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Araştırma Makalesi - Research Article

Düzenli Uzun Dalga Denkleminin Hiperbolik Tip Yürüyen Dalga Çözümleri

Hülya Durur[1*], Asıf Yokuş[2], Doğan Kaya[3]

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ÖZ

Bu çalışmanın temel amacı $(1/G')$ -açılım yöntemi kullanılarak Düzenli Uzun Dalga (RLW) denklemi için yürüyen dalga çözümlerini elde etmektir. Elde edilen çözümlerde sabitlere özel değerler verilerek 3 boyutlu, 2 boyutlu ve kontur grafikleri sunulmuştur. Bu grafikler Düzenli Uzun Dalga denkleminin özel bir çözümüdür ve denklemin durağan bir dalgasını temsil etmektedir. Bu makalede sunulan çözümleri ve grafikleri bulmak için bilgisayar paket programı kullanılmaktadır.

Anahtar Kelimeler- $(1/G')$ ***-Açılım Yöntemi, Düzenli Uzun Dalga Denklemi, Yürüyen Dalga Çözümü, Tam Çözüm.***

[1]*Sorumlu yazar iletişim: hulyadurur@ardahan.edu.tr, http:// orcid.org/0000-0002-9297-6873
Ardahan Üniversitesi, Mühendislik Fakültesi, Bilgisayar Mühendisliği Bölümü, Ardahan, 75000, Türkiye.
[2]İletişim: asfyokus@yahoo.com, http:// orcid.org/0000-0002-1460-8573
Fırat Üniversitesi, Fen Fakültesi, Aktüerya Bölümü, Elazığ, 23100, Turkey.
[3]İletişim: dkaya36@yahoo.com, http:// orcid.org/0000-0003-4773-1313
İstanbul Ticaret Üniversitesi, Matematik Bölümü, Üsküdar, İstanbul, Turkey.

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Hyperbolic Type Traveling Wave Solutions of Regularized Long Wave Equation

ABSTRACT

The main goal of this study is to obtain the traveling wave solutions for Regularized Long Wave (RLW) equation by using $(1/G')$-expansion method. By giving special values to the constants in the solutions obtained, 3D, 2D, and contour graphics are presented. These graphics are a special solution of the (RLW) equation, and they represent a stationary wave of the equation. A computer package program is used to find the solutions and graphics presented in this article.

Keywords- $(1/G')$ ***-Expansion Method, Regularized Long Wave Equation, Traveling Wave Solution, Exact Solution.***

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I. INTRODUCTION

Traveling wave solutions of nonlinear evolution equations (NLEEs) play an active role in applied mathematics and mathematical physics. Variety methods have been used to find the analytical solutions of such equations (NLDEs). Some of these are F-expansion technique [1], Laplace transform method [2], $(1/G')$-expansion method [3-5], the residual power series method [6,7], modified exp -expansion function method [8-10], (G'/G)-expansion method [11,12], Adomian's decomposition method [13], collocation method [14,15], sub equation method [16,17], Homotopy perturbation method [18], new sub equation method [19] and so on [28-35].

In this study, authors investigated the analytical solutions of the (RLW) equation. Consider the form of the (RLW) equation [20],

$$u_t + u_x + a\left(u^2\right)_x - bu_{xxt} = 0, \quad a,b \in R, \tag{1}$$

the function u searched here depends on the variables *x* and *t.*

There are many studies in the literature on RLW equation. Some of the studies are as follows: Solitary wave solutions of the generalized RLW equation are obtained [21], solutions of generalized RLW equation are obtained using variational iteration method [22], solutions of the one-dimensional (RLW) equation are obtained using a lumped Galerkin method based on quadratic B-spline finite elements numerical [23], approximate numerical solution for the nonlinear (RLW) equation are obtained applying adomian decomposition method [24], numerical solution of the one-dimensional (RLW) equation is obtained using linearized implicit finite difference method [25], the solution of the (RLW) equation is presented [26], and solitary wave solutions of the two-dimensional RLW equation are obtained [27].

II.MATERIAL AND METHOD

A. $(1/G')$ *-Expansion Method*

We get general form of NLPDEs

$$\sigma\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0. \tag{2}$$

Here, let $u = u(x,t) = u(\xi), \quad \xi = kx + wt,$ where k, w are constants. We can be converted into the following nonlinear ODE for $u(\xi)$:

$$F\left(u, u', u'', u''', \ldots\right) = 0. \tag{3}$$

The solution of Eq. (3) is assumed to have the form

$$u(\xi) = a_0 + \sum_{i=1}^{n} a_i \left(\frac{1}{G'}\right)^i, \tag{4}$$

where a_i, $(i = 0,1,\ldots,n)$ are constants, n is a fixed number to be calculated by the principle of homogeneous balance according to the properties of each equation and $G = G(\xi)$ provides the following second-order IODE

$$G'' + \lambda G' + \mu = 0, \tag{5}$$

where λ and μ are constants to be determined after,

$$\frac{1}{G'(\xi)} = \frac{1}{-\frac{\mu}{\lambda} + A\cosh[\xi\lambda] - A\sinh[\xi\lambda]}, \tag{6}$$

where A is integral constant. If the desired derivatives of the Eq. (4) are calculated and substituting in the Eq. (3), a polynomial with the argument $(1/G')$ is attained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. The equation is solved using a package program and put into place in the default Eq. (3) solution function. Finally, the solutions of Eq. (1) are found.

III. SOLUTIONS OF THE RLW EQUATION

The traveling wave transmutation $\xi = kx + wt$, allows us to convert Eq. (1) into an ODE for $u = u(\xi)$,

$$(k+w)u' + 2akuu' - bk^2wu''' = 0, \tag{7}$$

integrating Eq. (7) once with respect to ξ and by setting the integration constant to zero, we attain

$$(w+k)u + aku^2 - bk^2wu'' = 0. \tag{8}$$

In Eq. (8), we achieve balancing term $n = 2$ and in Eq. (4), the following situation is obtained:

$$u(\xi) = a_0 + a_1\left(\frac{1}{G'}\right) + a_2\left(\frac{1}{G'}\right)^2, \quad a_2 \neq 0, \tag{9}$$

where a_1 and a_2 cannot be zero at the same time. Replacing Eq. (9) into Eq. (8) and the coefficients of the algebraic Eq. (1) are equal to zero, can attain the following algebraic equation systems

$$\begin{aligned}
&Const : ka_0 + wa_0 + aka_0^2 = 0,\\
&\left(\frac{1}{G'[\xi]}\right)^1 : ka_1 + wa_1 - bk^2w\lambda^2a_1 + 2aka_0a_1 = 0,\\
&\left(\frac{1}{G'[\xi]}\right)^2 : -3bk^2w\lambda\mu a_1 + aka_1^2 + ka_2 + wa_2 - 4bk^2w\lambda^2a_2 + 2aka_0a_2 = 0,\\
&\left(\frac{1}{G'[\xi]}\right)^3 : -2bk^2w\mu^2a_1 - 10bk^2w\lambda\mu a_2 + 2aka_1a_2 = 0,\\
&\left(\frac{1}{G'[\xi]}\right)^4 : -6bk^2w\mu^2a_2 + aka_2^2 = 0.
\end{aligned} \tag{10}$$

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When the resulting system Eq. (10) is solved by a package program or manually, we can present the following situations.

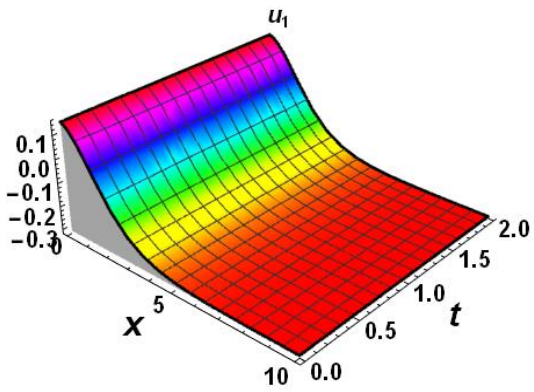
Case1.

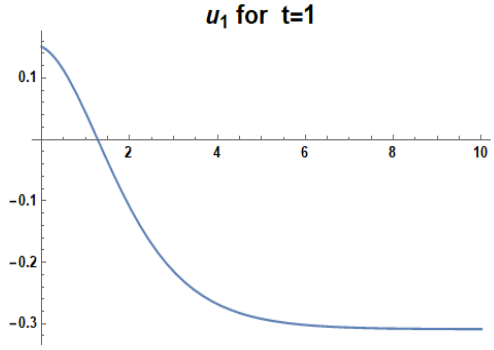
$$a_0 = -\frac{bk^2\lambda^2}{a\left(1+bk^2\lambda^2\right)}, a_1 = -\frac{6bk^2\lambda\mu}{a\left(1+bk^2\lambda^2\right)}, a_2 = -\frac{6bk^2\mu^2}{a\left(1+bk^2\lambda^2\right)}, w = -\frac{k}{1+bk^2\lambda^2}, \tag{11}$$

replacing values Eq. (11) into Eq. (9) and we have the following hyperbolic type solution for Eq. (1)

$$u_1(x,t) = -\frac{bk^2\lambda^2}{a\left(1+bk^2\lambda^2\right)} - \frac{6bk^2\mu^2}{a\left(1+bk^2\lambda^2\right)\left(-\frac{\mu}{\lambda}+A\cosh\left[\lambda\left(kx-\frac{kt}{1+bk^2\lambda^2}\right)\right]-A\sinh\left[\lambda\left(kx-\frac{kt}{1+bk^2\lambda^2}\right)\right]\right)^2}$$

$$-\frac{6bk^2\lambda\mu}{a\left(1+bk^2\lambda^2\right)\left(-\frac{\mu}{\lambda}+A\cosh\left[\lambda\left(kx-\frac{kt}{1+bk^2\lambda^2}\right)\right]-A\sinh\left[\lambda\left(kx-\frac{kt}{1+bk^2\lambda^2}\right)\right]\right)}. \tag{12}$$





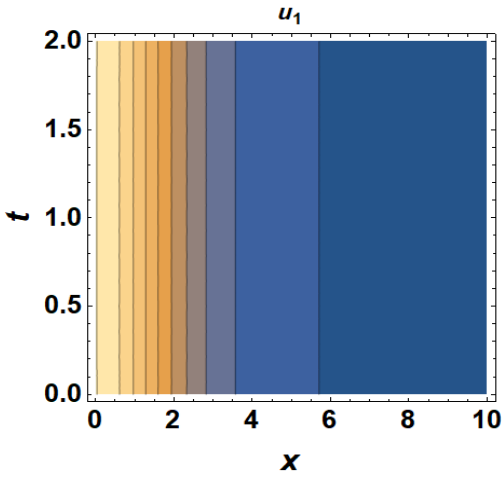


Figure 1. Respectively 3D, 2D, and contour graphics for $A=-2$, $b=2$, $k=3$, $\mu=1$, $\lambda=0.3$, $a=2$, $t=1$ values of Eq.(12).

Case2.

$$a_0=0, a_1=\frac{6\sqrt{b}\sqrt{w}\sqrt{k+w}\mu}{a}, a_2=\frac{6bkw\mu^2}{a}, \lambda=\frac{\sqrt{k+w}}{\sqrt{bk}\sqrt{w}}, \tag{13}$$

replacing values Eq. (13) into Eq. (9) and we have the following hyperbolic type solution for Eq. (1):

$$u_2(x,t)=\frac{6bkw\mu^2}{a\left(-\frac{\sqrt{bk}\sqrt{w}\mu}{\sqrt{k+w}}+A\cosh\left[\frac{\sqrt{k+w}(tw+kx)}{\sqrt{bk}\sqrt{w}}\right]-A\sinh\left[\frac{\sqrt{k+w}(tw+kx)}{\sqrt{bk}\sqrt{w}}\right]\right)^2}+\frac{6\sqrt{b}\sqrt{w}\sqrt{k+w}\mu}{a\left(-\frac{\sqrt{bk}\sqrt{w}\mu}{\sqrt{k+w}}+A\cosh\left[\frac{\sqrt{k+w}(tw+kx)}{\sqrt{bk}\sqrt{w}}\right]-A\sinh\left[\frac{\sqrt{k+w}(tw+kx)}{\sqrt{bk}\sqrt{w}}\right]\right)}. \tag{14}$$

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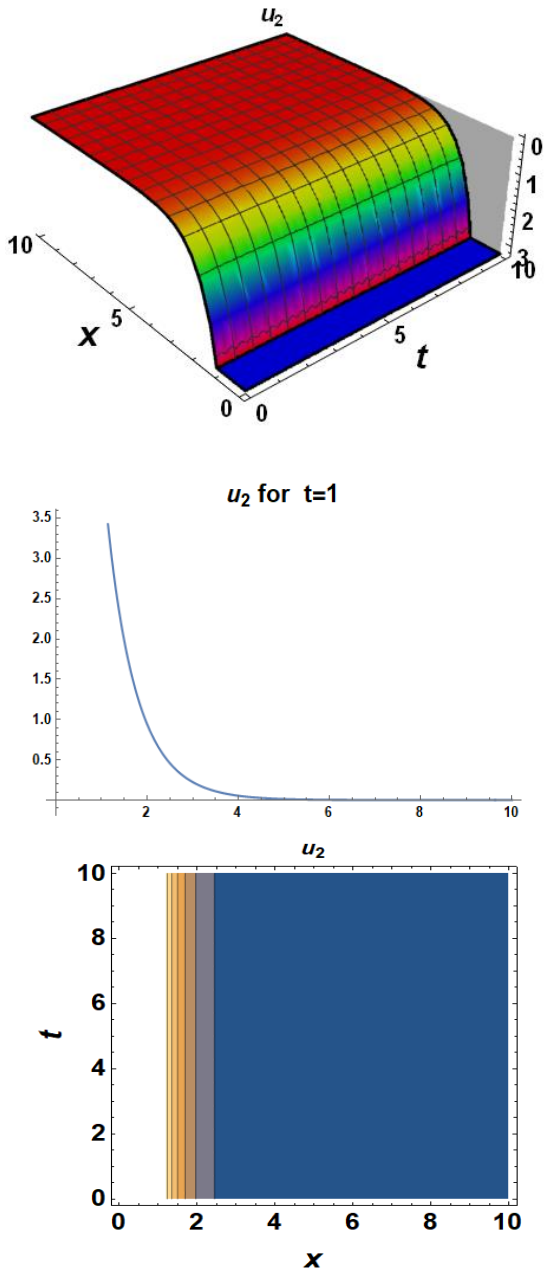


Figure 2. Respectively 3D, 2D, and contour graphics for $A = 3, b = 1, k = 2, \mu = 2, a = 0.2, w = 2, t = 1$ values of Eq.(14).

IV. CONCLUSIONS

In this paper, we obtain analytical solutions of RLW equation using $(1/G')$ -expansion method. 3D, 2D, and contour graphics are presented by giving special values to the constants in the obtained solutions. Each method offers solutions to nonlinear partial differential equations with different properties. It produces a hyperbolic solution function in the $(1/G')$ -expansion method. Solutions of this type have an important place especially in studies with shock wave structure and asymptotic behavior. This article shows that this method is efficient and practically suitable for applications in attaining analytical solutions for the RLW equation. With the aid of

computer package program, we guarantee the correctness of the solutions found by returning them to the original equation. We hope it will be useful for other studies in applied sciences.

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