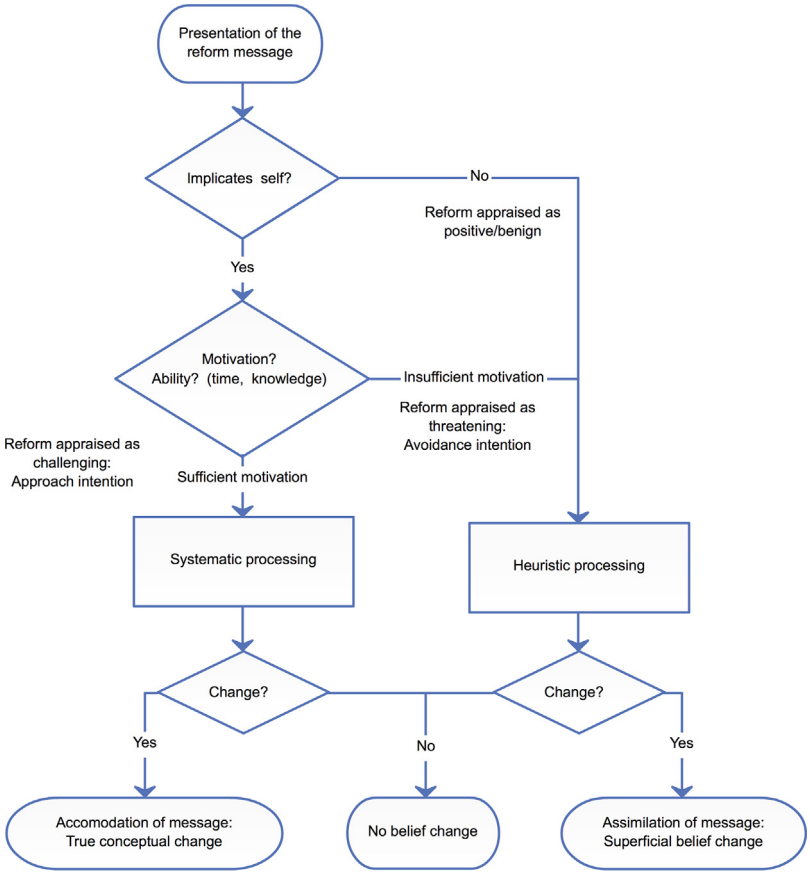


Fig. 1. Gregoire's model in a simplified flowchart.

7. Discussion

A conclusion in this study is that the way the mathematical competence reform has been implemented has not been very successful. Fifteen years after the reform was initiated the classroom practice is still dominated by carrying out procedures. There are several factors that may hinder an impact of a reform attempt. For example, teachers may not perceive that resources, such as time and support, are sufficient for a successful systematic processing of the message and for developing teaching that is aligned with it (Charalambous & Philippou, 2010; Gregoire, 2003). It may also be difficult and a slow process, even in a supportive environment, to develop such teaching (Schoenfeld, 2010). But the reason that comes out strongly in this study is the teachers' difficulties in identifying the meaning of the message of mathematical competencies conveyed in the national curriculum documents and the national tests.

The difficulties in identifying the message may be connected to the way the goals are presented. The competence goals are presented explicitly in the national curriculum documents, but in a rather vague way. This vagueness is represented in two ways. Firstly, concepts pertaining to the competencies are not very well described, which has the consequence that they are difficult to interpret. Secondly, the concepts are embedded along with many other types of goals and purposes, which makes the importance of the competence goals somewhat unclear. This means that the teachers' professional identity may not be affected by a lack of focus on these specific goals. As a consequence many teachers seem to have made a heuristic processing and just assimilated the message and carried on teaching in a similar way as before, content with the belief that their teaching is in alignment with the national curriculum documents.

The national tests also carry the message since different tasks require different competencies for their solution, and the solution requirements are spread over the competencies (Boesen, 2006). But a problem seems to be that this message is implicit since it is embedded in what is required to solve the tasks. A common suggestion to boost reform is to use high-stakes tests to assist curriculum documents. Proponents of such tests as a vehicle for implementing reform argue, "what you test is what you get." The underlying assumption is that these tests influence what the teachers do, so by designing the tests so that they reflect the curricula goals teachers will revise their teaching to make it more aligned with the tests. Empirical evidence backing this as a general conclusion is however lacking (Cimbricz, 2002; Mehrens, 2002). It seems that such tests influence practice under certain conditions, but not under others. For example, in some circumstances the tests may affect the choice of content, but this does not necessarily mean that the teaching practice is affected in other ways (Firestone, Mayrowetz, & Fairman, 1998; Grant, 2000, 2001). In the present study we see that the conditions given to teachers were not sufficient for affecting practice with respect to competency-related activities. Some teachers thought that the tests had some impact on content. But even though they did notice that the national test tasks were somewhat different than the tasks in the textbooks, most of them could not pinpoint the characteristics of the differences more than on a surface level. So since they did not see exactly what the test tasks were aiming at they could not revise their teaching practice accordingly. From this it also follows that the stakes of the tests was probably not a decisive factor, since the stakes does not make it more possible to identify the message. Thus, for external tests to be a vehicle for implementing a complex reform message, such as the competence reform, they may have to be complemented by other ways of clearly specifying how the reform message in the curriculum documents have been operationalized in the tests. It could be noted here that the Swedish National Agency of Education every now and then releases the secrecy of a national test, so the teachers have several tests available to look at. In addition, the tests are accompanied by a categorization of the tasks in competence categories. This does not seem to have been sufficient.

Another way of assisting curricula reform is by in-service training. This was also included in the activities of the Swedish teachers when the reform was launched. At every school the teachers were expected to spend time together formulating local interpretations of the curriculum documents for every subject, including mathematics. However, these activities were not directed at specific parts of the curriculum. The teachers could perform the activities as they saw fit, in some cases guided by their principals. Thus, teachers who did not experience their superficial interpretation of the competence message as a threat to their professional identity may very well have spent their time focusing on other aspects of the curriculum documents than the competence message. It seems that in schools where a lot of time has been spent on these activities, sometimes including reading other documents about the reform and involving external experts, this has led to examples of impact on the classroom practice. However, as a whole, in the absence of guidance on how to identify and interpret the terminology and the idea of the reform, these teacher discussions do not seem to have been sufficient to enhance teachers' understanding of the reform message. It may be that lack of reform expertise in the discussions has been an inhibiting factor for acquiring sufficient functional knowledge of the competencies through these discussions.

It thus seems that when a reform message is presented in a non-conspicuous way and terms that are difficult to interpret, like problem solving and reasoning, are not clearly defined and operationalized they are easily filtered and assimilated in an interpretation that coincides with the way teaching is already performed. This has been exemplified with individual teachers before (Ball, 1990; Cohen, 1990), and with larger samples using questionnaires and teacher logs (Charalambous & Philippou, 2010). But the present study is based on data from nearly two hundred teachers fairly representative of a whole nation, and both questionnaires and more in-depth qualitative approaches, such as interviews and classroom observations, have been used to collect data. Based on this data the study indicates that assimilation of the message is a widespread phenomenon and a significant hindrance for reform realization.

In summary, if we want a complex reform message to have real impact in our classrooms, teachers need sufficient conditions and knowledge to successfully work with how to interpret and teach in accordance with the message. One part

of these conditions is how the message is presented. Our study shows how decisive the presentation of a complex reform message is for the reform to have an impact on classroom practice. Characteristics of a non-sufficient presentation (including national curriculum documents and national tests) are described. The analysis was carried out on a national reform including ideas similar to the NCTM Standards. The ideas of competency goals are spreading into national curricula in many places around the world (see Section 1). Thus, the conclusions may be useful for several countries working with similar reforms.

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