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Mathematical competencies: a case study on semiotic systems and argumentation in an Italian High School

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ABSTRACT

This paper describes the main findings of a national action-research project, promoted by Regional School Office of Campania (South of Italy), aimed at improving numeracy and mathematical literacy through supporting schools' activities. The main focus is on processes underlying the creation of capabilities in representation, reasoning and communication. They have been investigated through creation of appropriate activities on the management of different semiotic systems and argumentation. About 400 fifteen-years- old students have been involved in the experimentation, divided into 20 high schools classes, in the Campania Region (South of Italy). The activities were designed to improve students' capabilities to solve real world problems, according to the OECD's Programme for International Student Assessment (PISA) framework. We investigated if the experimental activities might influence the acquisition of representation and communication skills and improve the PISA levels. We showed the project's rationale, its methodological choices, and theoretical framework. A protocols' analysis on conversion between various semiotic systems is also shown, mainly in two different schools' types (a Professional Institute and a Scientific High School). Finally, a quantitative analysis of the improvement of the PISA levels has been carried out, together with a discussion about the connection between the improvement of the mathematical skills and the realized activities.

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KEYWORDS
Argumentation; semiotics; mathematical literacy; mathematical capabilities

1. Introduction

In this paper, we present a case study about some innovative activities, co-planned by Mathematics Education researchers and teachers, to engage the students to manage different semiotic systems. Our main idea is that if the students have some semiotic activities, planned to take into account their main difficulties, they can improve also the Mathematics skills.

These activities are included into the European project 'Numero Ergo Sum: mathematical competencies in an European dimension', (Capone, Del Regno, & Tortoriello, 2018;

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Capone, Coppola, Dello Iacono, & Tortoriello, 2018) promoted by the Regional School Office of Campania in the South of Italy (called USR from now on), Campania Region and the Action and Cohesion Plane (PAC), realized in collaboration with Mathematics Education research group of Mathematics' Department of University of Salerno. The title is inspired by the Cartesian proposition 'Cogito Ergo Sum': thinking, argumenting and doubting through mathematics is the first step to build knowledge and skills in our liquid, complex, global society.

The project aims to improve in numeracy and mathematical literacy of 13–15 years old students, according to the Programme for International Student Assessment (PISA) framework. The PISA is a worldwide study by the Organisation for Economic Co-operation and Development (OECD) in member and non-member nations, intended to evaluate educational systems by measuring 15-year-old school pupils' scholastic performance on mathematics, science, and reading.

About 400 students were involved in our experimentation, from different high schools in Campania, in South of Italy.

The PISA approach to 'literacy' is competency-based and it deals with the capacity of students to apply mathematics knowledge and skills in key subject areas: formulation of a mathematical problem, use of mathematical outcomes and their interpretation in specific contexts.

In particular, the project is focused on the importance of linguistic aspects in understanding mathematical concepts, the role of semiotic representations in mathematics learning processes and the analysis of the potentialities of mathematical logic for the development of demonstrative skills.

In agreement with the PISA framework, we mainly focused on three matematical capabilities: communication, reasoning and representing (Turner, Dossey, Blum, & Niss, 2013).

Nevertheless, the understanding of a concept in mathematics presupposes the coordination of more semiotic representation registers (Duval, 2006), in particular treatments, i.e. the transformation of representations that happen within the same register, and conversions, which are transformations of reprentations that consist of changing a register without changing the objects being denoted.

Within this theoretical framework, main activities were designed by the research group to improve management of different semiotic registers, conversions between different semiotic representations and treatments within the same representation.

Our main contribution to the study of semiotic systems has been to provide students, through specific designed tasks, some critical means, both cultural and cognitive, as suggested in Duval (1995), so that students themselves can modify, in a more conscious way, their initial semiotic representation about a mathematical object: forcing them to provide different semiotic representations of the same object, students will be more aware of the same object and will be able to use it in different contexts.

Research has been focused to answer the following research questions:

(a) May the activities, focused on the management of different semiotic registers and on mathematical argumentation, influence the acquisition of representation and communication skills (PISA, OECD, 2012)?

(b) Is the level of linguistic competence related to errors in the management of different semiotic registers?

In particular, we analysed main difficulties emerged in the transition between the different semiotic registers and in the written argumentations.

During the development of the project, we highlighted an improvement in communication and capacity. A quantitative results analysis seems to show that designed activities improve mathematical skills, too.

2. The project

Numero ergo sum is an action-research project to analyse Campanian students' learning assessment about numeracy or Mathematical literacy, defined in PISA 2012 as follow:

> Mathematical literacy is an individual's capacity to formulate, use, and understand mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizens.

About 400 fifteen years old students were the active protagonists both in educational action and in the research-action process.

The students were divided into 20 classes from classical, scientific, technical institutes, professional institutes, linguistic high schools and scientific high schools, all located in Campania but equally distributed throughout the Region.

Each class has been associated with a tutor and a researcher from University.

The activities were proposed simultaneously in all the classes which joined the project. The sample of the selected classes is heterogeneous both by geographical location and by curriculum.

All didactic activities were carried out during the curricular hours in a fairly wide time frame, from March to May, one day a week for a total of 30 h in each class. A class teacher, with whom educational intervention strategies and procedures were previously agreed, was co-present with the university researcher.

The main focus was on the basic skills, in accordance with the OECD – PISA guidelines:

- Using appropriate variables, symbols, diagrams, and standard models to represent a real-world problem;
- identifying appropriate strategies for problem solving;
- analysing data and interpreting them by developing deductions and discounts on them, also by graphic representations;
- using consciously the computing tools and the potentialities of specific computer-based applications.
- using symbolic/formal language of arithmetic and algebra;
- comparing and analysing geometric figures by identifying invariants and relationships.

3. Theoretical background

The theoretical framework within which the project was conceived and developed mainly refers to the role of the semiotic representations in the mathematics learning processes, in a vygotskian approach.

Our semiotic framework is enlarged, in agreement with the idea of semiotic bundle (Arzarello, 2006), where the drawings and extra-linguistic modes of expressions have been also included. The students' errors and difficulties could also be explained by semiotic conflicts, i.e. as disparities between the student's interpretation and the meaning in the mathematics institution, as in the onto-semiotic approach (Godino, Batanero, & Font, 2007).

Our starting point is Sfard's theory (Sfard, 2001), based on the concept 'mathematics as a discourse'.

Representation, Vision, and Visualization as cognitive functions in mathematical thinking have been analysed according to Raymond Duval (Duval, 1995).

Finally, the linguistic competence in the sense of the functionalist school, which is also adopted in the Italian National Guidelines, according to Pier Luigi Ferrari (Ferrari, 2004) completes our theoretical framework.

In our work, the theoretical assumptions in the OECD PISA framework has been also considered, in particular mathematical capabilities such as representation, reasoning and communication (Niss, 2003; PISA, OECD, 2012; Turner et al., 2013).

3.1. Theoretical framework: semiotic representations in mathematics learning processes

Among the theories that have a central role in communication in mathematics, there is Sfard's theory that considers 'mathematics as a discourse' (Sfard, 2001). Sfard's discursive framework highlights the importance of communication and engagement in pre-established cultural practices in human development. In her framework, Sfard (2008) considers cognitive processes and interpersonal communication as facets of the same phenomenon and views thinking as communication (not necessarily verbal or visible). She rejects the dichotomies between thought and language as well as the separation between individual and social aspects of learning inherent in the constructivist approaches. According to this view, which is based on Vygotskian thought (Vygotskij, 1992), the language influences thought. For this reason, it is essential to educate students to argue (through both written and oral language) the processes implemented in the mathematical activity. A lot of difficulties of students with comprehension of mathematics depends on the mathematical processes and cognitive functioning underlying them and on the various registers of semiotic representations (Duval, 2006). In fact, the difference between the cognitive activity required for mathematics and that required for other domains of knowledge that does not develop a set of complex concepts and the development of semiotic representations was an essential condition for the development of mathematical thought. No kind of mathematical processing can be performed without using a semiotic system of representation, because mathematical processing always involves substituting some semiotic representation for another. From an epistemological point of view there is a basic difference between mathematics and the other domains of scientific knowledge. Mathematical objects, in contrast to phenomena of astronomy, physics, chemistry, biology, etc., are never

accessible by perception or by instruments (microscopes, telescopes, measurement apparatus). The only way to have access to them and deal with them is using signs and semiotic representation (Duval, 2006). Therefore, in order to support understanding of mathematical concepts, the ability to pass from one representation to another is crucial. According to Duval, the understanding of a concept in mathematics presupposes the coordination of more (at least two) semiotic representation registers. The paramount activities of semiotic representations are treatments and conversions.

Treatments are transformations of representations that happen within the same register: for example, carrying out a calculation while remaining strictly in the same notation system for representing the numbers, solving an equation or system of equations, completing a figure using perceptual criteria of connectivity or symmetry, etc. That gives prominence to the intrinsic role of semiotic systems in mathematical processes. The treatments, which can be carried out, depend mainly on the possibilities of semiotic transformation, which are specific to the register used (Duval, 2006). Conversions are transformations of representation that consist of changing a register without changing the objects being denoted: for example, passing from the algebraic notation for an equation to its graphic representation, passing from the natural language statement of a relationship to its notation using letters, etc. The treatments depend on the semiotic register of representation used. One of the criteria for choosing a semiotic register with respect to another is the power of the treatments that it allows to do (Duval, 2006). Conversion from the epistemological point of view is to choose a semiotic register in which the treatment can be more efficient and cannot be reduced to a treatment. From a cognitive point of view, however, conversion stimulates the understanding of mathematical concepts. In fact, two distinct representations of the same object, in general, do not have the same content. Thus, the coordination of semiotic registers, the ability to handle multiple representations of the same concept, and the ability to pass from one to the other, is a necessary condition for achieving the ability not to identify a concept with one of its representations. This ability is critical to mathematical comprehension and, moreover, is not natural in students, but needs to be stimulated and supported.

In addition, the training goal is to enhance the argumentative skills in mathematics and the production of written texts, according to Morgan (2002), which supports the importance of writing to learn. In fact, the difference between oral language and written language does not only concern the linguistic organization of texts but also cognitive functions (Duval, 2000; Ferrari, 2004).

3.2. Theoretical assumptions in the OECD PISA framework

The PISA Mathematics framework has been the starting point for our experimentation.

For purposes of the assessment and learning, the PISA 2012 (PISA, OECD, 2012) definition of mathematical literacy can be analysed in terms of three interrelated aspects: the *processes*, and the capabilities that underlie those processes; the *contents* and the *contexts* in which the assessment items are located.

Seven fundamental mathematical capabilities are related to the processes:

- *Communication*
- *Mathematising*

- *Devising strategies for solving problems*
- *Using symbolic, formal, and technical language and operation*
- *Using mathematical tools*
- *Reasoning and argument*
- *Representing*

We focused on the following capabilities:

- *Communication*: Mathematical literacy involves *communication*. It includes reading, decoding, and making sense of statements, questions, tasks, objects, images, or animations to form a mental model of the situation. found, the problem solver may need to present the solution, and perhaps an explanation or justification, to others.
- *Reasoning and argument*: This capability involves logically rooted thought processes that explore and link problem elements so as to make inferences from them, check a justification that is given, or provide a justification of statements or solutions to problems.
- *Representing*: Create a mathematical representation of real-world information.

3.3. The role of language

The role of language in mathematics learning is a critical topic, and it is usually dealt with from a variety of theoretical perspectives. A lot of research in Mathematics education highlight that some difficulties in learning mathematics are connected to limited linguistic competencies (Ferrari, 2004).

A controversial issue is the relationships between communication processes and the development of thinking. The verbal language is widely used in mathematical activities. Therefore, language problems in mathematics learning are not just about symbolic language, but they must be considered in a broader perspective. In the opinion of some researchers thinking and communication are closely linked, whereas others regard them as quite independent processes. Our experience appears to confirm Ferrari's ideas that communication processes and the development of thought are strongly linked. (Ferrari, 2004).

4. Methodology

The project was divided into three distinct steps and many actors were involved, each fit to different roles and responsibilities:

First step: preparation. The methodological approach of research-action was adopted, basing the planning on different territorial and educational curriculum needs. The selection and processing of the materials took place with the support of the tutors of the selected institutions. The critical (transversal) and objective difficulties proposed by single questions, analysis of language and distractors of understanding were taken into account.

Second step: implementation. During this step, extensive use of the active learning was involved, starting from situations that enable the need of mathematizable problems and the practice of heuristic procedures: situations naturally suitable to conjecturing, assuming

conditions, posing and solving problems that can bring about logical and deductive reasoning. Some strategies were implemented, including Rogers' 'non-directive learning', 'research-action', 'teaching to think', 'heuristic teaching-learning' or the more, hands-on problem solving, Ausubel's 'bifactorial teaching-learning' model (Ausubel, 1967) or the more recent, Bruner-inspired 'meaningful teaching' or 'mastery strategies', or learning by – doing and peer-education.

When the project group conceived the problems, it considered that:

- the problem must be accessible and firmly rooted in mathematical materials familiar to children.
- the problem is developmental; it activates and trains new modes of thinking and stretches existing skills.
- the problem is revealing, and it opens new mathematical vistas or sheds new light on existing objects, structures and processes of mathematics.

In the management of the activities, tools and techniques for teaching have been used: working on the difficulties, scaffolding the language, calculation/procedure from resources to energy saving, training for the development of metacognitive skills (building maps and forms), guided exercise, autonomous exercise (Capone, Rogora, & Tortoriello, 2018).

In class' activities, most mathematical questions have been proposed as a verbal problem, i.e. tasks presented in written verbal form, possibly supplemented by mathematical symbolism (Gerofsky, 1996) where the text is understood as a system of signs adopted for the communication of the task. We have tried to structure the activity by highlighting two fundamental aspects related to a mathematical problem: cognitive content and editorial organization (Duval, 1993). According to Laborde (1995) the variables not directly associated with the language are:

- Syntactic complexity
- Clarity of the layout, punctuation and syntactic structures
- Density of the sentence
- Difference between the form of the information presented and its form fit to the processing to get to resolution.
- Degree of explanation of intermediate objects useful for resolution
- Order of information provided

Students were divided into groups of four or five students. Collaboration could be considered as a catalyst for the development of certain problem-solving skills (reasoning) because the learner will use the techniques and strategies of reasoning which were assimilated during the work with his classmates or with the teacher even when facing alone a problem similar to one already seen in group. Moreover, logical memory, voluntary attention and the formation of concepts first develop at interpsychological level, as a consequence of social interaction with the group of peers and adults in which it is inserted, and then, later on, at intrapsychological level (Vygotskij, 1992).

Third step: dissemination of results and monitoring. Protocols returned by students have been analysed and compared between them by type of items and curriculum.

The materials produced were spread through various channels including seminars, reports, and scientific publications.

5. Protocols' analysis

According to O'Halloran (2005), the activities proposed to students can be classified by semiotic conversion. Here we focused on two main groups of semiotic systems in mathematics: symbolic language and figural language. So, three kinds of activities were proposed, in conformity with this scheme:

- Activities with symbols. The proposed activities were aimed at developing in students the awareness that the symbols depend on the context and that this awareness presume an agreement in the community of recipients (Coppola, Di Martino, Pacelli, & Sabena, 2015).
- Activities with graphs. The activities were focused on processing and interpretation of tables or matrices, seen as data representations, on processing and reading of graphs (histograms, aerograms, diagrams, Cartesians, etc.) to display information even more clearly and legibly than tables.
- Activities with figures. These activities are about the conversion between the figurative system and the symbolic system and vice versa. For example, activities were proposed so that students had to identify, among various expressions, the one corresponding to the area of a figure drawn and, vice versa, identify figures corresponding to a given algebraic expression.

In the following, there are some significant protocols of activities where transition between different semiotic registers has been evidenced. We focused on protocols about activities carried out at a Scientific High School, option Applied Sciences, and a Professional Institute.

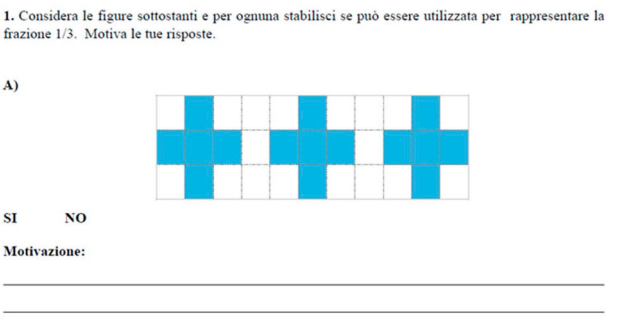
The first activity is on the fractions. The purpose of this activity is to overcome the limits hidden in the didactic contract (Brousseau, 1983) and the typical misconceptions (Sbaragli & Santi, 2011) in geometry and arithemtic and to enhance conversion skills between semiotic registers, in particular between symbolic register and figural register. The content categories are quantity, space and form. The knowledge and the mathematical concepts treated are fractions and area of plane figures. The most involved fundamental mathematical skills (Turner et al., 2013) are: representation, elaboration of strategies for problem solving, use of a symbolic, technical and formal language and operations. The involved processes are use of concepts, facts, procedures and mathematical reasoning.

Each question, in addition to the mere mathematical calculation, required a motivation to the answer.

In Figure 1, there is an examplum of item, with some answers. Texts in the figures are in Italian, nevertheless we show in the paper an English translation of them.

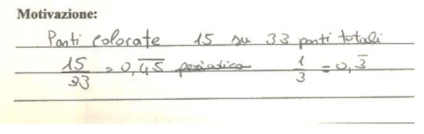
Explanation was not always provided because students are not used to explain the process they followed to get to results. In particular, at the Professional Institute only 20% of students gave a motivation.

Often, students gave tautological motivations or explainations with no meaning: 'Because the relationship is not 1/3'.



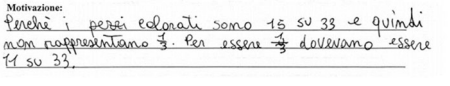
The asked question was: "*May the coloured figures represent the 1/3 Fraction? Give a motivation to your answer*".

Student 1's Answer



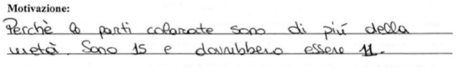
Coloured parts are 15 on a total of 33 parts, so that $\frac{15}{33} = 0,\overline{45}$, *which is different from* $\frac{1}{3} = 0,\overline{3}$

Student 2's Answer



Because coloured parts are 15 on 33, so they aren't 1/3. In order to be 1/3 they should be 11/33.

Student 3's Answer



Because the coloured parts are more than one half: they are 15 and they should be 11.

Figure 1. Example of item on the fractions, where conversion skills between symbolic register and figural register are evidenced. Some students' answers: student 1 at Scientific High School, students 2 and 3 at a Professional High School.

Some answers, also shown in Figure 1, are the following ones:

Student 1, at Scientific High School: *Coloured parts are 15 on a total of 33 parts, so that* $(15/33) = 0,\overline{45}$, *which is different from* $(1/3) = 0,\bar{3}$.

Student 2, at Professional Institute: *Because coloured parts are 15 on 33, so they aren't 1/3. In order to be 1/3 they should be 11/33.*

Let's observe that student 2 first use a tautological motivation and, after that, he tries to give an explanation, but he doesn't answer to the initial question 'Is it 1/3? Why not?', he implicitly changes the question in 'How many parts have to be coloured to be equal to 1/3?'

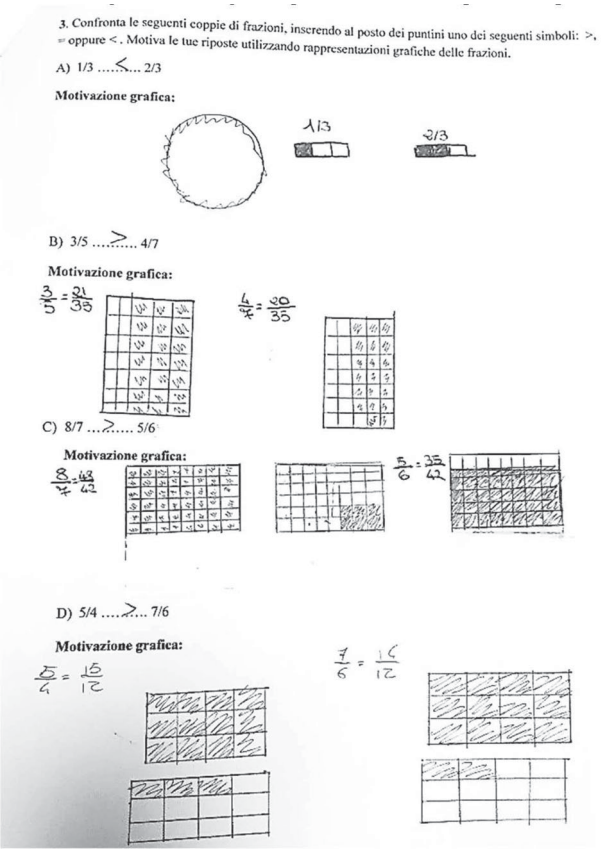
Sometimes, students use a wrong word, although their concept was quite correct. For example:

Student 3, at Professional Institute (Figure 1(c)): *Because the coloured parts are more than one half: they are 15 and they should be 11.*

Student 3 rightly comprehended that coloured parts should be 11, but he uses 1/2 instead of 1/3 in its argumentation.

The following protocol was about the same activity, the Question was:

> Compare between them the fractions, using symbols $<$ and $>$. Give a motivation of your answer by using a graphic representation.



"Compare between them the fractions, using symbols < and >. Give a motivation of your answer by using a graphic representation."

Figure 2. Example of item on fractions and some student's answers, using a non-verbal register.

Students, even without arguments, managed to well explain what was asked, using a non-verbal register, as you can see in Figure 2, even though the question was not simple to interpret.

There were no significant differences in the answers to this item between the two schools.

The following protocols concern the 'Let's talk about formulas' activity. The purpose of this activity is the strengthening of communication skills and conversion between semiotic registers: verbal register – symbolic register. The categories involved are change and relationships. The knowledge and mathematical concepts required are monomials and polynomials, linear equations, linear systems of two equations in two unknowns. The fundamental mathematical skills (Turner et al., 2013) most involved are: communication, representation, use of symbols, technical and formal language. The processes involved are the use of concepts, facts, procedures and mathematical reasoning.

Students were asked to translate a mathematical text into formulas: in Figure 3 there is the translaction from text to the formula of cubes difference.

Students encounter greater difficulties because they are not always able to identify the main part of the sentence. Even more complex for the students is to write a text

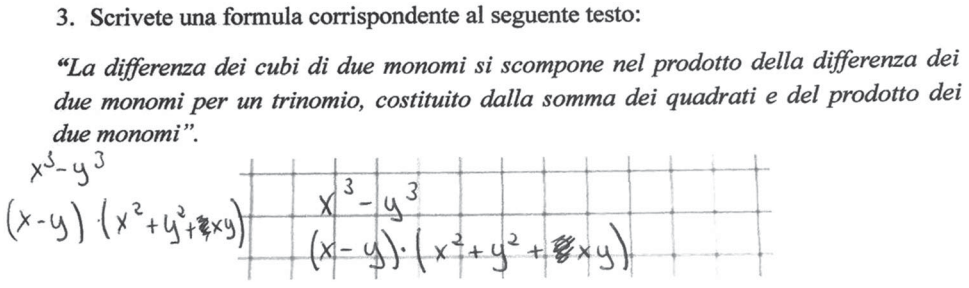
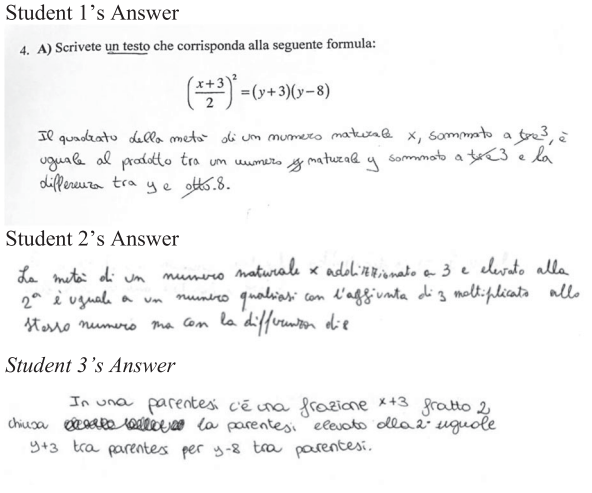


Figure 3. Example of item in 'change and relationships' categories: translation from text to cubes difference formula, where the conversion between verbal register and symbolic register has been evidenced.



"Square of the half of a number, x, added to 3 is equal to the product between a number y plus 3 and the difference between y and 8."

"Half of a number plus 3 and squared is equal to any number added to three crossed to the same number but with a difference of 8."

"In a bracket ther is a fraction x+3 divided by 2, close bracket and square, equal to y+3 within brackets, cross y-8, within brackets.

Figure 4. Example of item in 'change and relationships' categories: students are asked to translate formulas in text. Some students' answers: student 1 at Scientific High School, students 2 and 3 at a Professional High School.

corresponding to a formula (see Figure 4) or a sequence of instructions that allow to write a formula (Figure 5).

Student 1, at Scientific High School, answered: 'Square of the half of a number, x, added to 3 is equal to the product between a number y plus 3 and the difference between y and 8.'

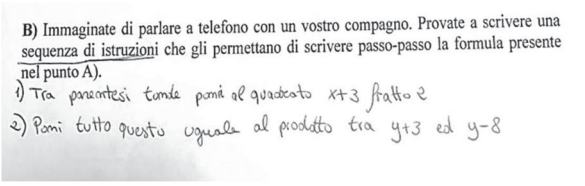
Student 2, at Professional School, wrote: 'Half of a number plus 3 and squared is equal to any number added to three crossed to the same number but with a difference of 8.' Some grammatical Italian errors are also evidenced in his text.

Student 3, at Professional School: 'In a bracket ther is a fraction $x+3$ divided by 2, close bracket and square, equal to $y+3$ within brackets, cross $y-8$, within brackets'.

In the next request (Figure 5), students have to give some instructions, as to a friend on the phone, in order to re-write the previous formula step by step.

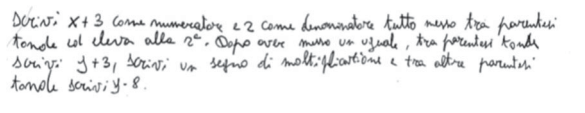
Some students, at professional School, re-wrote the previous answer, shown in Figure 4.

Analysing the protocols of Scientific High School and Professional Institute, the experimental evidence presents some differences compared to the ones just seen.



Student 1 *(scientific school) wrote*
1) in the round brackets square x+3 divided 2"
2) "After that, put all of this equal to product between y+3 and y-8.

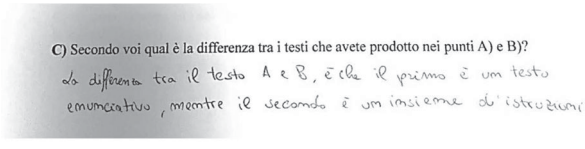
Student 2's Answer



Student 2 *(professional school): "Write x+3 as a numerator and 2 as a denominator, close all of that between brackets and square. After that, write equal and between brackets write y+3, a multiplication sign and between further brackets write y-8.*

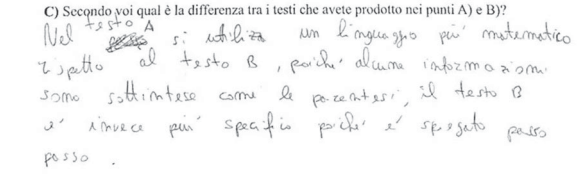
Figure 5. Students are asked to individuate a sequence of instruction to write a formula.

Student 1's Answer



Student1 *(Scientific School): Difference between text A and text B is that the 1st one is an enunciative text, whereas the second one is a series of instructions.*

Student 2's Answer



Student 2 *(Scientific School): in the test A a "more mathematical" language has been used with respect to text B, because some information is tacit, i.e. the brackets; text B is more specified, because it is explained step by step.*

Figure 6. Students are asked to find the difference between the two texts written by themselves (see Figures 4 and 5): some scientific high school students' answers.

In particular, with reference to this 'Let's talk about formulas' activity, the students of the Professional Institute managed to translate the text into a formula, even though with some difficulty, but no student was able to rightly move from one formula to another text that describes it.

Students have difficulty in thinking about mathematics as a discipline in which the end of the process is the result of logical reasoning and procedure.

It is confirmed when they were asked to identify the difference between the two texts in Figure 4 and in Figure 5. All students at Professional School wrote 'There are no differences between the two texts'.

Some students (see Figure 6), at scientific high school, did nothing but repeat what was already inherent in the question.

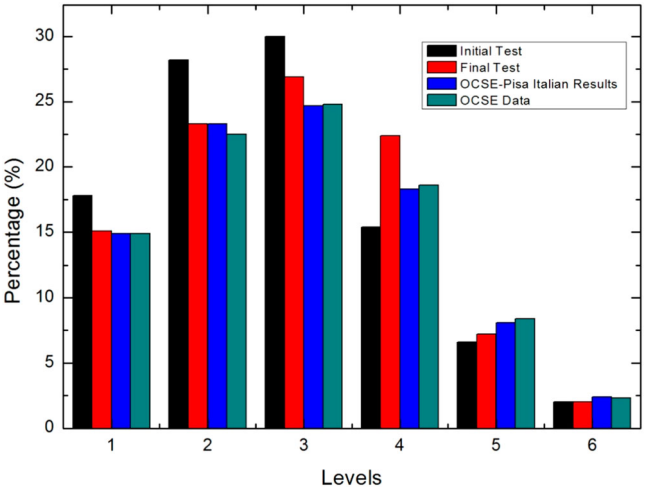


Figure 7. A quantitative analysis on skill levels: percentage of students in the corresponding OCSE levels, resulting in the initial (1st bar) and final (2nd bar) test, compared with OCSE-Pisa (3rd bar) and OCSE (4th bar) results.

Besides the management of the different semiotic registers in the study of geometry, we should also consider the misconceptions generated by an imbalance between the conceptual and the figural components, which have been evidenced in several activities. A geometric figure also includes mental representations of spatial properties. The geometric figures, therefore, represent mental constructions that simultaneously possess conceptual and figural properties. Psychological constraints prevent the integration between spatial properties (forms, positions, sizes) and conceptual qualities (abstraction, generality, perfection). Often the iconic representation is identified with the geometrical concept (D'Amore, 2003). Cognitive activity lies in the recognition of objects represented through different representations that must explain each other (Duval, 2006). There is sometimes confusion between 'figure' and 'design' because the figure is used both to indicate the abstract and ideal mathematical entity and its specific graphic representation, that is the design of the figure itself.

6. Skill levels: a quantitative analysis

In order to highlight if there is a correspondence between the realized activities and the skills levels, an initial and a final test has been carried out. Both the tests were similar: they were composed by 16 questions, each of them including two or three items, scheduled in different mathematical content knowledges: change and relationships, space and shape, quantity, uncertainty and data. Questions have been also chosen with difficulty increasing levels (1–6), in agreement with OCSE (PISA, OECD, 2012).

Both the tests have been evaulated in agreement with OCSE-PISA levels grid (PISA, OECD, 2012).

The quantitative analysis is summarized in Figure 7, where the tests obtained results have been compared between them and with OCSE Data and OCSE PISA results, too.

It is possibile to observe that the effect of the realized activities is a positive trend toward OCSE data, whereas in the initial test a more evident discrepancy was releaved, mainly in the levels 1–3.

7. Conclusions

Some studies show how main difficulties in learning mathematics can be attributed to poor linguistic skills (Ferrari, 2004) and to difficulties in management and in the coordination of various semiotic registers (Duval, 2006).

The aim of our experimentation was a trial to support the development of skills in mathematics, through some innovative activities, conceived to stimulate the students to change from a register to another one (treatments), or from a representation to another one, in the same semiotic register (conversions), and to argue in Mathematics.

The project involved about 400 fifteen-year-old students from Campania region, coming from technical and professional institutes, psycho-pedagogical high schools, classical and scientific high schools.

The theoretical framework at the base of our activities was the Sfard theory (Sfard, 2001), where the language is regarded as a constructor of meanings, and the role of different semiotic systems in the Mathematics learning and teaching (Arzarello, 2006; Duval, 2006; Godino et al., 2030).

The project was framed in the context of OECD-PISA research. We mainly focused on representation and communication skills (Turner et al., 2013).

From an observation of the results of the test produced by the students it seemed that difficulties emerged in the transition between the different semiotic registers. The students encountered difficulties both in moving from a graphical or symbolic register to a textual register, and vice versa. In this last step, the cognitive barriers have been more evident. The written argumentations also confirmed the difficulties in argumentation producing texts on the part of the students and this especially concerned students with poor linguistic skills, who completely omitted the written arguments especially in the first activities. During the development of the project we highlighted an improvement in communication and capacity.

The protocol analysis shows that, in some class contexts in which the students started out from situations of general very low linguistic skills, with no habit of writing in mathematics and the argumentation in general (see for example the Professional Institutes), students had more difficulty when changing semiotic registers. On the contrary, for the classes and contexts in which there were more linguistic competences from the beginning, we could see a process of evolution towards more and more complete arguments and the use of a more and more rigorous language from the scientific point of view.

We also analysed the skill levels of the students, comparing the results of an initial and a final test with the OECD Data and the Italian PISA data, and a positive trend has been evidenced as the effect of the realized activities.

Our proposal, as a future work, is to include, in the curricular activities for the mathematic learning and teaching, some activities mainly focused on the management of different semiotic registers and representations (both conversions and treatments), similarly to what we planned and experimented, in order to increase mathematic skills.

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