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Warehouse location selection by fuzzy multi-criteria decision making methodologies based on subjective and objective criteria

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ABSTRACT

This paper proposes three new extended fuzzy multi-criteria decision making methodologies capable of handling subjective and objective factors for the evaluation and selection of warehouse location. A warehouse location selection problem involves both subjective as well as objective criteria. The concept of fuzzy set theory is integrated with the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS), Simple Additive Weight (SAW) and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) methods to assess subjective criteria in terms of subjective factor measures. A classical normalization technique is employed to assess the objective criteria in terms of objective factor measures. Subjective factor measures and objective factor measures are integrated by the Brown and Gibson model to calculate the warehouse location selection index. The proposed methods are illustrated with two examples of warehouse location selection. A comparative study of the results and a sensitivity analysis are carried out. The study finds that the proposed methodologies are useful and effective fuzzy multi-criteria decision making tools for the evaluation and selection of warehouse location in a supply chain.

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1. Introduction

Global competition in the industrial market has posed a significant challenge to the very survival of companies. This threat led companies to realize that they cannot survive by staying as standalone identities, but have to work in close coordination with other organizations in the form of a network of various business entities, widely known as a supply chain. Supply chain performance primarily depends upon the cohesive and coordinated functioning of its major drivers, e.g. inventory, transportation, facilities and information. To optimize a supply chain network, facilities such as factories, warehouses, Distribution Centers (DCs) and retail outlets must be strategically located (Simchi-Levi, Kaminsky, & Simchi-Levi, 2003). Proper warehouse location improves the profitability of the company and reduces supply chain risk and uncertainty. Suitable selection of the warehouse location improves the level of customer service and customer satisfaction, thereby increasing the company's competitive advantage. These opportunities and threats of Warehouse Location Selection (WLS) tipped it as a strategic investment decision that deserves to be treated with importance and to receive the attention of the research community (Ozcan, Celebi, & Esnaf, 2011).

In spite of having huge strategic dimensions, warehouse location selection has no structured or programmed process (Simchi-Levi et al., 2003). Such long-term decisions have immense impact on organization as they are costly and difficult to reverse. Significantly, a committee comprising high-level managers of the organization along with external consultants play a crucial role in WLS. The committee pays the utmost attention to chosing the right combination of selection criteria for appropriate warehouse location. The selection criteria can be broadly classified into three categories (Liang & Wang, 1991) having critical, objective and subjective attributes. The committee considers multiple criteria/attributes of various categories in order to select the best warehouse location. Often these criteria are conflicting by nature. Hence, warehouse location selection is a Multi-Criteria Decision Making (MCDM) problem. MCDM is an approach to ranking and selecting the best from a set of feasible alternatives. MCDM can be classified into the Multi-Attribute Decision Making (MADM) or Multi-Objective Decision Making (MODM) approaches. The objective of MADM is finite and implicit, whereas the objective of the MODM approach is infinite and explicit. The main advantage of MCDM models is their ability to consider a large number of selection criteria. The MCDM approach can also be divided into two categories, viz. classical MCDM and Fuzzy MCDM (FMCDM). In classical MCDM, decisions are made under certainty on the basis of objective (quantitative) criteria (Zeleny, 1982). Decision making under uncertainty on the basis of subjective (qualitative) criteria deserves the application of FMCDM. (Zadeh, 1965) published an important evolutionary seminal paper on the modern concept of uncertainty.

MCDM problems are solved by various techniques or tools such as: the Total Sum (TS) method; the Simple Additive Weighting (SAW) method; the Analytical Hierarchy Process (AHP); the Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) method (Brauers, 2004); Data Envelopment Analysis (DEA); outranking approaches (Elimination and Et Choice Translating Reality [ELECTRE] and Preference Ranking Organization Method for Enrichment Evaluation [PROMETHEE]); and compromise approaches (Technique for Order Preference by Similarity to the Ideal Solution [TOPSIS]

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and Visekriterijumsko Kompromisno Rangiranje [VIKOR]). Korpela and Tuominen (1996) presented an AHP based integrated approach to warehouse site selection considering both qualitative and quantitative aspects. Liang (1999) proposed a fuzzy multiple attribute decision making (FMADM) method to identify the optimal alternative based on ideal and anti-ideal point concepts. Chen (2001) developed a new FMADM approach for the DC location selection problem based on a stepwise ranking procedure.

Kahraman, Ruan, and Ibrahim (2003) included four different fuzzy Multi-Attribute Group Decision Making (FMAGDM) approaches, which they extended to select the best facility location by taking into account quantitative and qualitative criteria. That paper revealed the Fuzzy Analytical Hierarchy Process (FAHP) as the most complex method out of the four methods proposed for solving the facility selection problem. Bhattacharya, Sarkar, and Mukherjee (2004) proposed a method for selecting plant location under an MCDM environment with certainty. Chen, Yuan, and Lee (2007) presented a two-phase fuzzy decision making method for locating warehouses in a supply chain. Chou, Chang, and Shen (2008) presented a Fuzzy Simple Additive Weighting System (FSAWS) for solving facility location selection problems by using objective/subjective attributes under Group Decision Making (GDM) conditions. The authors integrated Fuzzy Set Theory (FST), the Factor Rating System (FRS) and SAW to solve selection problems combining both vaguely defined subjective attributes and precisely defined objective attributes. Wang and Lee (2009) developed a fuzzy TOPSIS approach based on subjective weights and objective weights, wherein subjective weights are determined by the decision makers, whereas objective weights are determined by Shannon's entropy measure (Shannon & Weaver, 1947).

Demirel, Demirel, and Kahraman (2010) applied the multi-criteria choquet integral method to a real warehouse location selection problem for a big Turkish logistic firm. Rao and Patel (2010) proposed a subjective and objective integrated multiple attribute decision making method wherein the authors introduced ranked value judgments on an 11 point fuzzy conversion scale to assess qualitative attributes and a statistical variance method to determine the objective weights. Sadi-Nezhad and Damghani (2010) applied the Fuzzy TOPSIS (FTOPSIS) method based on a modified preference ratio method and an efficient fuzzy distance measurement for the assessment of traffic police centers performance. The efficient fuzzy distance measurement was used to measure the distance between two fuzzy numbers in terms of a fuzzy number (not a crisp value). The ranking method based on preference ratio concept provides relative ranking order of the alternatives instead of an absolute one. Shahanaghi and Yazdian (2010) addressed location and distribution decisions in supply chains with fleet sizing considering both tangible and intangible criteria. The authors applied TOPSIS for ranking the locations and used those ratings in the Multi-Objective Mixed-Integer Programming (MOMIP) to determine the optimal solution for the problem.

Jabalameli, Hosseininezhad, and Naini (2013) proposed a risk management model for the plant location problem under a fuzzy environment. Dey, Bairagi, Sarkar, and Sanyal (2013) proposed a hybrid fuzzy technique for the selection of warehouse location in a supply chain under a utopian environment. Farahani, Hekmatfar, Fahimnia, and Kazemzadeh (2014) presented a comprehensive review of over 40 years of hierarchical facility location modeling efforts, identified the gaps in the current literature and suggested the directions for future modeling efforts. Roh, Pettit, Harris, and Beresford (in press) considered the pre-positioning of warehouses for humanitarian relief organizations from both macro and micro perspectives, and analysed the managerial implications of those decisions. Karmaker and Saha (2015) carried out research work on the optimization of warehouse location selection through fuzzy multi-criteria decision making methods. Cebi and Otay (2015) presented multi-criteria and multi-stage facility location selection under an interval type-2 fuzzy environment: a case study of a cement factory. Parameshwaran, Kumar, and Sravanakumar (2015) presented an integrated approach for the optimal selection of robots by considering both objective and subjective criteria. Rao, Goh, Zhao, and Zheng (2015) presented a new Fuzzy Multi-Attribute Group Decision Making (FMAGDM) technique based on a linguistic 2-tuple for the location selection of city logistics centers from a sustainability perspective.

Previous studies have shown that conventional MCDM methods are limited in their ability to solve real-life warehouse location problems, especially those based on subjective and objective criteria. Many researchers have applied AHP for subjective assessment of criteria weights. But AHP inherits computational complexity and is a time-consuming process. This problem is aggravated further as the number of criteria increases. Objective analysis based on complex mathematical treatments such as linear programming or mixed integer programming is also a time-consuming process. Established MCDM methods also face difficulties in handling both tangible and intangible information at the same time. Moreover, in real-time case studies, paucity of weighting information is a hard reality. These limitations encourage us to propose new, simple, straightforward, effective FMCDM techniques to solve warehouse location selection problems. No study yet has proposed applying FMCDM methods, e.g. FTOPSIS, FSAW and FMOORA, independently for subjective assessment, together with a normalization technique for objective assessment, to solve warehouse location problems.

The novelty of the proposed research work is the extension of the FTOPSIS, FSAW and FMOORA methods for warehouse location selection to integrate the subjective factor measure (SFM) and the objective factor measure (OFM) within a proper objective decision-making approach. The proposed methodologies can also deal effectively with decision problems without prerequisite information concerning objective criteria weights. Though each algorithm can solve the problem independently, comparative analysis of the results among them demonstrates the validation of the methodologies.

The objective of this research paper is to increase the efficiency and effectiveness, and in turn the profitability, of a supply chain by aiding and guiding managers towards a strategic and optimum solution to the warehouse location selection problem. The objective is achieved by performing following steps:

(1) shortlisting the key parameters of warehouse location, evaluation and selection incorporating experts' experience and opinions into decision making;
(2) computation of the subjective and objective weights of the respective criteria;
(3) applying the FTOPSIS, FSAW and FMOORA techniques through a group decision making process to calculate the SFM;
(4) conversion of the total cost of an alternative, i.e. the Objective Factor Cost (OFC), into the OFM of that alternative by a classical normalization technique;

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(5) integration of the SFM and the OFM of the alternatives by application of the Brown and Gibson model (1972) to determine the Warehouse Location Selection Index (WLSI); and
(6) comparative study of the results along with sensitivity analysis of warehouse location selection problems while implementing and investigating the proposed FMCDM approaches.

The remainder of the paper is organized as follows. Section 2 summarizes the concept of fuzzy set theory. Section 3 defines the importance of criteria in warehouse location selection. Section 4 proposes three algorithms for warehouse location selection. Section 5 illustrates two real-life problems on warehouse location selection with the application of three proposed methodologies. Section 6 carries out an in-depth discussion, comparative study of results and sensitivity analysis. Finally, Section 7 provides conclusions and some essential remarks.

2. Fuzzy set theory

Bellman and Zadeh (1970) were first to propose decision making techniques in a fuzzy environment. The concept of fuzzy set theory when applied in a multiple criteria decision making environment is called the Fuzzy Multi-Criteria Decision Making (FMCDM) approach. The FMCDM approach can assist decision makers in evaluating, ranking and selecting the best alternative in a fuzzy environment. Human beings prefer subjective to objective assessment of vague, ambiguous and imprecise data. In many real-life problems where objective factors along with crisp sets are insufficient and inadequate, subjective assessment along with fuzzy sets are employed for the evaluation of alternatives. This implies the application of Fuzzy Set Theory (FST) to capture the importance of criteria and to evaluate the performance of alternatives in a fuzzy environment. This section very briefly reviews the basic fuzzy definitions espoused by this research work. They are as follows.

Definition 2.1: A *Triangular Fuzzy Number* (TFN) $\tilde{p}$ is defined by a triplet (p_1, p_2, p_3), and its *membership function* is defined as follows:

$$\mu_{\tilde{p}}(x) = \begin{cases} 0, & x < p_1 \\ \frac{x-p_1}{p_2-p_1}, & p_1 \le x \le p_2 \\ \frac{x-p_3}{p_2-p_3}, & p_2 \le x \le p_3 \\ 0, & x > p_3. \end{cases} \quad (1)$$

Definition 2.2: If $\tilde{p} = (p_1, p_2, p_3)$ and $\tilde{q} = (q_1, q_2, q_3)$ are two triangular fuzzy numbers, the *distance* between the fuzzy numbers is measured by

$$d(\tilde{p}, \tilde{q}) = \sqrt{\frac{1}{3}\left[(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2\right]} \quad (2)$$

This method of measuring the distance between two TFNs is known as the *vertex method*.

Fuzzy set theory is utilized extensively in decision making under uncertainty for warehouse location selection. The selection of location alternatives primarily depends upon the relevant criteria under the circumstances. The importance of these criteria in warehouse location selection is described next in Section 3.

3. Importance of criteria in warehouse location selection

Warehouse location selection is a complex multi-criteria decision making approach. The objective of the Multi-Criteria Decision Making (MCDM) tool is to select the best alternative from a set of feasible alternatives. The alternatives are evaluated, ranked and selected on the basis of the nature and importance of the selection criteria. The selection criteria can be broadly classified into three categories, i.e. critical, objective and subjective criteria. The selection of criteria and the evaluation of the importance of their weights make proportional impacts on performance measures while making decisions. Therefore, the procedure for evaluating the weights of criteria has a direct impact on the outcome of the methodology. Hence, the categories of criteria need to be briefly discussed in order to make a proper appraisal of the alternative locations.

3.1. Critical criteria

A critical criterion is one that must be satisfied before further assessment of alternatives. The importance of this criterion may preclude the location regardless of other conditions that might exist. For example, an aluminum smelting plant would not consider a location where low cost and plentiful electrical energy are not available.

3.2. Subjective criteria

The characteristics of subjective (qualitative) criteria are that they are ill-informed, ambiguous, imprecise and uncertain in nature and are often expressed in terms of linguistic variables that are measured by human perception, feelings and experience (Zadeh, 1965; Zimmermann, 1991). The availability of labor, the proximity of warehouses to markets and customers, etc. are considered as the potential subjective criteria for warehouse location selection. The FMCDM approach is useful for modeling this imprecision and uncertainty. The evaluation of weights of subjective criteria involves the diverse opinions of the decision makers based on their expertise and preferences. Some mathematical methods such as the eigenvector method or the weighted least squares method may help to evaluate the overall subjective assessment of the decision makers. The effect of all the subjective criteria may be summarized as a Subjective Factor Measure (SFM). In these algorithms, the SFM is evaluated independently by three fundamentally superior, indisputable and benchmark FMCDM tools, i.e. FTOPSIS, FSAW and FMOORA.

3.3. Objective criteria

Objective criteria are those that are measurable and quantitative such as cost, weight, speed, distance, lifecycles measured in suitable units, reliability, etc. Objective criteria are normally expressed as crisp numbers. In many real-life problems where objective criteria along with crisp sets are insufficient and inadequate, subjective assessment along with fuzzy sets are employed for the evaluation of alternatives. Generally, the entropy method and multiple objective programming help to determine the weights of objective criteria. An objective factor in the cost category (in the non-benefit category sense) is called the Objective Factor Cost (OFC). In this paper, the OFC represents the total cost of each alternative. Total cost is the

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sum of all cost elements of that alternative. The normalized assessment of the objective factor cost (total cost) is called the Objective Factor Measure (OFM), which is determined by a classical normalization technique.

3.4. Integration of critical, subjective and objective criteria

To ensure the proper evaluation of the weights of any alternative locations, both the subjective and objective approaches must be integrated in a correct comparison. The Brown and Gibson (1972) location model is a simple and viable tool developed for evaluating alternative locations using certain objective and subjective criteria. The model can be expressed as

$$\mathrm{LSI}_i = \mathrm{CFM}_i\left[\alpha \times \mathrm{OFM}_i \oplus (1-\alpha) \times \mathrm{SFM}_i\right], \tag{3}$$

where LSI_i is the location selection index for the ith alternative, CFM_i is the critical factor measure for the ith alternative and the value of $\mathrm{CFM}_i = 1$ or 0. Critical factors have the effect of eliminating the alternative location from consideration in itself, regardless of other conditions that might exist. SFM_i is the subjective factor measure of the ith alternative and its value lies in the range $0 \le \mathrm{SFM}_i \le 1$. OFM_i is the objective factor measure of the ith alternative, and its value lies in the range $0 \le \mathrm{OFM}_i \le 1$.$\alpha$ is the objective factor decision weight or coefficient of attitude having value in the range $0 \le \alpha \le 1$. The value of α depends upon the decision making attitude towards objective criteria. This factor also determines the relative importance of subjective and objective criteria in the overall evaluation and selection of any alternative. Higher values of α indicate dominance of the OFM. The most preferable value of α is 0.33.

The model also considers that some location factor is critical and its nature may preclude the location of a facility (warehouse) at a particular region. The fulfillment of a critical criterion is essential for further evaluation and selection of any warehouse location, i.e. $\mathrm{CFM}_i = 1$. Hence, Equation (3) is modified to

$$\mathrm{WLSI}_i = \left[\alpha \times \mathrm{OFM}_i \oplus (1-\alpha) \times \mathrm{SFM}_i\right]. \tag{4}$$

The subjective factor measure and the objective factor measure are combined with a proper coefficient of variable attitude to evaluate the WLSI. This index acts as the key parameter of warehouse location selection, which is incorporated in the newly developed three algorithms proposed next in Section 4.

4. Proposed FMCDM methodologies

In this paper, three newly developed algorithms are proposed for the evaluation and selection of warehouse locations. Suppose a committee comprising p decision makers assesses m feasible warehouse location alternatives considering n selection criteria. The committee is assumed homogeneous as the decision makers have equal importance in the decision making process though having different ages, genders, expertise, etc. The newly proposed FMCDM methodologies are described below.

4.1. Extended FTOPSIS method

TOPSIS was expounded and propounded by Hwang and Yoon (1981). According to the Fuzzy TOPSIS method, the best alternative is the one that is closest to the Fuzzy Positive Ideal Solution (FPIS) and farthest from the Fuzzy Negative Ideal Solution (FNIS). The stepwise extended FTOPSIS algorithm is explained below.

Step 1: Construct a fuzzy decision matrix with performance measures of alternatives as represented below:

$$M_i = \begin{array}{c|ccccc} A_i & D_1 & \cdots & D_k & \cdots & D_p \\ \hline C_1 & \tilde{x}_{11} & \cdots & \tilde{x}_{1k} & \cdots & \tilde{x}_{1p} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_j & \tilde{x}_{j1} & \cdots & \tilde{x}_{jk} & \cdots & \tilde{x}_{jp} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_n & \tilde{x}_{n1} & \cdots & \tilde{x}_{nk} & \cdots & \tilde{x}_{np} \end{array} \tag{5}$$

where $\tilde{x}_{jk}$ denotes the performance rating of alternativeA^{i} with respect to criterion C^{j}, assigned by the kth decision maker. Here $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p$. It is noted that the number of such decision matrices is equal to the number of alternatives.

Step 2: Construct a fuzzy weight matrix consists of weights awarded by each decision maker to each criterion as expressed below:

$$M_i = \begin{array}{c|ccccc} A_i & D_1 & \cdots & D_k & \cdots & D_p \\ \hline C_1 & \tilde{x}_{11} & \cdots & \tilde{x}_{1k} & \cdots & \tilde{x}_{1p} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_j & \tilde{x}_{j1} & \cdots & \tilde{x}_{jk} & \cdots & \tilde{x}_{jp} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_n & \tilde{x}_{n1} & \cdots & \tilde{x}_{nk} & \cdots & \tilde{x}_{np} \end{array} \tag{6}$$

where $\tilde{w}_{jk} = \left(a_j^k, b_j^k, c_j^k\right)$ is the weight of the jth criterion, awarded by the kth decision maker.

Step 3: Construct the average fuzzy decision matrix. The average fuzzy performance rating $(\tilde{r}_{ij})$ is computed by using the following equation:

$$\tilde{r}_{ij} = \frac{1}{p}\left(\sum_{k=1}^{k=p} \tilde{x}_{ij}^k\right) \tag{7}$$

where $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p$.The average fuzzy rating is a triangular fuzzy number that can be expressed as $\tilde{r}_{ij} = \left(\bar{\alpha}_{ij}, \bar{\beta}_{ij}, \bar{\gamma}_{ij}\right)$.

Step 4: Construct the average fuzzy weight matrix. The average fuzzy weight $(\bar{w}_j)$ is calculated by using the following equation:

$$\bar{w}_j = \frac{1}{p}\left(\sum_{k=1}^{k=p} \tilde{w}_{jk}\right), \tag{8}$$

where $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p$.

Step 5: Form the Weighted Average Fuzzy Decision Matrix (WAFDM). The weighted average fuzzy performance rating (an element of the WAFDM) is determined by using the following equation:

$$\tilde{s}_{ij} = \bar{w}_j \otimes \tilde{r}_{ij}. \tag{9}$$

Step 6: Calculate the normalized weighted fuzzy decision matrix by using the following equation:

$$\tilde{t}_{ij} = \frac{\tilde{s}_{ij}}{\sqrt{\sum_{i=1}^{i=m} s_{ij}'^2}}, \tag{10}$$

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Where S'_{ij} denotes the upper value of the TFN for the jth criterion and $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.

Step 7: Set the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS). In general, FPIS and FNIS are taken as $v_{ij}^{*} = (1, 1, 1)$ and $v_{ij}^{-} = (0, 0, 0)$, respectively.

Step 8: Compute the separation measures of each alternative from FPIS and FNIS.

$$S_i^{+} = \sum_{j=\mathrm{SC}} \sqrt{\left(v_{ij}^{*} - \tilde{t}_{ij}\right)^2}, i = 1, 2, ..., m;\ j = 1, 2, ..., n; \quad (11)$$

$$S_i^{-} = \sum_{j=\mathrm{SC}} \sqrt{\left(v_{ij}^{-} - \tilde{t}_{ij}\right)^2}, i = 1, 2, ..., m;\ j = 1, 2, ..., n; \quad (12)$$

where SC stands for Subjective Criteria.

Step 9: Determine the Relative Closeness (RC) or Closeness Coefficient (CC) of each alternative using the equation

$$\mathrm{CC}_i = \frac{S_i^{-}}{S_i^{+} + S_i^{-}}, \quad (13)$$

where $i = 1, 2, ..., m$. The Closeness Coefficient $\left(\mathrm{CC}_i\right)$ is the assessment of the ith alternative with respect to all subjective criteria. Hence, the closeness coefficient or relative closeness is called the Subjective Factor Measure (SFM′).

Step 10: The normalized Subjective Factor Measure (SFM) for the ith alternative is given by

$$\mathrm{SFM}_i = \frac{\mathrm{SFM}'}{\sum_{i=1}^{m}\left(\mathrm{SFM}'_i\right)}, \quad (14)$$

where $i = 1, 2, ..., m$ and $\sum_{i=1}^{i=m} \mathrm{SFM}_i = 1$.

Step 11: Calculate the Objective Factor Measure (OFM). The OFM of the alternative in the benefit category sense is calculated by a classical normalization technique:

$$\mathrm{OFM}_i = \left[\mathrm{OFC}_i \times \left(\sum_{i=1}^{m} \mathrm{OFC}_i^{-1}\right)\right]^{-1}, \quad (15)$$

where OFC_i is the Objective Factor Cost (total cost) for the ith alternative and OFM_i is the Objective Factor Measure for the ith alternative, where $i = 1, 2, m$.

Step 12: Evaluate the overall warehouse location selection index (WLSI^{i}) by integrating both the SFM and the OFM using Equation (4), which gives

$$\mathrm{WLSI}_i = \left[\alpha \times \mathrm{OFM}_i \oplus (1 - \alpha) \times \mathrm{SFM}_i\right], \quad (16)$$

where the value of α depends upon the decision maker's attitude towards the objective criteria. This objective decision making attitude can effectively deal with the decision problems where objective criteria weights are unavailable.

Step 13: Arrange the alternatives in descending order of their Warehouse Location Selection Index (WLSI^{i}).

Step 14: Select the best alternative associated with the highest WLSI^{i}.

4.2. Extended FSAW method

The Fuzzy Simple Additive Weighting (FSAW) is commonly used as FMADM method, proposed to solve warehouse location selection problems. FSAW calculates the weighted sum (total score) of each alternative with respect to all subjective criteria and may be called the subjective factor measure. The stepwise extended FSAW algorithm is described below.

Step 1: Construct a fuzzy decision matrix $\left(\tilde{x}_{ij}\right)_{m\times n}$, where $\tilde{x}_{ij}$ denotes the fuzzy performance rating of alternative A^{i} with respect to criterion C^{j} by the kth decision maker.

Step 2: Normalize the fuzzy performance ratings of alternative i with respect to subjective criterion j by the following formula:

$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{i=m} x_{ij}'^{2}}}, \quad (17)$$

where x'_{ij} denotes the upper value of the TFN for the jth criterion and $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.

Step 3: Compute the Subjective Factor Measure (SFM′) for alternative i using the following formula:

$$\mathrm{SFM}'_i = \sum_{j\in\mathrm{SC}} \left(\tilde{w}_j \otimes \tilde{r}_{ij}\right), \quad (18)$$

where SC stands for Subjective Criteria and $\tilde{w}_j$ is the fuzzy weight of the jth criterion.

Step 4: Normalize the Subjective Factor Measure as SFM for the ith alternative by

$$\mathrm{SFM}_i = \frac{\mathrm{SFM}'_i}{\sum_{i=1}^{m}\left(\mathrm{SFM}'_i\right)}, \quad (19)$$

where $i = 1, 2, ..., m$ and $\sum_{i=1}^{m} \mathrm{SFM}_i = 1$.

Step 5: Compute the Objective Factor Measure (OFM) for alternative i in the benefit category sense using the following formula:

$$\mathrm{OFM}_i = \left[\mathrm{OFC}_i \times \left(\sum_{i=1}^{m} \mathrm{OFC}_i^{-1}\right)\right]^{-1}. \quad (20)$$

Step 6: The Warehouse Location Selection Index (WLSI) for each alternative is calculated by the following formula:

$$\mathrm{WLSI}_i = \left[\alpha \otimes \mathrm{OFM}_i \oplus (1 - \alpha)\mathrm{SFM}_i\right]. \quad (21)$$

Step 7: Select the best alternative associated with the highest WLSI.

4.3. Extended FMOORA method

Brauers (2004) first introduced the MOORA method in order to solve various complex and conflicting decision making problems. In MOORA, normalized performance measures are added in the case of maximization (for beneficial criteria) and subtracted in the case of minimization (for non-beneficial criteria). In this algorithm, the subjective factor measure is the score of each alternative with respect to all subjective criteria. The stepwise extended Fuzzy MOORA (FMOORA) algorithm is described below.

Step 1: Construct a fuzzy decision matrix $\left(\tilde{x}_{ij}\right)_{m\times n}$, where $\tilde{x}_{ij}$ denotes the fuzzy performance rating of alternative A^{i} with respect to criterion C^{j} by the kth decision maker.

Step 2: Normalize the performance rating of alternative i with respect to subjective criterion j by the following formula:

$$x_{ij}^{*} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{i=m} x_{ij}''^{2}}}, \tag{22}$$

where x_{ij}'' denotes the upper value of the TFN for the jth criterion and$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.

Step 3: Compute the Subjective Factor Measure (SFM′) for alternative i using the following formula:

$$\mathrm{SFM}_i' = \sum_{j \in \mathrm{SC}} \tilde{w}_j \otimes x_{ij}^{*}, \tag{23}$$

where SC stands for Subjective Criteria and $\tilde{w}_j$ is the fuzzy weight of the jth criterion.

Step 4: Normalize the Subjective Factor Measure as SFM for the ith alternative by

$$\mathrm{SFM}_i = \frac{\mathrm{SFM}_i'}{\sum_{i=1}^{m} \left(\mathrm{SFM}_i'\right)}, \tag{24}$$

where $i = 1, 2, \ldots, m$.

Step 5: The Objective Factor Measure (OFM′) of the alternative in the non-benefit category sense is calculated by the following formula:

$$\mathrm{OFM}_i' = \left[\mathrm{OFC}_i \times \left(\sum_{i=1}^{m} \mathrm{OFC}_i^{-1}\right)\right]. \tag{25}$$

Step 6: Normalize the Objective Factor Measure as OFM for the ith alternative by

$$\mathrm{OFM}_i = \frac{\mathrm{OFM}_i'}{\sum_{i=1}^{m} \left(\mathrm{OFM}_i'\right)}, \tag{26}$$

where $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} \mathrm{OFM}_i = 1$.

Step 7: The Warehouse Location Selection Index (WLSI) for each alternative is calculated by the following formula:

$$\mathrm{WLSI}_i = \left[(1-\alpha) \otimes \mathrm{SFM}_i - \alpha \otimes \mathrm{OFM}_i\right]. \tag{27}$$

Step 8: Select the best alternative with the highest WLSI.

Conceptual flow diagram describing the proposed methodologies is depicted in Figure 1. The above-mentioned methodologies are applied on two warehouse location selection problems and numerical illustrations of the same are described next in Section 5.

5. Numerical illustration

In this section, the three proposed algorithms are illustrated with two numerical examples to adjudge their desirability from the points of view of viability and feasibility. The solutions of the cited examples are shown in Subsections 5.1 and 5.2 as follows.

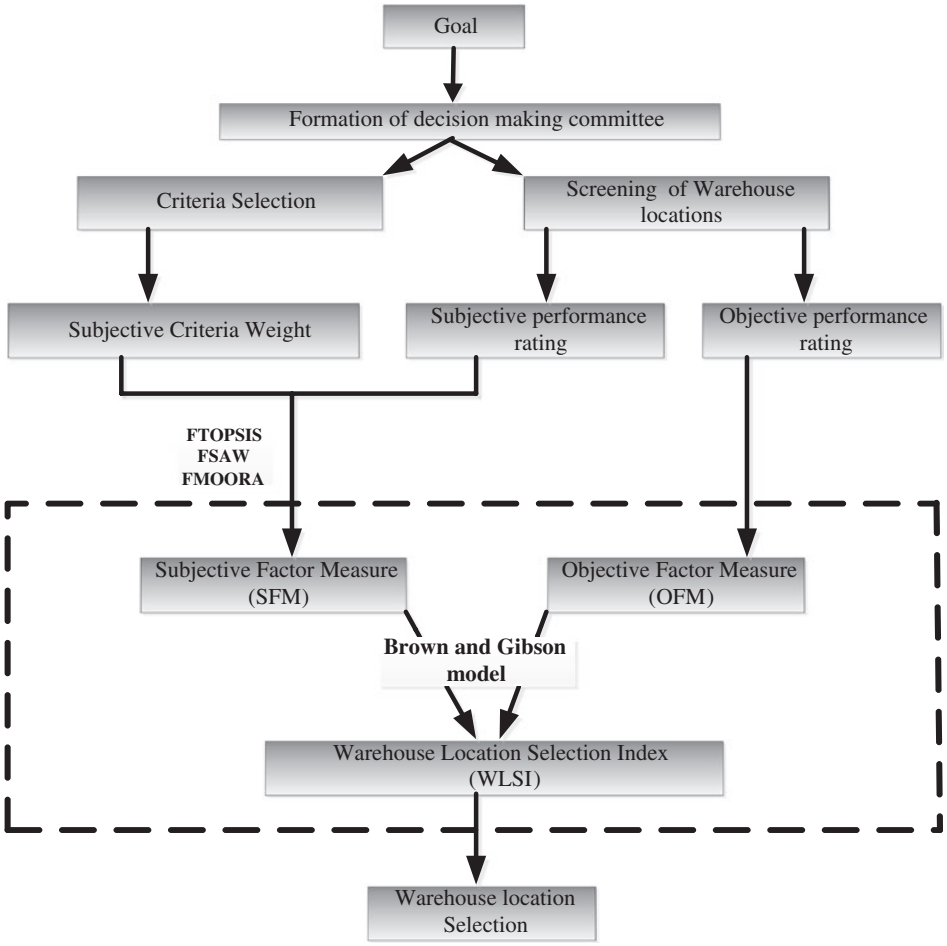


Figure 1. Conceptual flow diagram of the proposed methodologies.

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5.1. Example 1 and its illustration

A reputed South East Asian company desires to expand its base to increase its competitive advantages in the market. For its new plant, the company wants to select an appropriate seaport-based warehouse location primarily targeting the huge Indian market. To meet this specific requirement, the company has set up a committee of three Decision Makers (DMs), each from functionally dominant and diversified departments of the company. The departments of logistics, finance and marketing are the major functional units in the organization structure of the company. DMs having at least 10 years experience in their relevant fields were chosen from those departments. The committee is assumed to be homogeneous in spite of the DMs' difference in age, past experience, responsiveness, gender and reputation. The committee has prepared a set of questionnaires after exhaustive literature review and consultations with the top management of the company for probable warehouse locations suiting their strategic need. A questionnaire-based detailed survey is conducted of managers and supervisors of the company who are already stationed in various seaports of South East Asia. After preliminary screening, the committee short lists Chennai (A_1), Paradip (A_2), Mumbai (A_3) and Chittagaon (A_4) as four feasible warehouse location alternatives. Chennai and Paradip are seaports in India. Mumbai is another seaport and the commercial capital of India. Chittagaon is a seaport in Bangladesh. The committee has also prepared a set of questionnaires for relevant and requisite selection criteria after consulting with researchers in this domain. The committee identified six appropriate selection criteria for the evaluation and selection of the best warehouse location. The selection criteria are defined in the following way.

- *Availability of markets* (C_1). An adequate customer base, producer's plant and supplier base should be available close to the warehouse location.
- *Transportation facility* (C_2). Speedy transport facilities ensure timely supply of finished goods to customers. Transport mode, viz. rail, road, water and air, for the location depends on convenience and suitability.
- *Availability of land* (C_3). Apart from office buildings, open space is required for the storage of materials and finished products, parking of cars and trucks and internal vehicle movement.
- *Political stability* (C_4). Stable political decisions or political change in that region. It ensures government aid and subsidies – favorable industrial policies that attract the attention of many new entrepreneurs and vitally affect location selection.
- *Proximity to customers* (C_5). The location of the warehouse should be as close as possible to the customer base. This implies early fulfillment of customer requirements.
- *Cost* (C_6). Costs are a sacrifice of resources. They may or may not be in monetary units. Costs can be estimated for different location alternatives using standard economic analysis. Management has the objective of minimizing the combined relevant costs.

The first five criteria (C_1–C_5) are subjective in nature, whereas the last criterion (C_6) is objective. For assessing performance ratings, the DMs have consulted in detail with competent authorities at managerial level at each of the seaports. Few friendly (non-competitor) companies who frequently use these seaports for export and import of their products have also been consulted. These companies have provided some very important information regarding criteria evaluation. The information has been verified by cross checking and then incorporated into the current decision making problem. Performance ratings of alternatives and weights of the subjective criteria are expressed by seven degrees of linguistic variables due to the complex and fuzzy nature of the case problem. The linguistic terms of the ratings and weights along with their corresponding Triangular Fuzzy Numbers (TFNs) are shown in Tables 1 and 2, respectively. Membership functions of linguistic variables for ratings as well as weights are shown in Figures 2 and 3, respectively. A decision matrix based on all the subjective criteria is shown in Table 3. The decision makers assess the relative importance of the weights of all the subjective criteria as per their preference and expertise. A fuzzy weight matrix of the subjective criteria is awarded by each decision maker and shown in Table 4.

A decision matrix based on objective criteria involving different cost elements of alternatives is shown in Table 5. The

Table 1. Linguistic variables for the performance ratings of alternatives and their TFNs.

Linguistic variable	Performance rating
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium to Poor (MP)	(1, 3, 5)
Medium (M)	(3, 5, 7)
Medium to Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

Table 2. Linguistic variables for the importance of criteria and their TFNs.

Linguistic variable	Importance
Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0. 3)
Medium to Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium to High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

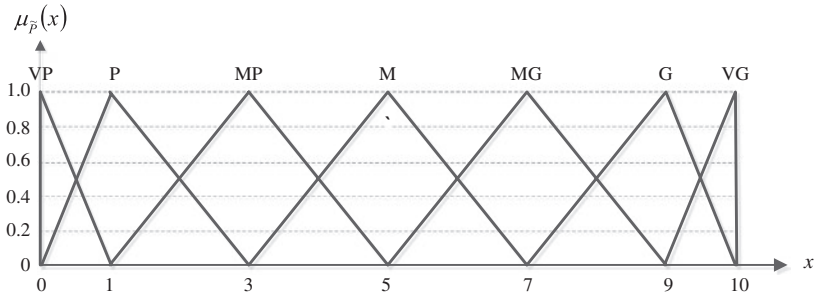


Figure 2. Membership functions of linguistic variables for ratings (Example 1).

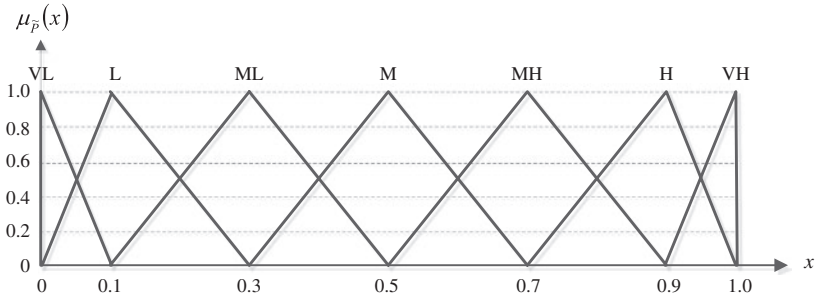


Figure 3. Membership functions of linguistic variables for weights (Example 1).

Table 3. Decision matrix with subjective criteria in linguistic terms assessed by three decision makers.

Alternatives	Criteria	Decision makers			Alternatives	Criteria	Decision makers		
		D_1	D_2	D_3			D_1	D_2	D_3
Chennai (A_1)	C_1	G	VG	MG	Paradip (A_2)	C_1	G	VG	MG
	C_2	VG	G	G		C_2	VG	G	MG
	C_3	MG	G	MP		C_3	P	G	VG
	C_4	MG	MG	P		C_4	VG	G	MG
	C_5	VP	MG	G		C_5	G	MG	VP
Mumbai (A_3)	C_1	G	VG	MG	Chittagaon (A_4)	C_1	G	VG	MG
	C_2	MG	G	P		C_2	VG	G	MG
	C_3	G	P	MG		C_3	G	P	MG
	C_4	MG	G	VG		C_4	MG	P	G
	C_5	P	G	VG		C_5	VP	G	MG

cost elements of the alternatives considered for this case study are taxes, insurance, material handling costs and lighting bills, which vary according the location. The costs of taxes, insurance and lighting are estimated after active consultation with the competent authorities of the respective alternative locations. The decision makers refer to relevant material handling design handbooks, manufacturer's catalogues and internet web sites for estimating material handling costs. The case problem is now solved by the three proposed FMCDM methodologies. Step-by-step solution and comparative analysis of results of the above problem are illustrated in Subsections 5.1.1, 5.1.2 and 5.1.3

5.1.1. Solution by the extended FTOPSIS method

Average fuzzy ratings of alternatives and average fuzzy weights under subjective criteria are computed using Equations (7) and (8), respectively, and shown in Table 6. Weighted fuzzy performance ratings under subjective criteria are calculated using Equation (9) as shown in Table 7. The normalized weighted fuzzy performances of alternatives under subjective criteria are computed using Equation (10) as presented in Table 8. For instance, the normalized weighted fuzzy performance value of alternative A_1 with respect to C_1 may be calculated as $\tilde{t}_{11}$ = (4.90/18.76, 7.46/18.76, 9.38/18.76), i.e. (0.261, 0.397, 0.5). Separation measures are determined using Equations (11) and (12), respectively. Separation measures are represented by the Euclidean distances of each alternative from the positive ideal solution and the negative ideal solution. The closeness coefficients SFM′, SFM, OFM and WLSI are calculated using Equations (13)–(16), respectively.

For instance, OFM_1 = 1/[1800(1/1800+1/2200+1/1850+1/1775)] = 0.2628. The value of the objective decision making attitude is considered to be 0.33. This value of the decision making attitude basically indicates the weights of the objective criteria.

Table 4. Fuzzy weight matrix of subjective criteria in linguistic terms assessed by three decision makers.

Criteria	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	L	H
C_3	H	MH	L
C_4	L	H	MH
C_5	MH	ML	H

Table 5. Decision matrix with objective criterion C_6.

	Cost components* ($\times 10^3$ US$/yr)					
Alternatives	Tax	Insurance	Material handling cost	Lighting	Objective Factor Cost (OFC) (=total cost)	Objective Factor Measure (OFM)
Chennai (A_1)	500	100	1000	200	1800	0.2628
Paradip (A_2)	600	150	1200	250	2200	0.2150
Mumbai (A_3)	450	75	1100	225	1850	0.2557
Chittagaon (A_4)	550	125	900	200	1775	0.2665

*Based on the analysis mentioned in Subsection 5.1.

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Table 6. Average fuzzy ratings of alternatives and average fuzzy weights under subjective criteria.

Weight	(0.70,0.86,0.97)	(0.53,0.67,0.77)	(0.40,0.57,0.73)	(0.40,0.57,0.77	(0.43,0.63,0.80)
Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	(7.00,8.67,9.67)	(7.67,9.33,10.00)	(4.33,6.33,8.00)	(3.33,5.00,7.00)	(4.67,5.33,6.67)
A_2	(7.00,8.67,9.67)	(7.00,8.67,9.67)	(5.33,6.67,7.67)	(7.00,8.67,9.67)	(4.00,5.33,6.67)
A_3	(7.00,8.67,9.67)	(4.00,5.67,7.33)	(4.00,5.67,7.33)	(7.00,8.67,9.67)	(5.33,6.67,7.67)
A_4	(7.00,8.67,9.67)	(7.00,8.67,9.67)	(4.00,5.67,7.33)	(4.00,5.67,7.33)	(4.00,5.33,6.67)

Table 7. Weighted fuzzy performance ratings under subjective criteria.

Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	(4.90,7.46,9.38)	(4.07,6.25,7.70)	(1.73,3.61,5.84)	(1.33,2.85,5.39)	(2.01,3.36,5.34)
A_2	(4.90,7.46,9.38)	(3.71,5.81,7.45)	(2.13,3.80,5.60)	(2.80,4.94,7.45)	(1.72,3.36,5.34)
A_3	(4.90,7.46,9.38)	(2.12,3.80,5.64)	(1.60,3.23,5.35)	(2.80,4.94,7.45)	(2.29,4.20,6.14)
A_4	(4.90,7.46,9.38)	(3.71,5.81,7.45)	(1.60,3.23,5.35)	(1.60,3.23,5.64)	(1.72,3.36,5.34)
SOS [a]	(96.04,222.38,351.93)	(48.55,120.99,202.03)	(12.67,48.36,122.72)	(20.01,67.41,171.79)	(15.20,51.48,123.07)
SQRT[b]	(9.8,14.912,18.76)	(6.968,11,14.21)	(3.56,6.95,11.08)	(4.474,8.21,13.11)	(3.9,7.17,11.09)

[a]Summation Of Squares.
[b]SQuare RooT.

Table 8. Normalized weighted fuzzy performance of alternatives under subjective criteria.

Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	(0.261,0.397,0.5)	(0.286,0.439,0.542)	(0.156,0.326,0.527)	(0.102,0.217,0.411)	(0.181,0.303,0.481)
A_2	(0.261,0.397,0.5)	(0.261,0.408,0.524)	(0.192,0.343,0.505)	(0.214,0.377,0.568)	(0.155,0.303,0.481)
A_3	(0.261,0.397,0.5)	(0.149,0.267,0.397)	(0.144,0.292,0.483)	(0.214,0.377,0.568)	(0.207,0.379,0.553)
A_4	(0.261,0.397,0.5)	(0.261,0.408,0.524)	(0.144,0.292,0.483)	(0.122,0.247,0.430)	(0.155,0.303,0.481)
FPIS	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)
FNIS	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Warehouse locations are arranged and ranked according to descending order of their warehouse location selection indices. As Chennai (A_1) has the highest selection index (0.2522), it is selected as the best warehouse location. The ranking order of the warehouse locations is found to be $A_1 > A_3 > A_4 > A_2$, i.e. Chennai > Mumbai > Chittagaon > Paradip. The computed results for those terms along with the ranking order of the alternatives by the extended FTOPSIS method are shown in Table 9.

5.1.2. Solution by the extended FSAW method

Normalized performance ratings, fuzzy scores, SFM′, SFMs, OFMs and WLSI are determined using Equations (17)–(21). Chennai (A_1) is selected as the best warehouse location as it has the highest selection index (0.2519). The ranking order of the warehouse locations found as $A_1 > A_3 > A_4 > A_2$, i.e. Chennai > Mumbai > Chittagaon > Paradip. Computed results of those terms and ranking of the alternatives by extended FSAW method are shown in Table 10.

5.1.3. Solution by the extended FMOORA method

Normalized performance ratings, fuzzy scores, SFM′, SFM, OFM′, OFM and WLSI are calculated using Equations (22)–(27). In extended FMOORA, the OFM of the alternative is considered in the cost category sense.

For instance, OFM'_1 = [1800(1/1800+1/2200+1/1850+1/1775)] = 3.81 and the normalized value of $OFM_1 = OFM'_1 / (OFM'_1+OFM'_2+OFM'_3+OFM'_4)$ = 3.81/ (3.81+4.65+3.91+3.75) = 0.2361.

Chennai (A_1) is selected as the best warehouse location as it has the highest selection index (0.08724). The ranking order of the warehouse locations is found to be $A_1 > A_3 > A_4 > A_2$, i.e. Chennai > Mumbai > Chittagaon > Paradip. The calculated results for these terms and the ranking of the alternatives by the extended FMOORA method are shown in Table 11.

5.2. Example 2 and its illustration

The proposed methodologies are also applied on another example, cited from Demirel et al. (2010). In the example, a homogeneous committee comprising five experts is formed by the said company. The experts chose four warehouse locations, i.e. Izmir, Izmit/Gebze, Zonguldak and Bursa, as feasible alternatives. The committee considered 5 main criteria and 16 sub-criteria for the evaluation and selection of the best

Table 9. S_i^+, S_i^-, CC, SFM′, SFMs, OFMs, WLSI and ranking of the alternatives by the extended FTOPSIS method.

Alternatives	S_i^+	S_i^-	CC	SFM′	SFM	OFM	WLSI	Rank
Chennai (A_1)	5.7954	3.1559	0.3526	0.3526	0.2469	0.2628	0.2522	1
Paradip (A_2)	5.5929	3.348	0.3745	0.3745	0.2622	0.2150	0.2467	4
Mumbai (A_3)	5.7707	3.1899	0.3560	0.3560	0.2493	0.2557	0.2514	2
Chittagaon (A_4)	5.8628	3.0865	0.3449	0.3449	0.2415	0.2665	0.2498	3

Table 10. Fuzzy score, SFM′, SFMs, OFMs, WLSI and ranking of the alternatives by the extended FSAW method.

Alternatives	Fuzzy score	SFM′	SFM	OFM	WLSI	Rank
Chennai (A_1)	(0.1622,0.2743,0.3977)	0.2780	0.2465	0.2628	0.2519	1
Paradip (A_2)	(0.1767,0.2966,0.4159)	0.2964	0.2628	0.2150	0.2470	4
Mumbai (A_3)	(0.1607,0.2795,0.4047)	0.2816	0.2496	0.2556	0.2516	2
Chittagaon (A_4)	(0.1556,0.2690,0.3915)	0.2720	0.2411	0.2665	0.2495	3

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Table 11. Fuzzy score, SFM′, SFMs, OFMs, WLSI and ranking of the alternatives by the extended FMOORA method.

Alternatives	Fuzzy score	SFM′	SFM	OFM	WLSI	Rank
Chennai (A_1)	(0.1622,0.2743,0.3977)	0.2780	0.2465	0.2361	0.08724	1
Paradip (A_2)	(0.1767,0.2966,0.4159)	0.2964	0.2628	0.2885	0.08083	4
Mumbai (A_3)	(0.1607,0.2795,0.4047)	0.2816	0.2496	0.2426	0.08720	2
Chittagaon (A_4)	(0.1556,0.2690,0.3915)	0.2720	0.2411	0.2328	0.08474	3

warehouse location. The selection criteria with their symbolic notation are shown in Table 12. The committee also assessed the importance of criteria and their sub-criteria and the ratings of the alternatives, which are represented in Table 13.

The cost (C') along with its sub-criteria ($C'_1 - C'_5$) are classified in the objective category. They are non-benefit criteria, hence lower values are desirable. The other criteria and their sub-criteria [LC (LC_1, LC_2); I (I_1, I_2, I_3); M (M_1, M_2 M_3); ME (ME_1, ME_2, ME_3)] are subjective and benefit criteria, for which higher values are desirable. In this problem, the decision makers represented the cost and their sub-criteria by subjective assessment due to inadequacy of information under a fuzzy environment. The above cited real-life problem is solved by the three proposed FMCDM methodologies. The solution of the above problem is illustrated in Subsections 5.2.1, 5.2.2 and 5.2.3.

5.2.1. Solution by the extended FTOPSIS method

The performance ratings of alternative locations with respect to all the criteria and their sub-criteria are expressed as linguistic variables as shown in Table 14. The performance ratings of the alternatives under all objective criteria are shown in Table 15. The fuzzy performance ratings of the alternatives under all subjective-based sub-criteria along with their weights is also shown in Table 16. Here, for instance, the weight of sub-criterion LC_1 is calculated as [(0.17, 0.29, 0.42) × (0.32, 0.50, 0.65)] = (0.05, 0.15, 0.27). The normalized weighted fuzzy decision matrix is computed using Equation (10) as presented in Table 17. Separation measures are determined using Equations (11) and (12), respectively. Separation measures are represented by the Euclidean distances of each alternative from the positive ideal solution and the negative ideal solution. The closeness coefficients, SFM′, SFM, OFM and WLSI are calculated using Equations (13)–(16), respectively.

For this example also, the objective decision making attitude is 0.33. Warehouse locations are arranged and ranked according to descending order of their warehouse location selection indices. Alternative Izmit/Gebze is selected as the best warehouse location as it has the highest selection index (0.270). The ranking order of the warehouse location found as Izmit/Gebze > Bursa > Izmir > Zonguldak. Computed results of those terms along with the ranking order of the alternatives by extended FTOPSIS method are shown in Table 18.

Table 12. Criteria and sub-criteria for warehouse location selection along with their linguistic variable importance weights.

Criteria	Weights of criteria in LV	Sub-criteria	Weights of sub-criteria in LV
Costs(C')	HI	Labor costs (C'_1)	M
		Transportation costs (C'_2)	HI
		Tax incentives and tax structure (C'_3)	SI
		Financial incentives (C'_4)	SI
		Handling costs (C'_5)	HI
Labor characteristics (LC)	SU	Skilled labor (LC_1)	M
		Availability of labor force (LC_2)	SU
Infrastructure (I)	HI	Existence of modes of transportation (I_1)	HI
		Telecommunication systems (I_2)	HI
		Quality and reliability of modes of transportation (I_3)	VI
Markets (M)	VI	Proximity to customers (M_1)	VI
		Proximity to suppliers or producers (M_2)	SI
		Lead times and responsiveness (M_3)	SI
Macro environment (ME)	SI	Policies of government (ME_1)	HI
		Industrial regulations, laws (ME_2)	SI
		Zoning and construction plans (ME_3)	SI

Table 13. Linguistic variables for the importance of criteria, sub-criteria and ratings of alternatives in a nine-point fuzzy scale (Delgado, Herrera, Herrera-Viedma, & Martínez, 1998).

Symbols of importance of weights	Definition of linguistic variables for the importance of criteria	Symbols of ratings of alternatives	Definition of linguistic variables for the rating of alternatives	Triangular Fuzzy Numbers[a] (TFN)
EU	Extra Unimportant	EL	Extra Low	(0,0,0)
VU	Very Unimportant	VL	Very Low	(0,0.015,0.07)
U	Unimportant	L	Low	(0.04,0.14,0.23)
SU	Slightly Unimportant	SL	Slightly Low	(0.17,0.29,0.42)
M	Middle	M	Middle	(0.32,0.5,0.65)
SI	Slightly Important	SH	Slightly High	(0.58,0.72,0.86)
HI	Highly Important	H	High	(0.72,0.85,0.97)
VI	Very Important	VH	Very High	(0.93,0.98,1.00)
EI	Extra Important	EH	Extra High	(1.00,1.00,1.00)

[a]Converted to triangular fuzzy numbers from trapezoidal fuzzy numbers.

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Table 14. Performance ratings of the alternatives in linguistic variables under all criteria and their sub-criteria.

	C′					LC		I			M			ME		
Location alternatives	C_1'	C_2'	C_3'	C_4'	C_5'	LC_1	LC_2	I_1	I_2	I_3	M_1	M_2	M_3	ME_1	ME_2	ME_3
Izmir (W_1)	SH	SH	M	SH	M	SL	SH	VH	EH	VH	M	H	M	SH	SH	M
Izmit/Gebze (W_2)	M	SH	M	SH	M	M	VH	VH	EH	VH	VH	VH	SH	M	SH	M
Zonguldak (W_3)	H	M	H	M	L	L	SH	H	VH	SH	H	H	SL	H	M	SL
Bursa (W_4)	H	SH	M	SH	SL	SL	H	H	VH	H	H	H	SH	H	SH	SL

Table 15. Performance ratings of the alternatives under objective criteria.

Alternatives	C_1'	C_2'	C_3'	C_4'	C_5'	OFC (=total cost)	OFM
Izmir	(0.58,0.72,0.86)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	3.14	0.2370
Izmit/Gebze	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	2.91	0.2557
Zonguldak	(0.72,0.85,0.97)	(0.32,0.50,0.65)	(0.72,0.85,0.97)	(0.32,0.50,0.65)	(0.04,0.14,0.23)	2.81	0.2648
Bursa	(0.72,0.85,0.97)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.17,0.29,0.42)	3.07	0.2424

Table 16. Fuzzy performance ratings of the alternatives under subjective based sub-criteria along with their weights.

Weight	(0.05,0.15,0.27)	(0.03,0.08,0.18)	(0.52,0.72,0.94)	(0.52,0.72,0.94)	(0.67,0.83,0.97)
Alternatives	LC_1	LC_2	I_1	I_2	I_3
Izmir (W_1)	(0.17,0.29,0.42)	(0.58,0.72,0.86)	(0.93,0.98,1.00)	(1.00,1.00,1.00)	(0.93,0.98,1.00)
Izmit/Gebze (W_2)	(0.32,0.50,0.65)	(0.93,0.98,1.00)	(0.93,0.98,1.00)	(1.00,1.00,1.00)	(0.93,0.98,1.00)
Zonguldak (W_3)	(0.04,0.14,0.23)	(0.58,0.72,0.86)	(0.72,0.85,0.97)	(0.93,0.98,1.00)	(0.58,0.72,0.86)
Bursa (W_4)	(0.17,0.29,0.42)	(0.72,0.85,0.97)	(0.72,0.85,0.97)	(0.93,0.98,1.00)	(0.72,0.85,0.97)
Cont'd					
(0.54,0.71,0.86)	(0.54,0.71,0.86)	(0.67,0.83,0.97)	(0.42,0.61,0.83)	(0.34,0.52,0.74)	(0.34,0.52,0.74)
M_1	M_2	M_3	ME_1	ME_2	ME_3
(0.32,0.50,0.65)	(0.72,0.85,0.97)	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.58,0.72,0.86)	(0.32,0.50,0.65)
(0.93,0.98,1.00)	(0.93,0.98,1.00)	(0.58,0.72,0.86)	(0.32,0.50,0.65)	(0.58,0.72,0.86)	(0.32,0.50,0.65)
(0.72,0.85,0.97)	(0.72,0.85,0.97)	(0.17,0.29,0.42)	(0.72,0.85,0.97)	(0.32,0.50,0.65)	(0.17,0.29,0.42)
(0.72,0.85,0.97)	(0.72,0.85,0.97)	(0.58,0.72,0.86)	(0.72,0.85,0.97)	(0.58,0.72,0.86)	(0.17,0.29,0.42)

Table 17. Normalized weighted performance of the alternative locations under subjective criteria.

Alternatives	LC_1	LC_2	I_1	I_2	I_3
Izmir (W_1)	(0.04,0.17,0.46)	(0.05,0.19,0.47)	(0.26,0.38,0.51)	(0.28,0.38,0.50)	(0.33,0.44,0.52)
Izmit/Gebze (W_2)	(0.07,0.29,0.71)	(0.08,0.25,0.54)	(0.26,0.38,0.51)	(0.28,0.38,0.50)	(0.33,0.44,0.52)
Zonguldak (W_3)	(0.01,0.08,0.25)	(0.05,0.19,0.47)	(0.20,0.33,0.49)	(0.26,0.38,0.50)	(0.21,0.32,0.45)
Bursa (W_4)	(0.04,0.17,0.46)	(0.06,0.22,0.52)	(0.20,0.33,0.49)	(0.26,0.38,0.50)	(0.26,0.38,0.51)
Cont'd					
M_1	M_2	M_3	ME_1	ME_2	ME_3
(0.11,0.23,0.36)	(0.23,0.36,0.50)	(0.15,0.30,0.45)	(0.17,0.30,0.49)	(0.16,0.31,0.53)	(0.13,0.32,0.59)
(0.32,0.44,0.55)	(0.30,0.41,0.51)	(0.28,0.43,0.60)	(0.09,0.21,0.37)	(0.16,0.31,0.53)	(0.13,0.32,0.59)
(0.25,0.38,0.53)	(0.23,0.36,0.50)	(0.08,0.17,0.29)	(0.21,0.36,0.56)	(0.09,0.22,0.40)	(0.07,0.19,0.38)
(0.25,0.38,0.53)	(0.23,0.36,0.50)	(0.28,0.43,0.60)	(0.21,0.36,0.56)	(0.16,0.31,0.53)	(0.07,0.19,0.38)

5.2.2. Solution by extended FSAW method

Normalized performance ratings, fuzzy scores, SFM′, SFM, OFM and WLSI are determined using Equations (17)–(21). Location alternative Izmit/Gebze is selected as the best warehouse location as it has the highest selection index (0.268). The ranking order of the warehouse locations is found to be Izmit/Gebze > Bursa > Izmir > Zonguldak. The computed results for those terms and the ranking order of the alternatives by the extended FSAW method are shown in Table 18.

5.2.3. Solution by the extended FMOORA method

Normalized performance ratings, fuzzy scores, SFM′, SFM, OFM′, OFM and WLSI are calculated using Equations (22)–(27). Izmit/Gebze is selected as the best warehouse location as it has the highest selection index (0.105). The ranking order of the warehouse locations found as Izmit/Gebze > Bursa > Izmir > Zonguldak. Calculated results of those terms and ranking order of the alternatives by extended FMOORA method are shown in Table 18.

These examples are solved with the help of Microsoft® Excel®. Some advanced software, such as MATLAB®, Minitab® and SCILAB, can be used for easy illustration of the developed method. This effort may give a prompt solution to the decision maker, which is extremely desirable. Next, a detailed discussion and comparative analysis of the results is carried out in Section 6, as follows.

Table 18. SFMs, OFMs, WLSI and ranking of the alternatives by the three proposed methodologies.

	Extended FTOPSIS				Extended FSAW				Extended FMOORA			
Alternatives	SFM	OFM	WLSI	Rank	SFM	OFM	WLSI	Rank	SFM	OFM	WLSI	Rank
Izmir (W_1)	0.247	0.237	0.244	3	0.247	0.237	0.244	3	0.247	0.263	0.080	3
Izmit/Gebze (W_2)	0.277	0.256	0.270	1	0.274	0.256	0.268	1	0.274	0.244	0.105	1
Zonguldak (W_3)	0.220	0.265	0.235	4	0.222	0.265	0.236	4	0.222	0.236	0.072	4
Bursa (W_4)	0.256	0.242	0.251	2	0.257	0.242	0.252	2	0.257	0.257	0.089	2

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Table 19. Comparison of results (ranking) of Example 1 obtained by the proposed methodologies.

	Proposed methodologies		
Alternatives	Extended FTOPSIS	Extended FSAW	Extended FMOORA
Chennai (A_1)	1	1	1
Paradip (A_2)	4	4	4
Mumbai (A_3)	2	2	2
Chittagaon (A_4)	3	3	3

6. Discussion, comparative study of results and sensitivity analysis

In order to demonstrate the applicability and potentiality, the proposed three methodologies are applied on two different benchmark problems of warehouse location selection. Detailed discussion, comparative study of the results and sensitivity analysis are mentioned in Subsections 6.1–6.4, as follows.

6.1. Example 1

Application of the extended FTOPSIS method on Example 1 gives the ranking order of the alternatives as $A_1 > A_3 > A_4 > A_2$, i.e. Chennai > Mumbai > Chittagaon > Paradip as shown in Table 19. The ranking of the alternatives for Example 1 by the extended FSAW method and the extended FMOORA method exactly match with the results of the extended FTOPSIS method, i.e. Chennai > Mumbai > Chittagaon > Paradip as shown in Table 19. A graphical representation of the comparative analysis on the results of Example 1 obtained by the proposed methodologies is shown in Figure 4.

6.2. Example 2

Application of the extended FTOPSIS method on Example 2 gives the ranking order of the location alternatives as Izmit/Gebze > Bursa > Izmir > Zonguldak as shown in Table 20**.** The ranking order of the alternatives for Example 2 both by the extended FSAW method and the extended FMOORA method exactly match with the results of the extended FTOPSIS method, i.e. Izmit/Gebze > Bursa > Izmir > Zonguldak as shown in Table 20. Demirel et al. (2010) applied the Choquet integral method on the same warehouse location selection problem and found the ranking of the alternatives as Izmit/Gebze > Izmir > Bursa > Zonguldak. The top ranking of the alternatives by different proposed methodologies exactly match with them as shown in Table 20. The last ranking of the alternatives by different proposed methodologies also exactly matches with the result of the cited paper. The intermediate ranking of the alternatives interchanges owing to the diverse evaluation procedures of the methodologies. Spearman's rank correlation coefficients are determined to test the strengths of the relationships between the proposed methods and the Choquet integral method. The value of the Spearman rank correlation coefficient in each comparison was found to be 0.80. This shows clearly that there exists a direct relationship between their ranking orders. A graphical representation of the comparative analysis of the results of Example 2 obtained by different methodologies is shown in Figure 5.

6.3. Sensitivity analysis

Warehouse location selection based on subjective and objective attributes is a complex decision making process. In the proposed algorithms, the warehouse location selection index (WLSI) is a function of objective decision making attitude (α). Here, objective decision making attitude behaves as a variable and its value varies from zero to one (0–1), which may yield different results. The exact choice of decision-making attitude by the decision makers of an organization is very important and significant to reflect the relative importance of the SFM and the OFM in calculating the WLSI. Hence, it is paramount to carry out a sensitivity analysis to study the effects of α on the evaluation of the performance of the whole process and in particular the performance of the alternatives. To carry out the sensitivity analysis of the location alternatives, different values of α are selected as follows: $\alpha = \{0, 0.1, 0.2, 0.3, 0.33, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Based on the specific objective decision making attitude, the WLSI values of the alternatives are calculated under the extended FTOPSIS, extended FSAW and extended FMOORA methods using Equations (16), (21)



Figure 4. Comparative ranking of warehouse locations obtained by various methods for Example 1.

Table 20. Comparison of results (ranking) of Example 2 obtained by various methods.

	Proposed methodologies			Demirel et al. (2010)
Alternatives	Extended FTOPSIS	Extended FSAW	Extended FMOORA	Choquet integral method
Izmir (W_1)	3	3	3	2
Izmit/Gebze (W_2)	1	1	1	1
Zonguldak (W_3)	4	4	4	4
Bursa (W_4)	2	2	2	3

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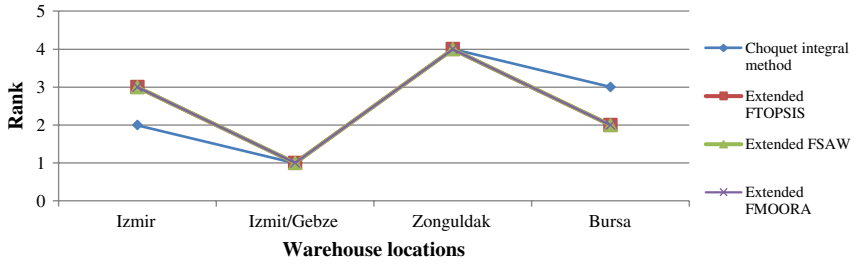


Figure 5. Comparative ranking of warehouse locations obtained by various methods for Example 2.

and (27), respectively. The ranking order of the alternatives based on the corresponding α and its ranges under the different proposed methods for both Examples 1 and 2 are shown in Table 21.

Sensitivity analysis for Example 1 under the proposed methods is depicted graphically in Figures 6–8, respectively. The graphical displays show all the information concerning how the WLSI of the alternatives changes in response to the potential shift of α value. One can see from Figure 6 that, under the proposed extended FTOPSIS method, the intersecting points of the WLSI of the alternatives are at $\alpha = \{0.24, 0.29, 0.42, 0.59\}$. Similarly, the intersecting points are at $\alpha = \{0.25, 0.30, 0.44, 0.59\}$ and $\alpha = \{0.22, 0.28, 0.46, 0.62\}$, respectively, under the extended FSAW and extended FMOORA methods as seen from Figures 7 and 8. It is clearly visible from Table 21 that the intersecting points nearly match, irrespective of the different proposed methodologies. The very small differences may be attributed to the different nature of the calculation procedures applied in the proposed MCDM methodologies. One can also see from Table 21 that the ranking order of the alternatives remains same in all those intersection points as well as in other intermediate ranges of α for all the proposed methods. Beyond the intersection points, the ranking order changes. Considering $\alpha = 0.33$, the baseline solution of Example 1 is $A_1 > A_3 > A_4 > A_2$. The baseline ranking order $A_1 > A_3 > A_4 > A_2$ changes remarkably in accordance with the shift in α value as shown in Table 21. It is also observed that for a moderately considerable range of $0.30 < \alpha < 0.42$ the ranking order of the alternatives remains $A_1 > A_3 > A_4 > A_2$ for all the proposed methods. Sensitivity analysis clearly suggests the Chennai (A_1) warehouse location as the best alternative as it has the highest WLSI for all the proposed methods for the cluster$0.30 < \alpha < 0.42$. The range$0.30 < \alpha < 0.42$ signifies the justification of the considered value of $\alpha = 0.33$ in the baseline solution for Example 1. Different cluster ranges of α values show different ranking orders of the alternatives as clearly shown in Table 21.

Similarly, the sensitivity analysis for Example 2 is graphically represented by Figures 9–11. The intersecting points found are at $\alpha = \{0.49, 0.61, 0.86\}$, $\alpha = \{0.47, 0.60, 0.85\}$ and $\alpha = \{0.48, 0.63, 0.87\}$, respectively, under the proposed methods. It is clearly visible from Table 21 that the intersecting points nearly match, irrespective of the different proposed methodologies. The very small differences may be attributed to the different nature of the calculation procedures applied in the proposed MCDM methodologies. One can also see that the ranking order of the alternatives remains the same at all those intersection points and in other intermediate ranges of α values as shown in Table 21. Beyond the intersection points, the ranking order changes. Considering $\alpha = 0.33$, the baseline solution for Example 2 is $W_2 > W_4 > W_1 > W_3$. The baseline ranking order $W_2 > W_4 > W_1 > W_3$ changes remarkably in accordance with the shift in α value as shown in Table 21. It is also seen that for a largely considerable range of $0.00 \le \alpha < 0.47$, the ranking order of the alternatives remains $W_2 > W_4 > W_1 > W_3$ under all the proposed methods. The sensitivity analysis clearly suggests warehouse location Izmit/Gebze (W_2) as the best alternative as it has the highest WLSI for all the proposed methods up to $\alpha < 0.47$. This range, $0.00 \le \alpha < 0.47$, also signifies the justification of the considered value of $\alpha = 0.33$ in the baseline solution for Example 2. A different cluster range of α values shows different ranking orders of the alternatives as clearly shown in Table 21. The preference for α is strictly a strategic decision of top management and tailor made to the said organization. In this regard, sensitivity analysis helps in visualizing the trend in

Table 21. Sensitivity analysis of the alternatives obtained by the proposed methods.

	Extended FTOPSIS		Extended FSAW		Extended FMOORA	
Examples	Alfa(α) value	Ranking order	Alfa(α) value	Ranking order	Alfa(α) value	Ranking order
Example 1	$0.00 \le \alpha < 0.24$	$A_2>A_3>A_1>A_4$	$0.00 \le \alpha < 0.25$	$A_2>A_3>A_1>A_4$	$0.00 \le \alpha < 0.22$	$A_2>A_3>A_1>A_4$
	$\alpha = 0.24$	$A_2=A_3>A_1>A_4$	$\alpha = 0.25$	$A_2=A_3>A_1>A_4$	$\alpha = 0.22$	$A_2=A_3>A_1>A_4$
	$0.24 < \alpha < 0.29$	$A_3>A_1>A_4>A_2$	$0.25 < \alpha < 0.30$	$A_3>A_1>A_4>A_2$	$0.22 < \alpha < 0.28$	$A_3>A_1>A_4>A_2$
	$\alpha = 0.29$	$A_3=A_1>A_4>A_2$	$\alpha = 0.30$	$A_3=A_1>A_4>A_2$	$\alpha = 0.28$	$A_3=A_1>A_4>A_2$
	$0.29 < \alpha < 0.42$	$A_1>A_3>A_4>A_2$	$0.30 < \alpha < 0.44$	$A_1>A_3>A_4>A_2$	$0.28 < \alpha < 0.46$	$A_1>A_3>A_4>A_2$
	$\alpha = 0.42$	$A_1>A_3=A_4>A_2$	$\alpha = 0.44$	$A_1>A_3=A_4>A_2$	$\alpha = 0.46$	$A_1>A_3=A_4>A_2$
	$0.42 < \alpha < 0.59$	$A_1>A_4>A_3>A_2$	$0.44 < \alpha < 0.59$	$A_1>A_4>A_3>A_2$	$0.46 < \alpha < 0.62$	$A_1>A_4>A_3>A_2$
	$\alpha = 0.59$	$A_1=A_4>A_3>A_2$	$\alpha = 0.59$	$A_1=A_4>A_3>A_2$	$\alpha = 0.62$	$A_1=A_4>A_3>A_2$
	$0.59 < \alpha \le 1.00$	$A_4>A_1>A_3>A_2$	$0.59 < \alpha \le 1.00$	$A_4>A_1>A_3>A_2$	$0.62 < \alpha \le 1.00$	$A_4>A_1>A_3>A_2$
Example 2	$0.00 \le \alpha < 0.49$	$W_2>W_4>W_1>W_3$	$0.00 \le \alpha < 0.47$	$W_2>W_4>W_1>W_3$	$0.00 \le \alpha < 0.48$	$W_2>W_4>W_1>W_3$
	$\alpha = 0.49$	$W_2>W_4>W_1=W_3$	$\alpha = 0.47$	$W_2>W_4>W_1=W_3$	$\alpha = 0.48$	$W_2>W_4>W_1=W_3$
	$0.49 < \alpha < 0.61$	$W_2>W_4>W_3>W_1$	$0.47 < \alpha < 0.60$	$W_2>W_4>W_3>W_1$	$0.48 < \alpha < 0.63$	$W_2>W_4>W_3>W_1$
	$\alpha = 0.61$	$W_2>W_4=W_3>W_1$	$\alpha = 0.60$	$W_2>W_4=W_3>W_1$	$\alpha = 0.63$	$W_2>W_4=W_3>W_1$
	$0.61 < \alpha < 0.86$	$W_2>W_3>W_4>W_1$	$0.60 < \alpha < 0.85$	$W_2>W_3>W_4>W_1$	$0.63 < \alpha < 0.87$	$W_2>W_3>W_4>W_1$
	$\alpha = 0.86$	$W_2=W_3>W_4>W_1$	$\alpha = 0.85$	$W_2=W_3>W_4>W_1$	$\alpha = 0.87$	$W_2=W_3>W_4>W_1$
	$0.86 < \alpha \le 1.00$	$W_3>W_2>W_4>W_1$	$0.85 < \alpha \le 1.00$	$W_3>W_2>W_4>W_1$	$0.87 < \alpha \le 1.00$	$W_3>W_2>W_4>W_1$.

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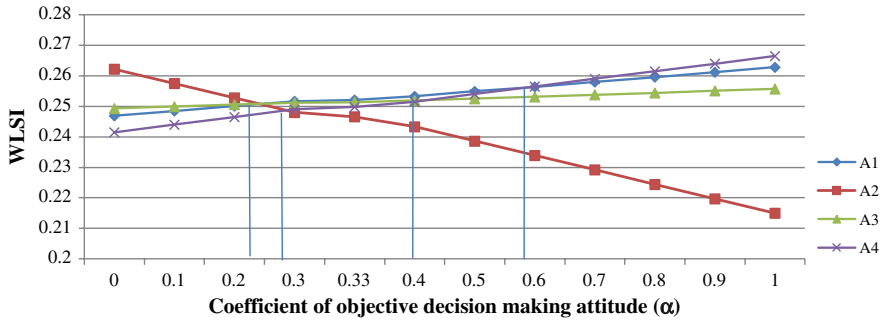


Figure 6. Sensitivity analysis of the alternatives for Example 1 based on the extended FTOPSIS method.

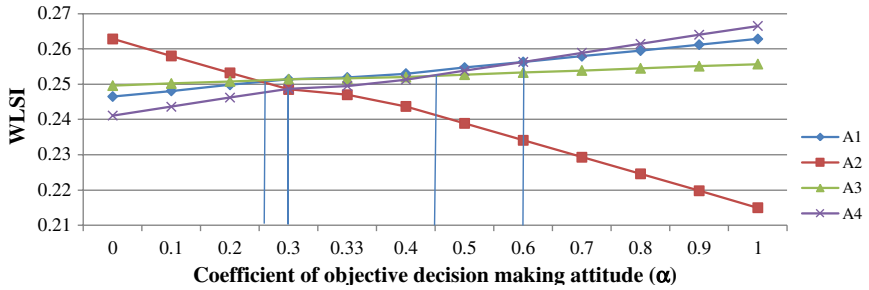


Figure 7. Sensitivity analysis of the alternatives for Example 1 based on the extended FSAW method.

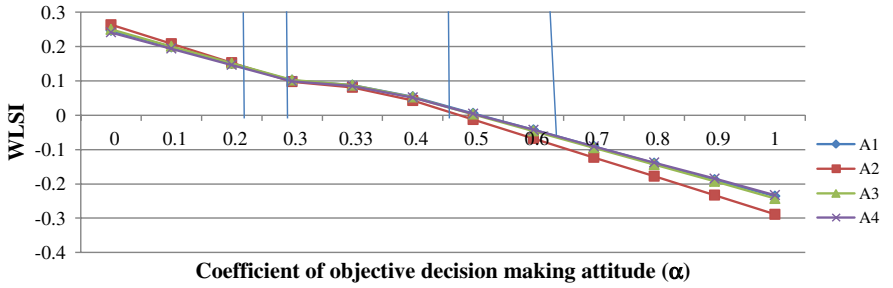


Figure 8. Sensitivity analysis of the alternatives for Example 1 based on the extended FMOORA method.

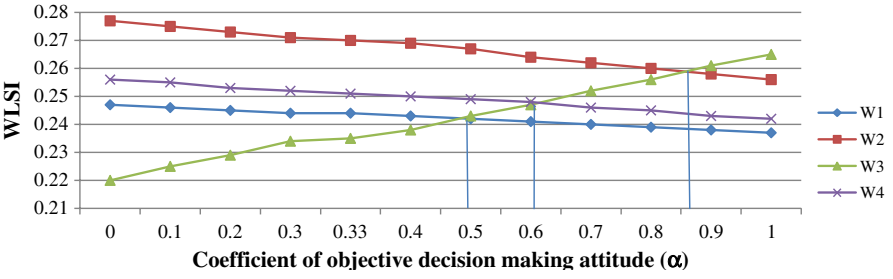


Figure 9. Sensitivity analysis of the alternatives for Example 2 based on the extended FTOPSIS method.

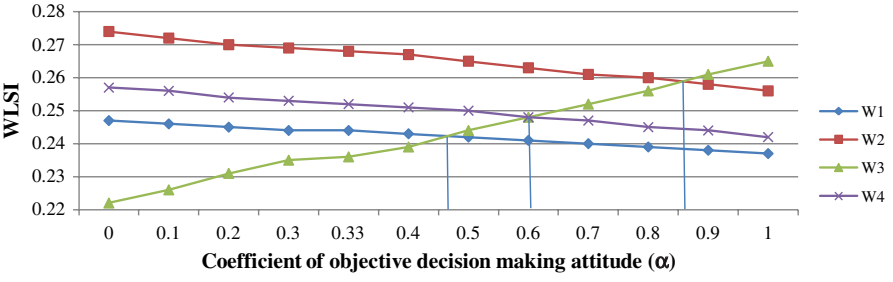


Figure 10. Sensitivity analysis of the alternatives for Example 2 based on the extended FSAW method.

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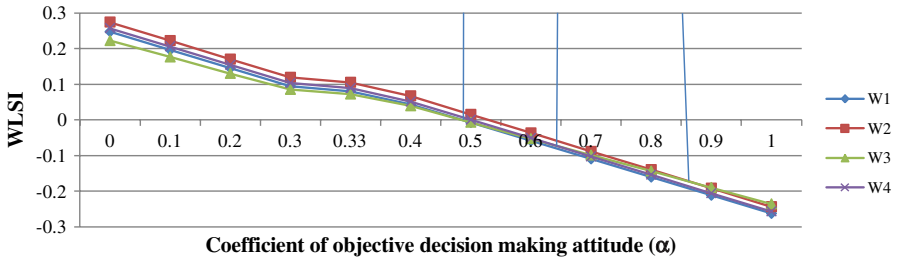


Figure 11. Sensitivity analysis of the alternatives for Example 2 based on the extended FMOORA method.

ranking order as α change and simultaneously demonstrates the robustness of the proposed methodologies.

6.4. Simplicity of the proposed methodologies in comparison with other methods

The proposed algorithms are extensions of the FTOPSIS, FSAW and FMOORA methods, to which a few simple steps have been added to accommodate the said work. Renowned researchers in this domain have shown that the classical SAW, MOORA and TOPSIS methods are basically simple, easy to comprehend and require less effort to implement due to their simpler evaluation techniques (Chou et al., 2008; Ozcan et al., 2011). The proposed algorithms possess the same inherent advantages and keep their simplicity and ability intact regardless of problem size, unlike the FAHP method. An AHP needs $(2n + 4)$ computational steps where n is the number of attributes (Kumar & Garg, 2010). To establish the simplicity of the proposed methods over other MCDM methods, e.g. fuzzy AHP, fuzzy Choquet integral, fuzzy LINMAP as well as the digraph and matrix approaches, their algorithms are analysed based on the growth of running time and the effect of input size. The asymptotic upper bound order of the algorithms (Cormen, Leiserson, Rivest, & Stein, 2009) is considered to confirm the degree of simplicity of the proposed algorithms theoretically. According to Big O based asymptotic analysis and other relevant parameters, the simplicity of the proposed algorithms is compared with the aforementioned MCDM methods as shown in Table 22.

Furthermore, to confirm the simplicity of the proposed algorithms over FAHP, programs were written for both FAHP and the proposed algorithms. These programs were executed on an Intel Core i3-2330M CPU with a 2.20 GHz processor

Table 22. Comparison of the simplicity of the proposed methods with that of some existing MCDM methods (Brauers, 2004; Demirel et al., 2010; Hwang & Yoon, 1981; Rao & Padmanabhan, 2006; Satty, 1980; Xia, Li, Zhou, & Wang, 2006).

Parameters	Proposed methods	Fuzzy AHP method	Fuzzy Choquet integral method	Fuzzy LINMAP method	Digraph and matrix approach
1. Core process	Integrating subjective factor measure and objective factor measure of alternatives with proper decision making attitude.	Forming hierarchical structure and pairwise comparison matrices.	General averaging operator that can represent the notion of importance of a criterion and interactions among criteria.	Pairwise comparisons of alternatives given by decision makers and generates the best alternative that has the shortest distance to the PIS.	Matrix equivalent of a digraph gives one-to-one taking all the attributes and their relative importance into account.
2. Computational complexity	$O(mnp)$,[a] i.e. polynomial.	$O(m^2np)$, i.e. polynomial.	$O(mnp)$, i.e. polynomial.	Exponential[b] in worst case.	$O(n^{2.273})$,[c] i.e. polynomial for a single decision maker.
3. Constant term in computational complexity	Constant term is independent of *m, n, p.*	Constant term is dependent on *n.*	Constant term is independent of *m, n, p,* but slightly high due to rigorous calculations.	Constant term is independent of *m, n, p.*	Constant term is independent of *m, n.*
4. Overall running time[d]	Minimum	Moderate	Moderate	Maximum	Moderate
5. Number of steps	Independent of input	Dependent on input(n)	Independent of input	Independent of input	Independent of input
6. Computer implementation	Easy to implement	Easy to implement	Difficult to implement	Very difficult to implement	Slightly difficult to implement
7. Mathematical calculation	Simple arithmetic and standard matrix operations	Simple arithmetic and standard matrix operations	Complicated arithmetic computations	Quite complex arithmetic operations[b]	Simple arithmetic and standard matrix operations
8. Number of outranking relations	1	$N(N-1)$ where N is the order of the matrix	1	$m(m-1)/2$	$n(n-1)$
9. Consistency check	Not required	Required	Not required	Required	Not required
10. Easy/Difficult to comprehend	Very easy	Easy	Very difficult	Very difficult	Easy

[a] m is the number of alternatives, n is the number of criteria and p is the number of decision makers.
[b] Owing to the presence of the classical simplex method to solve LPP.
[c] Calculation of the determinant of the $n \times n$ matrix by FAST matrix multiplication.
[d] depends upon computational complexity as well as constant term.

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speed and 4 GB of RAM. The programs were written in C++ and compiled with the help of g++. The same problem instance (four alternatives, six criteria and three decision makers) was solved using both programs. The computational running time taken by the above-mentioned different algorithms for the same problem instance is shown below:

Algorithm →	Extended FTOPSIS	Extended FSAW	Extended FMOORA	FAHP
Running time →	0.020 sec	0.016 sec	0.016 sec	0.030 sec

The Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) method employs linear programming techniques. In LINMAP, the equations formed by the linear programming model are solved using the existing Simplex® method software. The Simplex® method itself can be of exponential order. Since LINMAP employs the Simplex® method for solving the equations, hence we can conclude that LINMAP itself is of exponential order. Other methods mentioned here (the digraph and matrix approaches, the Choquet integral) are of polynomial order, hence, in terms of time as a constraint, LINMAP lags behind these methods. All the other methods also have more overall running time than the proposed methods as mentioned in Table 22. Conclusively, comparative analysis confirms the simplicity of the proposed methods over other considered methods.

The in-depth discussion, comparative study of results and sensitivity analysis conclude that the technique espoused herein thus validates the logic behind the approach, which complies with the strategies adopted by the earlier researchers. The focus in this section details some salient characteristics of the proposed algorithms as listed below.

(1) These algorithms are capable of handling subjective as well as objective attributes simultaneously in order to select the best warehouse location.
(2) The algorithms are very simple and the calculation procedure is easier and straightforward.
(3) This paper considers a large number of relevant selection criteria for warehouse location selection. Hence the selection of the best warehouse location using the proposed methodologies is more appropriate and acceptable.
(4) The real-life situation of warehouse location selection is vague and uncertain rather than clear and certain. Therefore, when dealing with the imprecise data originating from those real case scenarios of volatile industrial environment, it is more logical to apply the fuzzy set theory.
(5) Sensitivity analysis reveals the robustness of the proposed methodologies under diverse decision making attitudes.
(6) A comparative study of the results signifies the compatibility and applicability of the proposed methodologies. Hence the proposed methodologies can be treated as effective decision making aids for supply chain managers.

7. Conclusions and some essential remarks

Warehouse location selection is a strategic aspect of managerial decision making in any supply chain. In most cases, the success and survival of the supply chain depends upon the premeditated selection of warehouse location. Hence, the continuous and appropriate evaluation of selection procedures for warehouse location is essential in order to survive stiff competition in the market. The current work attempts to follow this up in a methodical manner. In real-life decision making processes, the decision maker is unable or unwilling to express his/her preferences precisely as crisp/numerical values and the assessment is very often expressed in linguistic terms. In such situations, fuzzy set theory is an appropriate contrivance to deal with this kind of problem. The proposed FMCDM approaches (extended FTOPSIS, extended FSAW and extended FMOORA) can handle both subjective as well as objective criteria. Integration of subjective and objective criteria determines the warehouse location selection index, which is considered to be the key selection parameter for warehouse location selection. The proposed new methodologies are applied on two warehouse location selection problems in order to show their viability, potentiality and applicability. Each approach individually reveals a unique warehouse location as the best one. Moreover, sensitivity analysis shows that the proposed methods have the capability to trade-off between the subjective and objective judgments while choosing competing location alternatives. The proposed methodologies also have the potential to deal effectively with such decision problems without prerequisite information on objective criteria weights. The discussion and comparative study of the results proves that the proposed methodologies are novel, useful and highly effective MCDM techniques that can be utilized as precise decision making tools by supply chain managers.

The proposed approaches can easily be extended to deal with other management decision making problems, e.g. supplier evaluation, personnel selection, etc. By computerizing the current FMCDM procedure, a large number of alternatives, criteria and decision makers' opinions can easily be considered and the ranking order of alternatives can be obtained automatically. Implementation of the proposed algorithm simplifies the computational work to solve the warehouse location selection problem. The proposed study has two limitations. Firstly, it cannot be applied to heterogeneous decision making environment, and secondly the algorithm assumes that the selection criteria are independent. Based on the findings of this investigation, the following suggestions are made for possible directions of future research.

(1) The proposed methodologies could be adapted for heterogeneous decision making environments.
(2) Other established MCDM techniques could be applied and their results compared with the current work.
(3) Consideration of mutual dependency among the selection criteria in the proposed methodologies could also be a topic for future research work.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Bellman, R. E., & Zadeh, L. A. (1970). Decision making in a fuzzy environment. *Management Science, 17*(4), B141–B164.

Bhattacharya, A., Sarkar, B., & Mukherjee, S. K. (2004). A new method for plant location selection: A holistic approach. *International Journal of Industrial Engineering: Theory, Application and Practice, 11*(4), 330–338.

Brauers, W. K. M. (2004). *Optimization methods for a stakeholder society: A revolution in economic thinking by multiobjective optimization*. Boston, MA: Kluwer Academic.

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Brown, P. A., & Gibson, D. F. (1972). A quantified model for facility site selection application to multi plant location problem. *AIIE Transactions, 4*(1), 1–10.

Cebi, F., & Otay, I. (2015). Multi-criteria and multi-stage facility location selection under interval type-2 fuzzy environment: A case study for a cement factory. *International Journal of Computational Intelligence Systems, 8*(2), 330–344.

Chen, C. T. (2001). A fuzzy approach to select the location of the distribution center. *Fuzzy Sets and Systems, 118*, 65–73.

Chen, C. L., Yuan, T. W., & Lee, W. C. (2007). Multi-criteria fuzzy optimization for locating warehouses and distribution centers in a supply chain network. *Journal of the Chinese Institute of Chemical Engineers, 38*, 393–407.

Chou, S. Y., Chang, Y.-H., & Shen, C.-Y. (2008). A fuzzy simple additive weighting system under group decision making for facility location selection with objective/subjective attributes. *European Journal of Operational Research, 189*, 132–145.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to algorithms* (3rd ed.). Cambridge, MA: MIT Press.

Delgado, M., Herrera, F., Herrera-Viedma, E., & Martínez, L. (1998). Combinig numerical and linguistic information in group decision making. *Information Sciences, 107*, 177–194.

Demirel, T., Demirel, N. C., & Kahraman, C. (2010). Multi-criteria warehouse location selection using Choquet integral. *Expert Systems with Applications, 37*, 3943–3952.

Dey, B., Bairagi, B., Sarkar, B., & Sanyal, S. K. (2013). A hybrid fuzzy technique for the selection of warehouse location in a supply chain under a utopian environment. *International Journal of Management Science and Engineering Management, 8*(4), 250–261.

Farahani, R. Z., Hekmatfar, M., Fahimnia, B., & Kazemzadeh, N. (2014). Hierarchical facility location problem: Models, classifications, techniques, and applications. *Computers & Industrial Engineering, 68*, 104–117.

Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making – method and application, a state-of- the-art survey*. New York, NY: Springer-Verlag.

Jabalameli, M. S., Hosseininezhad, S. J., & Naini, S. G. (2013). A risk management model for the plant location problem. *International Journal of Management Science and Engineering Management, 8*, 93–101.

Kahraman, C., Ruan, D., & Ibrahim, D. G. (2003). Fuzzy group decision-making for facility location selection. *Information Sciences, 157*, 135–153.

Karmaker, C. L., & Saha, M. (2015). Optimization of warehouse location through fuzzy multi-criteria decision making methods. *Decision Science Letters, 4*, 315–334.

Korpela, J., & Tuominen, M. (1996). A decision aid in warehouse site selection. *International Journal of Production Economics, 45*, 169–180.

Kumar, R., & Garg, R. K. (2010). Optimal selection of robots by using distance based approach method. *Robotics and Computer-Integrated Manufacturing, 26*, 500–506.

Liang, G. S. (1999). Fuzzy MCDM based on ideal and anti-ideal concepts. *European Journal of Operational Research, 112*, 682–691.

Liang, G. S., & Wang, M. J. J. (1991). A fuzzy multi-criteria decision-making method for facility site selection. *International Journal of Production Research, 29*, 2313–2330.

Ozcan, T., Celebi, N., & Esnaf, S. (2011). Comparative analysis of multi-criteria decision making methodologies and implementation of a warehouse location selection problem. *Experts Systems with Applications, 38*, 9773–9779.

Parameshwaran, R., Kumar, S. P., & Sravanakumar, K. (2015). An integrated fuzzy MCDM based approach for robot selection considering objective and subjective criteria. *Applied Soft Computing, 26*, 31–41.

Rao, C., Goh, M., Zhao, Y., & Zheng, J. (2015). Location selection of city logistics centers under sustainability. *Transportation Research Part D, 36*, 29–44.

Rao, R. V., & Padmanabhan, K. K. (2006). Selection, identification and comparison of industrial robots using digraph and matrix methods. *Robotics and Computer-Integrated Manufacturing, 22*, 373–383.

Rao, R. V., & Patel, V. K. (2010). A subjective and objective integrated multiple attribute decision making method for material selection. *Materials and Design, 31*, 4738–4747.

Roh, S., Pettit, S., Harris, I., & Beresford, A. (in press). The pre-positioning of warehouses at regional and local levels for a humanitarian relief organization. *International Journal of Production Economics*. doi:10.1016/j.ijpe.2015.01.015

Sadi-Nezhad, S., & Damghani, K. K. (2010). Application of a fuzzy TOPSIS method based on modified preference ratio and fuzzy distance measurement in assessment of traffic police centers performance. *Applied Soft Computing, 10*, 1028–1039.

Satty, T. L. (1980). *The analytic hierarchy process*. New York, NY: McGraw-Hill.

Shahanaghi, K., & Yazdian, S. A. (2010). Facility location and distribution decision in supply chains with fleet sizing considering both tangible and intangible criteria. *International Journal of Management Science and Engineering Management, 5*(4), 302–308.

Shannon, C. E., & Weaver, W. (1947). *The mathematical theory of communication*. Urbana, IL: The University of Illinois Press.

Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2003). *Designing & managing the supply chain*. New York, NY: McGraw-Hill.

Wang, T. C., & Lee, H. D. (2009). Developing a fuzzy TOPSIS approach based on subjective weights and objective weights. *Experts Systems with Applications, 36*, 8980–8985.

Xia, H.-C., Li, D.-F., Zhou, J.-Y., & Wang, J.-M. (2006). Fuzzy LINMAP method for multi attribute decision making under fuzzy environment. *Journal of Computer and System Sciences, 72*, 741–759.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zeleny, M. (1982). *Multiple criteria decision making*. New York, NY: McGraw-Hill.

Zimmermann, H. J. (1991). *Fuzzy set theory and its applications* (2nd ed.). Boston, MA: Kluwer Academic.