ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

FAST EVALUATION OF ELECTRIC FIELD GREEN'S FUNCTIONS AND THEIR VOLUMETRIC INTEGRALS

M.Sc. THESIS

Fjolla SHEHU

Department of Electronics and Communication Engineering

Telecommunication Engineering Programme

June 2012

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(504091390)

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Thesis Advisor: Prof. Dr. Ibrahim Akduman

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ELEKTRİK TİPİ GREEN FONKSİYONLARIN VE HACİMSEL İNTREGRALLERİNİN HIZLI HESAPLANMASI

YÜKSEK LİSANS TEZİ

Fjolla SHEHU
(504091390)

Elektronik ve Haberleşme Mühendisliği Anabilim Dalı

Telekomünikasyon Mühendisliği Programı

Tez Danışmanı: Prof. Dr. Ibrahim Akduman

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Fjolla SHEHU, a M.Sc. student of ITU Institute of Science and Technology 504091390 successfully defended the thesis entitled **"FAST EVALUATION OF ELECTRIC FIELD GREEN'S FUNCTIONS AND THEIR VOLUMETRIC INTEGRALS"**, which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor :	**Prof. Dr. Ibrahim Akduman** Istanbul Technical University	.....
Jury Members :	**Prof. Dr. Ali YAPAR** Istanbul Technical University	.....
	Prof. Dr. Lale TÜKENMEZ ERGENE Istanbul Technical University	.....
		.....

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To my family,

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Fjolla SHEHU
Telecommunication Engineer

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ABBREVIATIONS

DCIM	**:** Discrete Image Method
EM	**:** Electromagnetic
GPOF	**:** Generalized Pencil-of-function
HED	**:** Horizontal Electric Dipole
MoM	**:** Method of Moments
SIP	**:** Sommerfeld Integration Path
VED	**:** Vertical Electric Dipole

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FAST EVALUATION OF ELECTRIC FIELD GREEN'S FUNCTIONS AND THEIR VOLUMETRIC INTEGRALS

SUMMARY

Scattering problems hold wide and important place within the electromagnetic theory. Solutions of the problems of this kind can be applied directly to remote sensing, geophysics, medical imaging and other civilian or military applications. Generally scattering problems can be divided into two groups. The first one that is direct scattering problem examines the interaction of a body with a certain geometric and electromagnetic properties placed in a known medium with the waves evoked from a given source Thus, it tries to calculate the scattered field from the body. The second one that is inverse scattering problem collects information about an object with unknown physical or geometrical properties or placed in an inaccessible place by measuring the area scattered from it. One very important area of application for the second group of problems is microwave tomography which is an important developing technique of the last years.

In both direct and inverse scattering problems, the derivation of Green's functions of for the region where the object buried plays very important role. Almost all existing methods for solving the problem require knowing the Green's functions related to the medium. In cases when the dealing problem is about bodies with changes in 3-dimensional, or that the dimensions are finite Green's functions are in dyadic form. Dyadic Green's functions that are interpreted as the impulse response can be expressed either directly for field quantities or for vector and scalar potentials. Especially for 3-D direct scattering problems, dyadic Green's functions solved for potentials contain certain advantages. Because singularity of the integral expressions is usually weak and it is relatively easier to calculate these functions. On the other hand, although electric field dyadic Green's functions contain strong singularity, especially in solving inverse scattering problems usage of them is inevitable.

Depending on the nature of the problem dealt with, the dyadic Green's functions can be expressed in various forms. In infinite and homogeneous wide space these functions have analytical expressions. On the other hand, in many practical applications the region, where the object is placed, is composed of different homogeneous environments coming together. The most common situation of this kind of structures is space separated as the planar plates. In this case, as in empty space dyadic Green's functions cannot be easily expressed analytically. However, using the spectral dyadic Green's functions expressions, these functions may be displayed as a continuous spectrum of the planar and cylindrical waves. For this representation special integrals (Sommerfeld integrals) on complex domain are used. As it is often not possible to directly calculate these integrals also while calculating numerically there are plenty of challenges. Especially in case that the source and observation points are near each

other in case of high frequencies, usage of known numerical integration techniques to calculate the integrals is very poor in terms of computational burden and in term of getting with high precision.

The purpose of this study is to calculate quickly and effectively electric-type dyadic Green's function of the two layered space. In this aim DCIM, that is an effective method for the calculation of Dyadic Green's function and their integrals over cube-shaped cells, is utilized. DCIM is based on expressing each component in terms of spherical waves illuminated by finite number of discrete sources by writing the spectral representations as the sum of exponentials.

In this thesis, first of all, expressions for electric-type dyadic Green's functions are derived. In continuation, integrals of the dyadic Green's over of cuboid cells are calculated semi-analytically. Subsequently DCIM method is described in details. In order to emphasize the effectiveness and the validity of the method and the results obtained from direct numerical integration are used to compare with. In conclusion it is observed that the data of DCIM match with the data calculated by numerical integration and that there is significant decrease in computational time.

ELEKTRİK TİPİ GREEN FONKSİYONLARIN VE HACİMSEL İNTREGRALLERİNİN HIZLI HESAPLANMASI

ÖZET

Saçılma problemleri elektromanyetik teorinin içerisinde çok geniş ve önemli bir yer tutar. Uzaktan algılama, jeofizik, tıbbi görüntüleme ve diğer sivil veya askeri uygulamalarda bu türden problemlerin çözümleri direk uygulama alanına sahiptir. Genel olarak saçılma problemlerini iki gruba ayırmak mümkündür. Bunlardan ilki olan düz saçılma problemlerinde bilinen bir ortamda yer alan ve tüm elektromanyetik ve geometrik özellikleri belirli olan bir cismin, yine belirli bir kaynaktan uyarılmış olan dalgalarla etkileşimi incelenir. Böylece cisimden saçılan alan belirlenmeye çalışılır. İkinci grup olan ters saçılma problemleri ise fiziksel veya geometrik özellikleri bilinmeyen veya yanına yaklaşılamayan bir cisimden saçılan alanın ölçülmesiyle cisim hakkında bilgi elde edilmeye calışılır. Bu ikinci grup problemlerinin çok önemli bir uygulama alanı da, son yıllarda üzerinde çok yoğun çalışmalar yürütülmekte olan, mikrodalga tomografisidir.

İster düz ister ters olsun tüm saçılma problemlerinin çözümünde cismin içerisinde yer aldığı bölgeye ilişkin Green fonksiyonunun hesaplanması çok önemli bir problem oluşturur. Mevcut problem çözme yöntemlerinin hemen hepsinde ortama ilişkin Green fonksiyonunun bilinmesi gerekir. Ele alınan problemin 3-boyutlu değişime sahip olduğu cisim veya boyutlarının sonlu olduğu her durumda Green fonksiyonları dyadic yapısındadırlar. Bilindiği üzere birim dürtü yansıtı olarak yorumlanabilen Dyadik Green fonksiyonları, ya direkt olarak alan büyüklükleri için ya da vektör ve skaler potansiyeller için ifade edilebilirler. Özellikle 3-boyutta düz saçılma problemleri çözülürken potansiyellere ilişkin Dyadik Green fonksiyonlarının kullanılması belirli avantajlar içerir. Çünkü genelde bu fonksiyonların tekillikleri zayıf tekillikler olup integral ifadelerinin içerisinde bunların hesaplarını yapmak nispeten daha kolaydır. Öte yandan elektrik alana ilişkin Dyadik Green fonksiyonları kuvvetli tekillik içermesine rağmen özellikle ters saçılma problemlerinin çözümünde kaçınılmaz olarak kullanılmak zorundadırlar.

Ele alınan problemin yapısına bağlı olarak Dyadik Green fonksiyonları çeşitli şekillerde ifade edilebilirler. Homojen ve sonsuz geniş bir uzay halinde bu fonksiyonlar analitik ifadelere sahiptir. Öte yandan pek çok pratik uygulamada cismin içinde yer aldığı bölge homojen olmayıp değişik ortamların bir araya gelmesinden oluşmuştur. Bu türden yapıların en yaygın olarak karşılaşıldığı durum ise düzlemsel olarak tabaklara ayrılmış bir uzaydır. Bu halde Dyadik Green fonksiyonları boş uzayda olduğu gibi direk analitik olarak ifade edilemezler. Ancak Dyadik Green fonksiyonlarının spektral ifadeleri kullanılarak bu fonksiyonlar düzlemsel ve silindirik dalgaların sürekli bir spektrumu gibi gösterilebilirler. Bu gösterilimde kompleks düzlemde özel integraller (Sommerfeld integrali) kullanılır. Bu integrallerin

direk olarak hesaplanması çoğu zaman mümkün olmadığı gibi sayısal olarak hesaplanmalarında da pek çok güçlük vardır. Özellikle kaynak ve gözlem noktalarının yakın olması veya yüksek frekanslarda çalışılması durumunda, bu integralleri bilinen sayısal integrasyon teknikleri ile hesaplanması hem hesap yükü bakımından çok fazla hemde yüksek hassasiyette sonuçlar elde etmek bakımından çok yetersizdir.

Yukarıda belirtildiği gibi ters saçılma problemlerinin çok önemli bir kullanım alanı mikrodalga tomografisidir. Mikrodalga tomografisinde görüntüleme için kullanılan yöntemlerden bir tanesi kontrast kaynak metodudur. Bu metodun özelliği ortama ilişkin elektrik tipi Green fonksiyonlarının hesaplanmasını gerektirmesidir. Buna ek olarak bir de Green fonksiyonlarının hücresel integrallerinin hesaplanması gerekmektedir. Green fonksiyonlarının hesaplanması uzun süren bir işlemdir, ayrıca hücresel integrallerinin hesaplanması da işlem süresini yoğun ölçüde artırır.

Bu çalışmanın amacı 2-parçalı uzaya ilişkin elektrik tipi Dyadik Green fonksiyonunun hızlı ve etkin bir biçimde hesaplanmasıdır. Bu amaçla Dyadic Green fonksiyonu ve bunların küp şeklindeki hücreler üzerindeki integrallerinin hesabı için etkin bir yöntem olan ayrık karmaşk imaj yöntemi (DCIM) yöntemi kullanılmıştır. DCIM spektral gösterilimdeki bileşenlerin üstel terimlerin toplamı gibi yazılarak her bir bileşeni sonlu sayıda ayrık kaynaktan yayılan küresel dalgalar şeklinde ifade etmeye dayanır. Mevcut integral ifadesini bilinen Sommerfeld özdeşliğine benzeterek integrali toplama dönüştürmeye yardımcı olur.

Spektral düzlemde bileşenlerin üstel terimlerin toplamı olarak yazılması için 'Generalized pencil of function' (GPOF) yöntemi ile iki parçalı örnekleme yolu kullanılmıştır. Sommerfeld integralinin hesaplanması, tekil noktaların integrasyon yoluna yakın ya da üzerinde olması durumunda zorlaşır. Bu sorunu aşabilmek için integrasyon yolu, tekil noktaların yakın olmayacağı şekilde, yeniden düzenlenmiştir. İntegrasyon yolu iki parçaya ayrılıp, GPOF için gerekli koşul olan doğrusal örneklemeyi sağlayacak şekilde düzenlenmiştir. İlk once ilk parça üzerinde GPOF yaklaşımı uygulanıp sonra mevcut fonksiyondan çıkartılmıştır. Kalana GPOF diğer parça üzerinde uygunlanmıştır. Böylece hata en aza indirgenmiştir. Sommerfeld eşözdeşliğinin kullanılabilmesi için k_z kompleks düzlemine geçiş yapılması gerekir, bu yüzden k_ρ düzleminden kz düzlemine geçiş parametreleri belirlenmiştir.

Bu tezde öncelikle elektrik tipi Dyadic Green fonksiyonlarının boş uzay için ifadeleri elde edilmiştir. Devamında iki parçalı düzlem için Green fonksiyonları hesaplanmıştır. Boş uzayda ifadeler düzlemsel dalgalar cinsiden elde eldiği için, iki katmanlı uzay durumunda, gerekli yansıma katsayıları eklenerek Green fonksiyonu ifadeleri elde edilir. Bütün ifadeler spektral düzlemde yazılmıştır. Devamında yarı analitik olarak Dyadic Green fonksiyonlarının hücresel integralleri hesaplanmıştır. Hücresel integraller sıkça kullanılan yaklaşımlar yapılarak analitik olarak hesaplanmıştır. Devamında DCIM yöntemi detaylı olarak anlatılmıştır ve önceden bulunan Green fonksiyonu ifadelerine uygulanmıştır.

Yöntemin geçerliliğini ve olumlu etkisini vurgulamak amacıyla direk sayısal integralden elde edilen sonuçlar kullanılıp kıyaslama yapılmıştır. Örnekler hem elektrik tipi Dyadik Green fonksiyonlarının ifadeleri için, hem de Green fonksiyonlarının hücresel integralleri için gösterilmiştir. Sonuçlarda DCIM ile hesaplanan verilerin sayısal integral ile hesaplanan verilerle örtüştüğü ve hesaplama zamanının belirgin

miktarda kısaldığı gözlemlenmiştir. Hücresel integralin numerik integrasyon ile hesaplanması durumunda işlem zamanı çok artarken, DCIM ile kayda değer bir fark gözlenmemiştir.

1. INTRODUCTION

In the last decades computational electromagnetic has spread widely in many different areas. The analysis of electromagnetic scattering has various applications for detection and location of buried object which can be applied in many different cases. One of the most significant applications of electromagnetic inverse scattering that is becoming popular is microwave tomography. Microwave tomography is an emerging biomedical imaging technique where the tissues are stimulated at the microwave frequencies rather than X-rays as in the conventional imaging methods. While reducing the power levels absorbed by the tissues, microwave tomography may open up new possibilities in terms of improved imaging quality. There are numerous methods used for inverse scattering, however all of them need the calculation of Green's function. Green's function has played a significant role in the development of numerical approaches utilized for analysis of microwave scattering problems [1–4]. Based on these studies, it is safe to state that in order to compute adequately the analysis of printed structures in multi layered environment, the Green's function in the spatial domain is necessary. The general method for derivation of dyadic Green's functions is based on vector and scalar potentials instead of field quantities. This is due to the fact that the kernels of operators are weakly singular when the Green's functions are derived based the potentials, while they becomes hyper-singular when field quantities are used [5]. On the other hand, during the analysis of the inverse scattering problems, the usage of the electric type Green's function is indispensible. That requires volumetric discretization of the object under test that may be located in a homogeneous or in a layered homogeneous medium. One most used method for microwave tomography is " contrast source inversion " method [6, 7]. The fundamental property of this method is that it requires the electric field Green's function to be known. Additionally volumetric integrals of Green's function are required to be calculated. The calculation of Green's functions is time consuming operation and their volumetric integrals intensely increase the computational burden. Therefore, the focus of most recently studies is oriented in

their efficient computation. In order to apply the techniques for fast evaluation, the Green's functions for layered medium in the spectral and spatial domains need to be defined. It would be helpful to mention that spectral-domain Green's functions in layered media can be obtained analytically, and spatial-domain Green's functions are obtained by applying inverse Fourier transform or inverse Hankel transform integrals. In either case, the integration of complex and oscillatory functions over infinite domains is obligatory [8–11]. Referring to these details, one of the techniques used for betterment of computation that has attracted valuable interest in computational electromagnetic studies, is based on approximation of the spatial-domain Green's functions in closed forms [12–21]. The first configuration of the approach had some problems to overcome, which were resolved by superimposing the two-level sampling algorithm [16] and generalized pencil-of-function method for exponential approximation [22] on the initial method. Therefore, the number of samples demanded for the exponential approximation and the sensitivity to noise are reduced.

1.1 Purpose of Thesis

In this thesis the fast evaluation of dyadic Green's functions of electric field in a layered medium and their integrals over cuboid cells are considered. To this aim, the discrete complex image method (DCIM) [16] is employed. Chapter 2 starts with the derivation of dyadic Green's functions for free-space and layered media, and followed by the derivation of volumetric integrals of dyadic Green's function over cuboid cells. Chapter 3 describes evaluation of closed-form approximation of the Green's function in spatial domain and provides several examples for comparison of results obtained from numerical integration and fast evaluation, in order to show the efficiency of the method.

2. DERIVATION OF ELECTRIC FIELDS GREEN'S FUNCTIONS FOR TWO HALF SPACES MEDIA AND DERIVATION OF THEIR VOLUMETRIC INTEGRAL

2.1 General Overview of Dyadic Green's Function

To describe Green's Function, one can say that Green's Function in electromagnetic problems is equivalent to the impulse response in circuit and system problems. In circuit and system theory, when the impulse response of a time-invariant linear system is known, the output to an arbitrary input can be calculated by taking the convolution of the impulse response and input. On other hand, since the convolution operation in time domain corresponds multiplication in frequency domain, the output of the frequency domain can be computed by multiplying Fourier transforms of the input and the impulse response. In physical terms, Fourier transformation makes it possible to write the frequency content of signals by the use of functions, which form an orthogonal set such as sines, cosines, complex exponentials, etc., in time domain. Thus, by taking the Fourier transform of input and impulse response, the process of convolution is reduced to multiplication of the frequency domain representations of the input signal and the impulse response of the system. To conclude, in aim to estimate the output signal of the system, the Fourier transform must be performed. The impulse input for circuits and systems is defined in time domain, $\delta(t)$, as the input signals in such systems are usually functions of time. However, in EM problems, since Green's functions in EM problems play the role of the impulse response in system theory, point source, which is an impulse source in space, plays the role of the impulse function in time.

2.2 The Derivation of Green's Function of Electrical Fields for Free Space

It is known that there is a relation between fields (electric and magnetic) and potentials (vector and scalar), using the same relations Green's functions of electric and magnetic fields can be derived from the Green's functions of potentials as well. In this aim,

starting with the four Maxwell's equations,

$$\bigtriangledown \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.1}$$

$$\bigtriangledown \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J} \tag{2.2}$$

$$\bigtriangledown \cdot \vec{D} = \rho \tag{2.3}$$

$$\bigtriangledown \cdot \vec{D} = 0 \tag{2.4}$$

(Faraday-Maxwell law (2.1), Generalized Ampere's law (2.2),Gauss's law (2.3) and Law of conversation of magnetic flux (2.4)) the expression of electric and magnetic fields in terms of potentials are derived as in below [8]:

$$E(\vec{r}) = -i\omega \cdot A(\vec{r}) - \bigtriangledown \phi(\vec{r}) \tag{2.5}$$

$$\mu H(\vec{r}) = \bigtriangledown \times \phi(\vec{r}) \tag{2.6}$$

where $\phi(\vec{r})$ and $A(\vec{r})$ are the scalar and vector potentials, respectively, and they are related by the Lorentz gauge as

$$\bigtriangledown \cdot A(\vec{r}) = -i\omega\varepsilon\mu\phi(\vec{r}) \tag{2.7}$$

In this work time convention is used as $e^{-i\omega t}$, and μ and ε are the permeability and the permittivity of the medium, respectively. Substituting (2.7) into (2.5) results in the following electric field expression in terms of the vector potential:

$$E(\vec{r}) = i\omega \left[\bar{\bar{I}} + \frac{1}{k^2} \bigtriangledown \bigtriangledown\right] \cdot A(\vec{r}) \tag{2.8}$$

Where $\bar{\bar{I}} = \vec{x}\vec{x} + \vec{y}\vec{y} + \vec{z}\vec{z}$ is called the idem factor or idem diad. From relations shown in (2.6) and (2.8) it is clearly seen that if vector potential is known, Green's functions for the electric and magnetic fields can be calculated by simple differentiations. Therefore, in order to obtain Green's functions for the fields, Green's functions for potentials must be calculated first. In this aim the source term is assumed to be an electric current dipole oriented in an arbitrary direction $\vec{\alpha}$, represented as $\vec{J} = Il\delta\left(\vec{r} - \vec{r'}\right)\vec{\alpha}$ and the governing equation is the inhomogeneous Helmholtz equation.

$$\bigtriangledown^2 A(\vec{r}) + k^2 A(\vec{r}) = -\mu J(\vec{r}) \tag{2.9}$$

Hence the solution of the inhomogeneous Helmholtz equation for the given source is achieved as:

$$A\left(\vec{r},\vec{r'}\right)=\vec{\alpha}\mu Il\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{4\pi\left|\vec{r}-\vec{r'}\right|} \tag{2.10}$$

where $\vec{r}$ and $\vec{r'}$ are the observation and source points, respectively and $k\left(=\omega\sqrt{\varepsilon\mu}\right)$ is the wave number of the medium. Once the vector potential is obtained, substituting (2.10) into (2.8), and (2.10) into (2.6) would result in the electric and magnetic fields, respectively, in a homogeneous, isotropic and unbounded medium.

$$\bar{E}\left(\vec{r},\vec{r'}\right)=i\omega\left[\bar{\bar{I}}+\frac{1}{k^2}\bigtriangledown\bigtriangledown\right]\cdot\vec{\alpha}\mu Il\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{4\pi\left|\vec{r}-\vec{r'}\right|} \tag{2.11}$$

$$\bar{H}\left(\vec{r},\vec{r'}\right)=\bigtriangledown\times\vec{\alpha}\mu Il\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{4\pi\left|\vec{r}-\vec{r'}\right|} \tag{2.12}$$

The electric fields Green's functions due to an arbitrary oriented electric dipole source are derived.

As discussed previously, in order to facilitate the calculation the transformation from spatial domain to spectral domain is applied by the Fourier transform. Once the Green's functions are obtained in spectral domain, the reversal to spatial domain must be accomplished. As mentioned before the transition from spectral domain to spatial domain is performed by inverse Fourier transform, which for EM problems is presented by Weyl identity:

$$\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}=\frac{i}{2\pi}\iint_{-\infty}^{\infty}\frac{e^{ik_x(x-x')+ik_y(y-y')+ik_z|z-z'|}}{k_z}dk_xdk_y \tag{2.13}$$

In EM problems there is another approach for the transition from spectral domain to spatial domain is Sommerfeld integration

$$\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}=\frac{i}{2}\int_{-\infty}^{\infty}\frac{k_\rho}{k_z}H_0^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.14}$$

where $H_0^{(1)}$ is the Hankel function of first kind and $\vec{k}=k_x\vec{e}_x+k_y\vec{e}_y+k_z\vec{e}_z$ thus

$$k_z=\sqrt{k^2-k_\rho^2}\Rightarrow k_\rho=\pm k\ and\ H_0^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)\Rightarrow k_\rho=0 \tag{2.15}$$

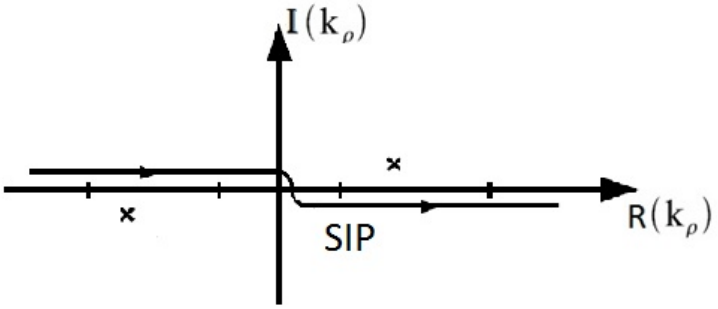


Figure 2.1: SIP for the loosy medium.

are the branch points and the Sommerfeld integration path (SIP) is given in Figure 2.1.. To obtain all components of Dyadic Green's Function, one should calculate the electric fields due to electric dipole oriented in each direction one by one. To this aim, first the fields created by an arbitrary directed electric dipole must be written for free space. The expressions of fields are obtained due to each direction of an electric dipole, which are vertical electric dipole (VED) (z-directed), horizontal electric dipole (HED)(x-directed) and horizontal electric dipole (HED) (y-directed).

2.2.1 Horizontal electric dipole (x-directed) in free space

In this section the expressions of components of electric and magnetic fields, due to an x-directed electric dipole in free space, are obtained.

By using the equations (2.11) and (2.12), the components of electric and magnetic fields due to an x-directed electric dipole are written as below.

$$E_x\left(\vec{r},\vec{r'}\right) = \frac{i\omega\mu Il}{4\pi k^2}\left(k^2+\frac{\partial^2}{\partial x^2}\right)\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.16}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \frac{iIl}{4\pi\varepsilon\omega}\frac{\partial^2}{\partial y\partial x}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.17}$$

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{iIl}{4\pi\varepsilon\omega}\frac{\partial^2}{\partial z\partial x}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.18}$$

$$H_x\left(\vec{r},\vec{r'}\right) = 0 \tag{2.19}$$

$$H_y\left(\vec{r},\vec{r'}\right) = \frac{Il}{4\pi}\frac{\partial}{\partial z}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.20}$$

$$H_z\left(\vec{r},\vec{r'}\right) = \frac{-Il}{4\pi}\frac{\partial}{\partial y}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.21}$$

By the help of Sommerfeld Integral (2.14), the spatial representation in terms of spectral representation of fields for free space for z components are established.

$$E_z\left(\vec{r},\vec{r'}\right) = \pm\frac{Il}{8\pi\omega\varepsilon}\cos\phi\int_{-\infty}^{\infty}k_\rho^2 H_1^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.22}$$

$$H_z\left(\vec{r},\vec{r'}\right) = \frac{-iIl}{8\pi}\cos\phi\int_{-\infty}^{\infty}\frac{k_\rho^2}{k_{1z}}H_1^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.23}$$

2.2.2 Horizontal electric dipole (y-directed) in free space

The definitions of electric and magnetic fields due to a y-directed electric dipole are obtained by expending the equations (2.11) and (2.12).

$$E_x\left(\vec{r},\vec{r'}\right) = \frac{iIl}{4\pi\varepsilon\omega}\frac{\partial^2}{\partial x\partial y}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.24}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \frac{iIl}{4\pi\varepsilon\omega}\left(k^2+\frac{\partial^2}{\partial y^2}\right)\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.25}$$

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{iIl}{4\pi\varepsilon\omega}\frac{\partial^2}{\partial z\partial y}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.26}$$

$$H_x\left(\vec{r},\vec{r'}\right) = \frac{-Il}{4\pi}\frac{\partial}{\partial z}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.27}$$

$$H_y\left(\vec{r},\vec{r'}\right) = 0 \tag{2.28}$$

$$H_z\left(\vec{r},\vec{r'}\right) = \frac{Il}{4\pi}\frac{\partial}{\partial x}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.29}$$

The final forms obtained by solving equations (2.26) and (2.29), are shown in following equations.

$$E_z\left(\vec{r},\vec{r'}\right) = \pm\frac{iIl}{8\pi\omega\varepsilon}\sin\phi\int_{-\infty}^{\infty}k_\rho^2 H_1^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.30}$$

$$H_z\left(\vec{r},\vec{r'}\right) = \frac{-iIl}{8\pi}\cos\phi\int_{-\infty}^{\infty}\frac{k_\rho^2}{k_{1z}}H_1^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.31}$$

2.2.3 Vertical electric dipole (z-directed) in free space

The expression of the electric field due to z-directed dipole is as below.

$$\vec{E}\left(\vec{r},\vec{r'}\right) = i\omega\mu\left[\bar{\bar{I}} + \frac{1}{k^2}\bigtriangledown\bigtriangledown\right]\vec{e}_z Il\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{4\pi\left|\vec{r}-\vec{r'}\right|} \tag{2.32}$$

Where the components of electric and magnetic fields are:

$$E_x\left(\vec{r},\vec{r'}\right) = \frac{i\omega\mu Il}{4\pi k^2}\frac{\partial^2}{\partial z\partial x}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.33}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \frac{i\omega\mu Il}{4\pi k^2}\frac{\partial^2}{\partial z\partial y}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.34}$$

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{i\omega\mu Il}{4\pi k^2}\left(k^2+\frac{\partial^2}{\partial z^2}\right)\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.35}$$

$$H_x\left(\vec{r},\vec{r'}\right) = \frac{Il}{4\pi}\frac{\partial}{\partial y}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.36}$$

$$H_y\left(\vec{r},\vec{r'}\right) = \frac{-Il}{4\pi}\frac{\partial}{\partial x}\left\{\frac{e^{ik\left|\vec{r}-\vec{r'}\right|}}{\left|\vec{r}-\vec{r'}\right|}\right\} \tag{2.37}$$

$$H_z\left(\vec{r},\vec{r'}\right) = 0 \tag{2.38}$$

By using the Sommerfeld Integration the spectral representation of (z-component) due to vertically directed (z-directed) electric dipole in free space is obtained as below.

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{-Il}{8\pi\omega\varepsilon}\int_{-\infty}^{\infty}\frac{k_\rho^3}{k_z}H_0^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z|z-z'|}dk_\rho \tag{2.39}$$

$$H_z\left(\vec{r},\vec{r'}\right) = 0 \tag{2.40}$$

Since these expressions are written for plane wave spectrum, one can obtain field expressions for a layered media by adding the appropriate reflection coefficients into the integrals in (2.39) and (2.40).

2.3 The Derivation of Green's Function of Electrical Fields in Layered Media

The next step is to obtain the expression of electric field for the two-layered media, where the case considered is when the observation and source points are both in the upper half space as shown in Figure.2.1.

As mentioned before one can obtain field expressions for a layered media by adding the appropriate reflection coefficients into the equations obtained for free-space considering that these expressions are written for plane wave spectrum.

Let us define a spectral representation for electric field. By taking in consideration that representation and if z-dependent component is known the transverse components are acquired by the equations shown below.

$$\vec{E}\left(\vec{r},\vec{r'}\right) = \int_{-\infty}^{\infty} \underline{\overline{E}}\left(k_\rho,\vec{r},\vec{r'}\right) dk_\rho \tag{2.41}$$

$$\underline{\overline{E}}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{k^2-k_z^2}\left[grad_t \frac{\partial \underline{\underline{E_z}}}{\partial z} - i\omega\mu\vec{e}_z \times grad_t \underline{\underline{H_z}}\right] + \underline{E}_z\left(k_\rho,\vec{r},\vec{r'}\right)\vec{e}_z \tag{2.42}$$

$$\underline{\overline{H}}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{k^2-k_z^2}\left[grad_t \frac{\partial \underline{\underline{E_z}}}{\partial z} - i\omega\mu\vec{e}_z \times grad_t \underline{\underline{E_z}}\right] + \underline{H}_z\left(k_\rho,\vec{r},\vec{r'}\right)\vec{e}_z \tag{2.43}$$

In equations (2.42) and (2.43) $grad_t$ is the transverse gradient and is defined as derivation in terms of x and y components. ($grad_t = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y$) [8].

2.3.1 Horizontal electric dipole (x-directed) in two half space media

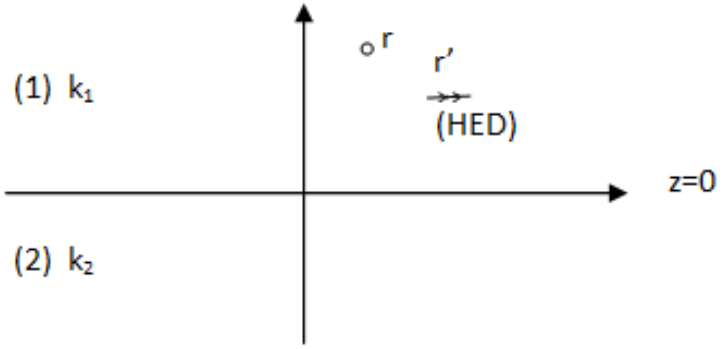


Figure 2.2: Horizontal electric dipole on the upper half space.

By putting the appropriate reflection coefficient in the equations (2.26) and (2.29), the spectral representations of electric and magnetic fields due to a horizontal x-directed

dipole in the layered media are written.

$$E_z\left(\vec{r},\vec{r'}\right)=\frac{iIl}{8\pi\omega\varepsilon_1}\cos\phi\int_{-\infty}^{\infty}k_\rho^2H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\left[\pm e^{ik_z|z-z'|}-R_{12}^{TM}e^{ik_{1z}(z+z')}\right]dk_\rho \tag{2.44}$$

$$H_z\left(\vec{r},\vec{r'}\right)=\frac{iIl}{8\pi\omega\varepsilon_1}\cos\phi\int_{-\infty}^{\infty}\frac{k_\rho^2}{k_{1z}}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\left[\pm e^{ik_z|z-z'|}-R_{12}^{TM}e^{ik_{1z}(z+z')}\right]dk_\rho \tag{2.45}$$

Where the reflection coefficients are defined as below.

$$R_{12}^{TM}=\frac{\varepsilon_2k_{1z}-\varepsilon_1k_{2z}}{\varepsilon_2k_{1z}+\varepsilon_1k_{2z}} \tag{2.46}$$

$$R_{12}^{TE}=\frac{\mu_2k_{1z}-\mu_1k_{2z}}{\mu_2k_{1z}+\mu_1k_{2z}} \tag{2.47}$$

By using the definitions in (2.36) and (2.37) the equations for other components are obtained and shown in following equations.

$$\underline{E}_x=\frac{1}{k_1^2-k_{1z}^2}\left[\frac{\partial^2\underline{E}_z}{\partial z\partial x}+i\omega\mu\frac{\partial\underline{H}_z}{\partial y}\right] \tag{2.48}$$

$$\underline{E}_y=\frac{1}{k_1^2-k_{1z}^2}\left[\frac{\partial^2\underline{E}_z}{\partial z\partial y}-i\omega\mu\frac{\partial\underline{H}_z}{\partial x}\right] \tag{2.49}$$

The equations in (2.39) and (2.40) are solved by taking into consideration the following definitions.

$$\frac{\partial\cos\phi}{\partial x}=\frac{-\sin^2\phi}{|\vec{\rho}-\vec{\rho}'|} \tag{2.50}$$

$$\frac{\partial\cos\phi}{\partial y}=\frac{-\sin\phi\cos\phi}{|\vec{\rho}-\vec{\rho}'|} \tag{2.51}$$

$$\frac{\partial\sin\phi}{\partial x}=\frac{-\sin\phi\cos\phi}{|\vec{\rho}-\vec{\rho}'|} \tag{2.52}$$

$$\frac{\partial\sin\phi}{\partial y}=\frac{\cos^2\phi}{|\vec{\rho}-\vec{\rho}'|} \tag{2.53}$$

$$\frac{\partial H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)}{\partial x}=k_\rho\cos\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)-\frac{1}{\left|\vec{\rho}-\vec{\rho}'\right|}\cos\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.54}$$

$$\frac{\partial H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)}{\partial y}=k_\rho\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)-\frac{1}{\left|\vec{\rho}-\vec{\rho}'\right|}\sin\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.55}$$

In whole study the $\cos\phi$ and $\sin\phi$ functions are defined as $\cos\phi = \frac{x-x'}{|\vec{\rho}-\vec{\rho}'|}$; $\sin\phi = \frac{y-y'}{|\vec{\rho}-\vec{\rho}'|}$ for $z < z'$ and $\cos\phi = \frac{x+x'}{|\vec{\rho}-\vec{\rho}'|}$; $\sin\phi = \frac{y+y'}{|\vec{\rho}-\vec{\rho}'|}$ for $z > z'$. Finally the x and y components are obtained for the field expression due to x-directed electric dipole.

$$E_x\left(\vec{r},\vec{r'}\right) = \int \frac{iIl}{8\pi\omega\varepsilon_1}\left[ik_{1z}e^{ik_{1z}|z-z'|} - ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho\cos^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]$$
$$+\frac{i\omega\mu}{8\pi}iIl\frac{1}{k_{1z}}\left[e^{ik_{1z}|z-z'|} - R_{12}^{TE}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho\sin^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]dk_\rho \qquad \textbf{(2.56)}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \int \frac{iIl}{8\pi\omega\varepsilon_1}\left[ik_{1z}e^{ik_{1z}|z-z'|} - ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]$$
$$+\frac{i\omega\mu}{8\pi}iIl\frac{1}{k_{1z}}\left[e^{ik_{1z}|z-z'|} - R_{12}^{TE}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]dk_\rho \qquad \textbf{(2.57)}$$

2.3.2 Horizontal electric dipole (y-directed) in two half space media

By following the same steps used in the above sections, the electric and magnetic field expressions due to a horizontal y-directed dipole are obtained.

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{iIl}{8\pi\omega\varepsilon_1}\sin\phi\int_{-\infty}^{\infty}k_\rho^2 H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\left[\pm e^{ik_{1z}|z-z'|} - R_{12}^{TM}e^{ik_{1z}(z+z')}\right]dk_\rho \qquad \textbf{(2.58)}$$

$$H_z\left(\vec{r},\vec{r'}\right) = \frac{iIl}{8\pi}\cos\phi\int_{-\infty}^{\infty}\frac{k_\rho^2}{k_{1z}}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\left[e^{ik_{1z}|z-z'|} + R_{12}^{TE}e^{ik_{1z}(z+z')}\right]dk_\rho \qquad \textbf{(2.59)}$$

Depending on the transverse components that are written in equations (2.42) and (2.43) other components are obtained.

$$E_x\left(\vec{r},\vec{r'}\right) = \int \frac{iIl}{8\pi\omega\varepsilon_1}\left[ik_{1z}e^{ik_{1z}|z-z'|} - ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$-\frac{i\omega\mu}{8\pi}iIl\frac{1}{k_{1z}}\left[e^{ik_{1z}|z-z'|} - R_{12}^{TE}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]dk_\rho \qquad \textbf{(2.60)}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \int \frac{iIl}{8\pi\omega\varepsilon_1}\left[ik_{1z}e^{ik_{1z}|z-z'|} - ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho \sin^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$+\frac{i\omega\mu}{8\pi}iIl\frac{1}{k_{1z}}\left[e^{ik_{1z}|z-z'|} - R_{12}^{TE}e^{ik_{1z}(z+z')}\right]$$
$$\left[k_\rho \cos^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]dk_\rho \qquad \textbf{(2.61)}$$

2.3.3 Vertical electric dipole (z-directed) in two half space media

Considering that, the problem is solved for the case where the observation and source points are both in the same half space as shown in Figure 2.3, the z-component of electric field due to a vertically oriented electric dipole is obtained as below:

$$E_z\left(\vec{r},\vec{r'}\right) = \frac{iIl}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty}\frac{k_\rho^3}{k_{1z}}H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\left[ik_{1z}e^{ik_{1z}|z-z'|} - R_{12}^{TM}e^{ik_{1z}(z+z')}\right]dk_\rho \qquad \textbf{(2.62)}$$

The z-components of electric and magnetic field in that space are obtained.

$$E_z\left(\vec{r},\vec{r'}\right) = \int \frac{-Il}{8\pi\omega\varepsilon_1}\frac{k_\rho^3}{k_{1z}}H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\left[ik_{1z}e^{ik_{1z}|z-z'|} - R_{12}^{TM}e^{ik_{1z}(z+z')}\right]dk_\rho \qquad \textbf{(2.63)}$$

$$H_z\left(\vec{r},\vec{r'}\right) = 0 \qquad \textbf{(2.64)}$$

$$E_x\left(\vec{r},\vec{r'}\right) = \int \frac{1}{k^2-k_z^2}\frac{\partial^2}{\partial z\partial x}\underline{E}_z\left(k_\rho,\vec{r},\vec{r'}\right)dk_\rho \qquad \textbf{(2.65)}$$

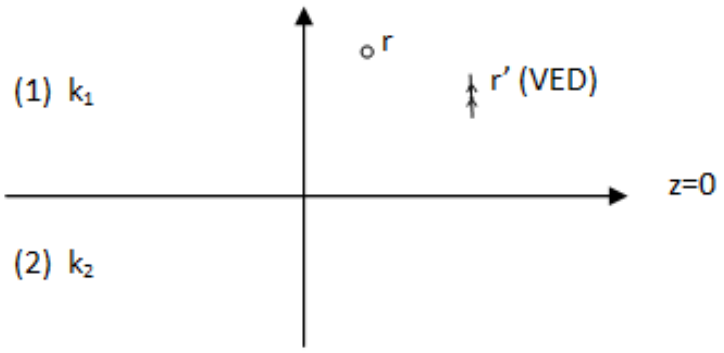


Figure 2.3: Vertical electric dipole on the upper half space.

$$E_y\left(\vec{r},\vec{r'}\right) = \int \frac{1}{k^2 - k_z^2}\frac{\partial^2}{\partial z \partial y}\underline{E}_z\left(k_\rho,\vec{r},\vec{r'}\right)dk_\rho \tag{2.66}$$

The spectral representations of x and y components in terms of spectral z-component are shown in (2.65) and (2.66), respectively. Using the representation of z-components in (2.61), and the spectral representations of z-component, in (2.63) and (2.64), the open expression of each component is obtained as written below.

$$E_x\left(\vec{r},\vec{r'}\right) = \int \frac{Il}{8\pi\omega\varepsilon_1}\frac{k_\rho^2}{k_{1z}}\cos\phi\left[ik_{1z}e^{ik_{1z}|z-z'|} + ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)dk_\rho \tag{2.67}$$

$$E_y\left(\vec{r},\vec{r'}\right) = \int \frac{Il}{8\pi\omega\varepsilon_1}\frac{k_\rho^2}{k_{1z}}\sin\phi\left[ik_{1z}e^{ik_{1z}|z-z'|} + ik_{1z}R_{12}^{TM}e^{ik_{1z}(z+z')}\right]H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)dk_\rho \tag{2.68}$$

$$E_z\left(\vec{r},\vec{r'}\right) = \int \frac{-Il}{8\pi\omega\varepsilon_1}\frac{k_\rho^3}{k_{1z}}\left[e^{ik_{1z}|z-z'|} + R_{12}^{TM}e^{ik_{1z}(z+z')}\right]H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)dk_\rho \tag{2.69}$$

2.4 The Dyadic Green's Function in Two Half Space Media

In the previous sections, the electric field expressions for the layered media due to an arbitrary electric dipole are obtained. As mentioned before in the case when Il is assumed to be 1 and when each tern is divided $i\omega\mu$.

The Dyadic Green Function for only reflected terms is shown in the equations below.

From equations (2.56), (2.57) and (2.44), the x components of Green's Functions are derived.

$$\underline{G}^R_{xx}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{8\pi k_1^2}\left(-ik_{1z}\right)R_{12}^{TM}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$+\frac{i}{8\pi}\frac{1}{k_{1z}}R_{12}^{TE}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\sin^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right] \quad \textbf{(2.70)}$$

$$\underline{G}^R_{xy}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{8\pi k_1^2}\left(-ik_{1z}\right)R_{12}^{TM}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$+\frac{i}{8\pi}\frac{1}{k_{1z}}R_{12}^{TE}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right] \quad \textbf{(2.71)}$$

$$\underline{G}^R_{xz}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{8\pi k_1^2}k_\rho^2\left(-R_{12}^{TM}\right)e^{ik_{1z}(z+z')}\cos\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) \quad \textbf{(2.72)}$$

From equations (2.60), (2.61) and (2.58), the y components of Green's Functions are derived.

$$\underline{G}^R_{yx}\left(k_\rho,\vec{r},\vec{r'}\right) = \frac{1}{8\pi k_1^2}\left(-ik_{1z}\right)R_{12}^{TM}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$-\frac{i}{8\pi}\frac{1}{k_{1z}}R_{12}^{TE}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos\phi\sin\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)-\frac{\sin 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right] \quad \textbf{(2.73)}$$

$$\underline{G}^R_{yy}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi k_1^2}\left(-ik_{1z}\right)R_{12}^{TM}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\sin^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)-\frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]$$
$$-\frac{i}{8\pi}\frac{1}{k_{1z}}R_{12}^{TE}e^{ik_{1z}(z+z')}$$
$$\left[k_\rho\cos^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)-\frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho}'\right|}H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right] \tag{2.74}$$

$$\underline{G}^R_{xz}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi k_1^2}k_\rho^2\left(-R_{12}^{TM}\right)e^{ik_{1z}(z+z')}\sin\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.75}$$

The z components of Green Functions are derived from equations (2.67), (2.68) and (2.69).

$$\underline{G}^R_{zx}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi k_1^2}k_\rho^2R_{12}^{TM}e^{ik_{1z}(z+z')}\cos\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.76}$$

$$\underline{G}^R_{zx}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi k_1^2}k_\rho^2R_{12}^{TM}e^{ik_{1z}(z+z')}\sin\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.77}$$

$$\underline{G}^R_{zz}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi k_1^2}\frac{k_\rho^3}{k_{1z}}R_{12}^{TM}e^{ik_{1z}(z+z')}H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.78}$$

2.5 Volumetric Integrals of Dyadic Green's Functions

In this section the procedure of calculating the volumetric integrals of Green's functions in a semi-analytically way is given. As mentioned in introduction chapter, in order to apply contrast source inversion method, the volumetric integrals of Green's function over cuboid cells, are required to be calculated.

For the sake of brevity, the procedure of integration is explained in details just for one of the Dyadic Green functions and then the final expressions for all terms are given. To make the calculations easier the order of the derivatives and integrals are changed. So first the volumetric integrals are calculated, then the derivatives are taken.

Let us write the expression of before the derivatives are evaluated.

$$\underline{G}^R_{xx}\left(k_\rho,\vec{r},\vec{r'}\right)=\frac{1}{8\pi}\frac{\partial^2}{\partial x^2}\left[\frac{1}{k_1^2k_\rho}\left(-ik_{1z}\right)R_{12}^{TM}e^{ik_{1z}(z+z')}H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right]$$
$$+\frac{i}{8\pi}\frac{\partial^2}{\partial y^2}\left[\frac{1}{k_{1z}k_\rho}R_{12}^{TE}e^{ik_{1z}(z+z')}H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right)\right] \tag{2.79}$$

When the volumetric integral is applied the equation becomes:

$$\iiint_{cuboid} g\left(\vec{r},\vec{r'}\right) dx'dy'dz' \tag{2.80}$$

where $g\left(\vec{r},\vec{r'}\right)$ is one component of the Dyadic Green's Function and is defined in terms of spectral representation as below.

$$g\left(\vec{r},\vec{r'}\right) = \int_{-\infty}^{\infty} \underline{g}\left(k_\rho,\vec{r},\vec{r'}\right) dk_\rho \tag{2.81}$$

By putting in equation (2.81) (instead of) and changing the order of integrals we obtain:

$$\iiint_{cuboid} \int_{-\infty}^{\infty} \underline{G}_{xx}^{R}\left(k_\rho,\vec{r},\vec{r'}\right) dk_\rho dx'dy'dz' = \int_{-\infty}^{\infty} \iiint_{cuboid} \underline{G}_{xx}^{R}\left(k_\rho,\vec{r},\vec{r'}\right) dx'dy'dz'dk_\rho$$
$$= \frac{1}{8\pi}\frac{\partial^2}{\partial x^2}\left[\int_{-\infty}^{\infty} \frac{1}{k_1^2 k_\rho}\left(-ik_{1z}\right) R_{12}^{TM} Int_1 Int_2 dk_\rho\right] + \frac{i}{8\pi}\frac{\partial^2}{\partial y^2}\left[\int_{-\infty}^{\infty} \frac{1}{k_{1z}k_\rho} R_{12}^{TE} Int_1 Int_2 dk_\rho\right] \tag{2.82}$$

The volumetric integrals ($Int_1 Int_2$)can be calculated analytically [23] and their results are given in equations below.

$$Int_1 = \int_{z_0-\frac{\Delta z}{2}}^{z_0+\frac{\Delta z}{2}} e^{ik_{1z}(z\pm z')} dz' = e^{ik_{1z}(z\pm z')} \Delta z \sin c\left(\frac{k_z \Delta z}{2}\right) \tag{2.83}$$

$$Int_2 = \int_0^{2\pi}\int_0^{a_e} H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\rho' d\rho' d\phi' = \begin{cases} \frac{2\pi a_e}{k_\rho} J_1\left(k_\rho a_e\right) H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) \\ \frac{4i}{k_\rho^2} + \frac{2\pi a_e}{k_\rho} J_0\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) H_1^{(1)}\left(k_\rho a_e\right) \end{cases} \tag{2.84}$$

where $a_e = \sqrt{\frac{\Delta x \Delta y}{\pi}}$ is the radius of a circle whose area is equivalent to the area of the one face of cuboid [23]. By taking into consideration the equation (2.53) and (2.54) the derivatives are estimated and the final form is achieved.

$$\iiint_{cuboid} \underline{G}_{xx}^{R}\left(k_\rho,\vec{r},\vec{r'}\right) dv = \frac{1}{8\pi k_1^2}\left(-ik_{1z}\right) R_{12}^{TM} e^{ik_{1z}(z+z')} M\left(k_{1z}\right)$$
$$\left[k_\rho \cos^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|} H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right]$$
$$+\frac{i}{8\pi}\frac{1}{k_{1z}} R_{12}^{TE} e^{ik_{1z}(z+z')} M\left(k_{1z}\right)$$
$$\left[k_\rho \sin^2\phi H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho}-\vec{\rho'}\right|} H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho'}\right|\right)\right] \tag{2.85}$$

where $M(k_{1z})$ is:

$$M(k_{1z}) = 2\pi a_e \Delta z \frac{J_1(k_\rho a_e)}{k_\rho} \sin c\left(\frac{k_z \Delta z}{2}\right) \tag{2.86}$$

The same procedure is repeated for each term and the final expressions are estimated.

$$\begin{aligned}\iiint_{cuboid} \underline{G}^R_{xy}\left(k_\rho, \vec{r}, \vec{r'}\right) dv = \frac{1}{8\pi k_1^2}(-ik_{1z}) R^{TM}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right] \\ + \frac{i}{8\pi}\frac{1}{k_{1z}} R^{TE}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right]\end{aligned} \tag{2.87}$$

$$\iiint_{cuboid} \underline{G}^R_{xz}\left(k_\rho, \vec{r}, \vec{r'}\right) dv = \frac{1}{8\pi k_1^2} M(k_{1z}) k_\rho^2 \left(-R^{TM}_{12}\right) e^{ik_{1z}(z+z')} \cos\phi H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) \tag{2.88}$$

$$\begin{aligned}\iiint_{cuboid} \underline{G}^R_{yx}\left(k_\rho, \vec{r}, \vec{r'}\right) dv = \frac{1}{8\pi k_1^2}(-ik_{1z}) R^{TM}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right] \\ - \frac{i}{8\pi}\frac{1}{k_{1z}} R^{TE}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \cos\phi \sin\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\sin 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right]\end{aligned} \tag{2.89}$$

$$\begin{aligned}\iiint_{cuboid} \underline{G}^R_{yy}\left(k_\rho, \vec{r}, \vec{r'}\right) dv = \frac{1}{8\pi k_1^2}(-ik_{1z}) R^{TM}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \sin^2\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right] \\ - \frac{i}{8\pi}\frac{1}{k_{1z}} R^{TE}_{12} e^{ik_{1z}(z+z')} M(k_{1z}) \\ \left[k_\rho \cos^2\phi H_0^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) - \frac{\cos 2\phi}{\left|\vec{\rho} - \vec{\rho'}\right|} H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right)\right]\end{aligned} \tag{2.90}$$

$$\iiint_{cuboid} \underline{G}^R_{xz}\left(k_\rho, \vec{r}, \vec{r'}\right) dv = \frac{1}{8\pi k_1^2} M(k_{1z}) k_\rho^2 \left(-R^{TM}_{12}\right) e^{ik_{1z}(z+z')} \sin\phi H_1^{(1)}\left(k_\rho \left|\vec{\rho} - \vec{\rho'}\right|\right) \tag{2.91}$$

$$\iiint_{cuboid} \underline{G}_{zx}^{R}\left(k_\rho,\vec{r},\vec{r'}\right)dv = \frac{1}{8\pi k_1^2} M\left(k_{1z}\right) k_\rho^2 R_{12}^{TM} e^{ik_{1z}(z+z')} \cos\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.92}$$

$$\iiint_{cuboid} \underline{G}_{zx}^{R}\left(k_\rho,\vec{r},\vec{r'}\right)dv = \frac{1}{8\pi k_1^2} M\left(k_{1z}\right) k_\rho^2 R_{12}^{TM} e^{ik_{1z}(z+z')} \sin\phi H_1^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.93}$$

$$\iiint_{cuboid} \underline{G}_{zz}^{R}\left(k_\rho,\vec{r},\vec{r'}\right)dv = \frac{1}{8\pi k_1^2} M\left(k_{1z}\right) \frac{k_\rho^3}{k_{1z}} R_{12}^{TM} e^{ik_{1z}(z+z')} H_0^{(1)}\left(k_\rho\left|\vec{\rho}-\vec{\rho}'\right|\right) \tag{2.94}$$

3. FAST EVALUATION OF GREEN'S FUNCTION AND ITS VOLUMETRIC INTEGRAL

3.1 Application of Discrete Complex Image Method

In this chapter a guideline for the Discrete Image Method (DCIM) is summarized. As discussed earlier, for the layered media the greens function are derived in closed form of spherical waves in spectral domain. In order to transform to spatial domain a well known Hankel transform is used, which is derived from Sommerfeld integration shown in (2.14).

$$g\left(r,r';k\right)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\tilde{g}\left(z,z';k_z\right)H_0^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)k_\rho dk_\rho \tag{3.1}$$

where $g\left(r,r';k\right)$ and $\tilde{g}\left(z,z';k_z\right)$ are the spatial and spectral representation of Green's function respectively and $H_0^{(1)}$ is zeroth order Hankel function of first kind. It is known that the numerical integration of Hankel transform is time consuming operation, thus to overcome this computational expenses a numerical method Discrete Image Method is applied to the problem. The main aim of DCIM is to write Green's function as a superposition of point sources with respect to complex distances, as result of which the integral can be evaluated as a form of Sommerfeld Identity.

Note that it is possible to extract (z,z') dependency of spectral representation, $\tilde{g}\left(z,z';k_z\right)$can be written as:

$$\tilde{g}\left(z,z';k_z\right)=f\left(k_z\right)\frac{i}{2k_z}e^{ik_z|z+z'|} \tag{3.2}$$

It is assumed that $f\left(k_z\right)$ can be approximated as:

$$f\left(k_z\right)\cong\sum_{n=1}^{N}a_n e^{b_n k_z} \tag{3.3}$$

Applying last changes to equation (3.1), equation given below is derived:

$$g\left(r,r';k\right)\cong\frac{1}{4\pi}\sum_{n=1}^{N}a_n\int_{-\infty}^{\infty}i\frac{k_\rho}{k_z}H_0^{(1)}\left(k_\rho\left|\rho-\rho'\right|\right)e^{ik_z z_n} \tag{3.4}$$

where $z_n = |z+z'| - ib_n$. As result the integral appearing in (3.1) is written as a form of well-known Sommerfeld identity (2.14). Consequently, the Green's function becomes

$$g\left(r,r';k\right) \cong \sum_{n=1}^{N} a_n \frac{e^{ikr_n}}{4\pi r_n} \tag{3.5}$$

where $r_n = \sqrt{(\rho-\rho')^2 + z_n^2}$ is known as the complex distance.

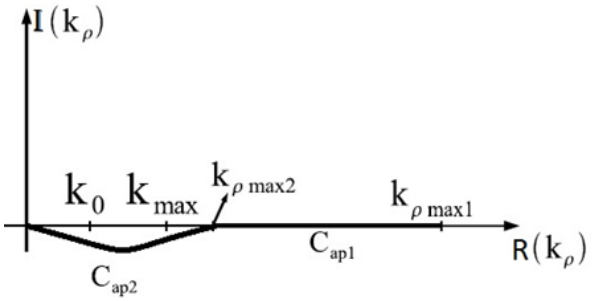


Figure 3.1: Paths of approximation for 2-level DCIM on k_ρ plane.

The approximation in (3.3) is achieved by applying Generalized Pencil of Function (GPOF) [24, 25] together with two level sampling path approaches. As mentioned in previous chapter one of the main difficulties while evaluating Sommerfeld integral is when branch points are close to or on the integration path. One way to overcome this complication is to modify the integration path. In this work the integration path is changed in such a way that singularities are not close any more. The approximation path for the integral on plane is shown in Figure 3.1, which is divided in two parts and is modified in such a way that it fulfils the requirement of linear sampling which is a must in order to use GPOF. In consideration of this requirement the sampling path on plane is determined as shown in Figure 3.2.

In order to use Sommerfeld Identity, transformation of k_ρ plane to k_z plane is needed during exponential approximation shown in (3.3), thus the following expressions are defined:

$$\begin{array}{lll} ForCap1 & k_z = ik\left[T_{02}+t\right] & 0 \le t \le T_{01} \\ ForCap1 & k_z = k\left[it + \left(1 - \frac{t}{T_{02}}\right)\right] & 0 \le t \le T_{02} \end{array} \tag{3.6}$$

where t is the linear sampling parameter. Using the equation (3.6) expressions for k_{zmax1} and k_{zmax2} are obtained as below:

$$\begin{array}{c} k_{zmax1} = ik\left(T_{01}+T_{02}\right) \\ k_{zmax2} = ik\left(T_{02}\right) \end{array} \tag{3.7}$$

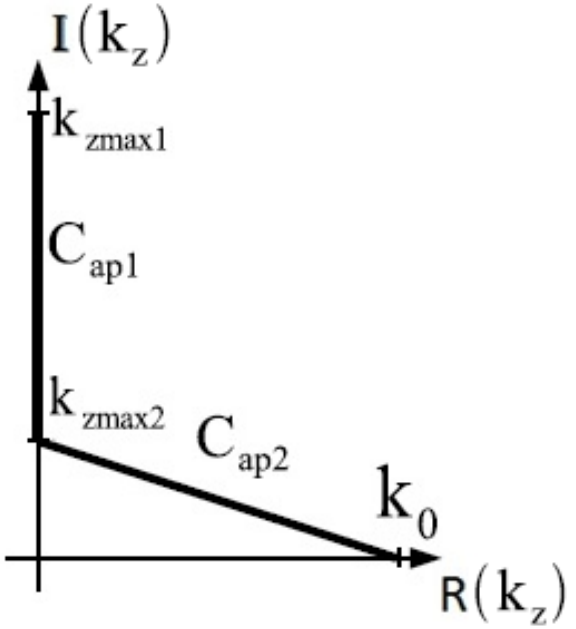


Figure 3.2: Paths of approximation for 2-level DCIM on k_z plane.

Values of $k_{\rho max1}$ and $k_{\rho max2}$ are estimated by equations below which are obtained from (3.7).

$$\begin{aligned} k_{\rho max1} &= k\sqrt{1+(T_{01}+T_{02})^2} \\ k_{\rho max1} &= k\sqrt{1+(T_{02})^2} \end{aligned} \tag{3.8}$$

First the sampling of $f(k_z)$, which part of spectral Green's function, is performed on path Cap1 and GPOF approximation is applied thus $f(k_z)$ is obtained as sum of exponentials.

$$f_1(k_z) = \sum_{n=1}^{N_1} a_{1n} e^{b_{1n}k_z} \tag{3.9}$$

where a_{1n} and b_{1n} are the coefficients and exponents obtained from GPOF method and N_1 is the number of exponentials used in the first part of the method. Once the exponentials on the first path are evaluated, they are subtracted from $f(k_z)$ and the remanding part is sampled through path Cap2 and approximated by GPOF and $f_2(k_z)$ is obtained.

$$f_2(k_z) = \sum_{n=1}^{N_2} a_{2n} e^{b_{2n}k_z} \tag{3.10}$$

where a_{2n} and b_{2n} are the coefficients and exponents obtained from GPOF method and N_2 is the number of exponentials used in the second part of the method. As result, $f(k_z)$ can be written as sum of exponentials.

$$f(k_z) = \sum_{n=1}^{N_1} a_{1n} e^{b_{1n}k_z} + \sum_{n=1}^{N_2} a_{2n} e^{b_{2n}k_z} \tag{3.11}$$

4. NUMERICAL RESULTS

In this chapter, in order to show the applicability and the effectiveness of the presented method, the comparison of results obtained by numerical integration and DCIM approximation are displayed. Initially the the analysis of the derivation of dyadic Green's functions for layered media are illustrated and it is followed by the presentation of the analysis of evaluation of volumetric integral of dyadic Green's function over cuboids. The configuration that is considered for examples is two layered media as shown in Figure 3.1.

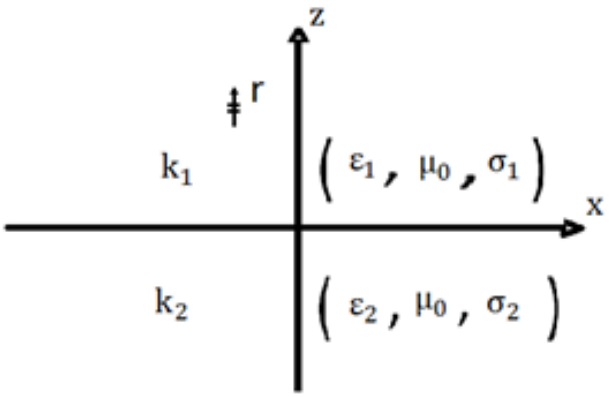


Figure 4.1: Geometry of the problem.

4.1 Analysis of Fast Evaluation of Electric Field Green's Functions

The operating Frequency is 1 GHz and electric permittivity and magnetic permeability for the upper medium are 1 and 1 respectively and for the lower medium are 4 and 1 respectively. The conductivity for upper medium is 0 and for lower medium is 0.2. The locations of source is (0.05, 0, 0.5) and observation points are and (-2, 1, 0.8) - (2, 1, 0.8).

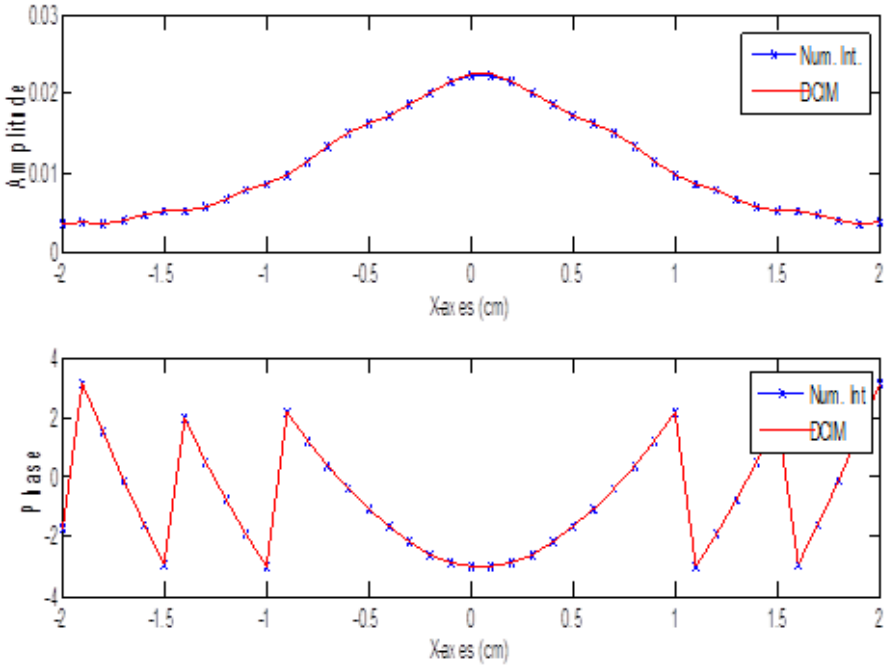


Figure 4.2: Gxx component of Dyadic Green's Function.

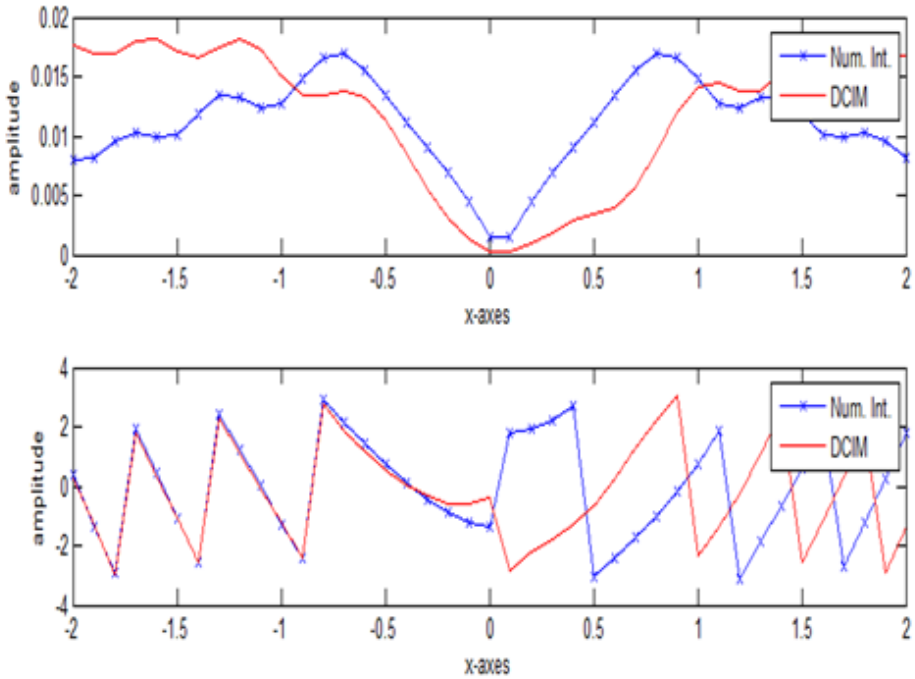


Figure 4.3: Gxy component of Dyadic Green's Function.

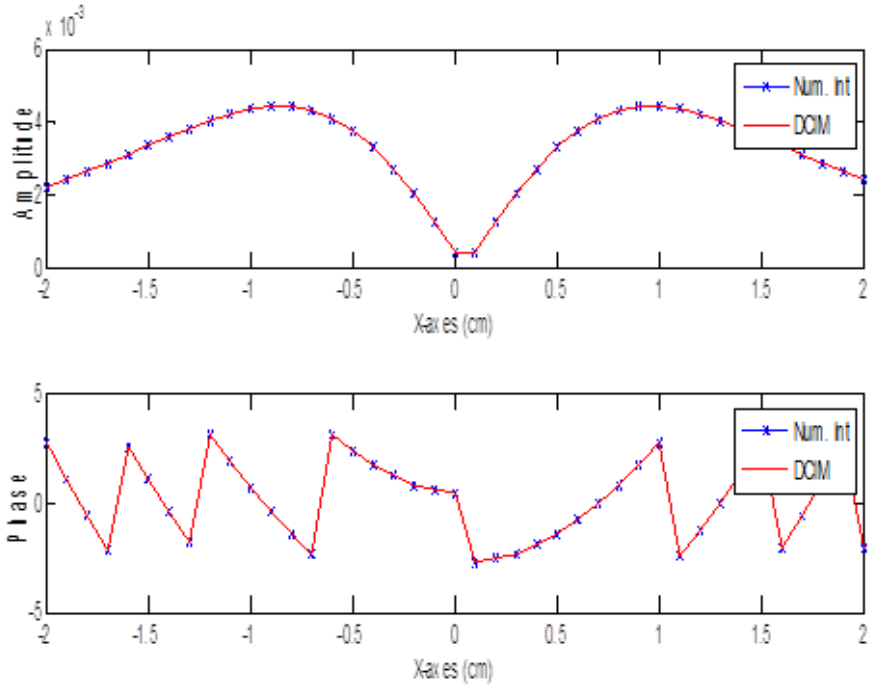


Figure 4.4: Gxz component of Dyadic Green's Function.

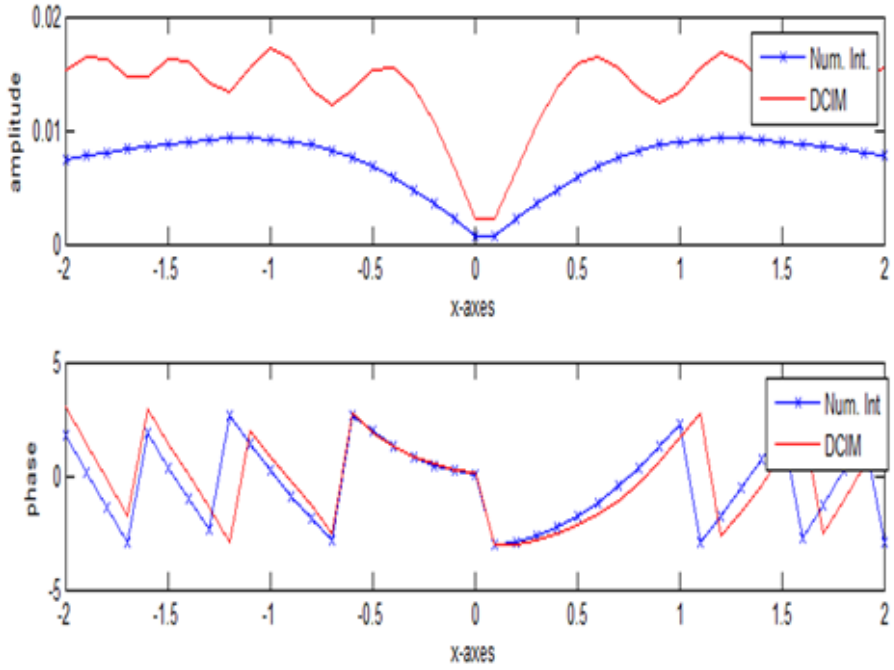


Figure 4.5: Gyx component of Dyadic Green's Function.

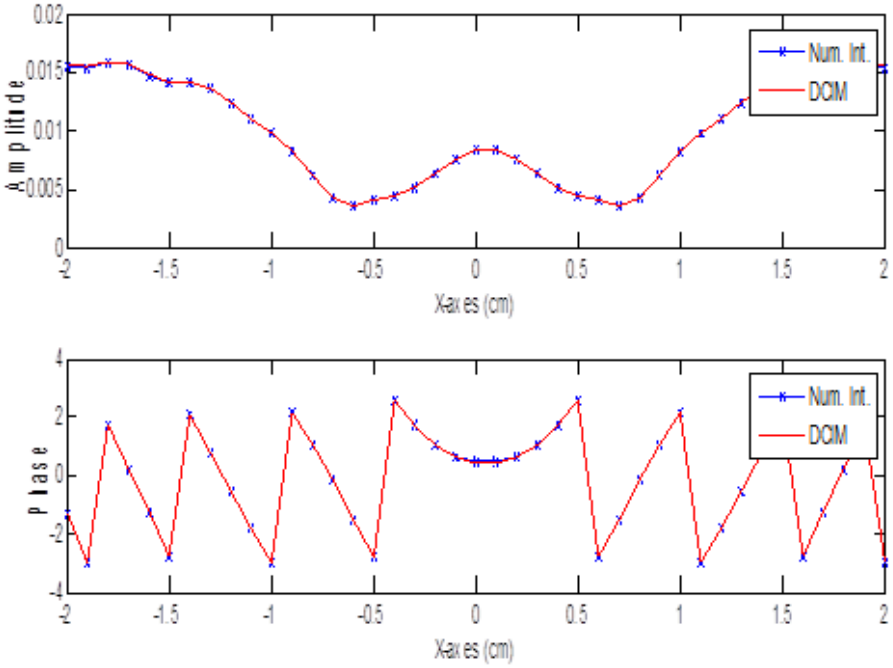


Figure 4.6: Gyy component of Dyadic Green's Function.

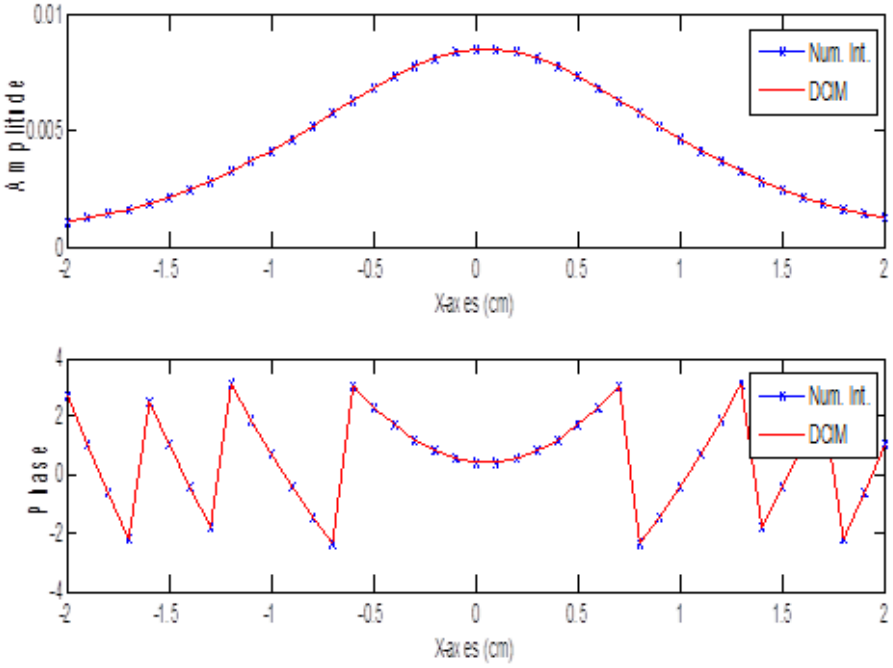


Figure 4.7: Gyz component of Dyadic Green's Function.

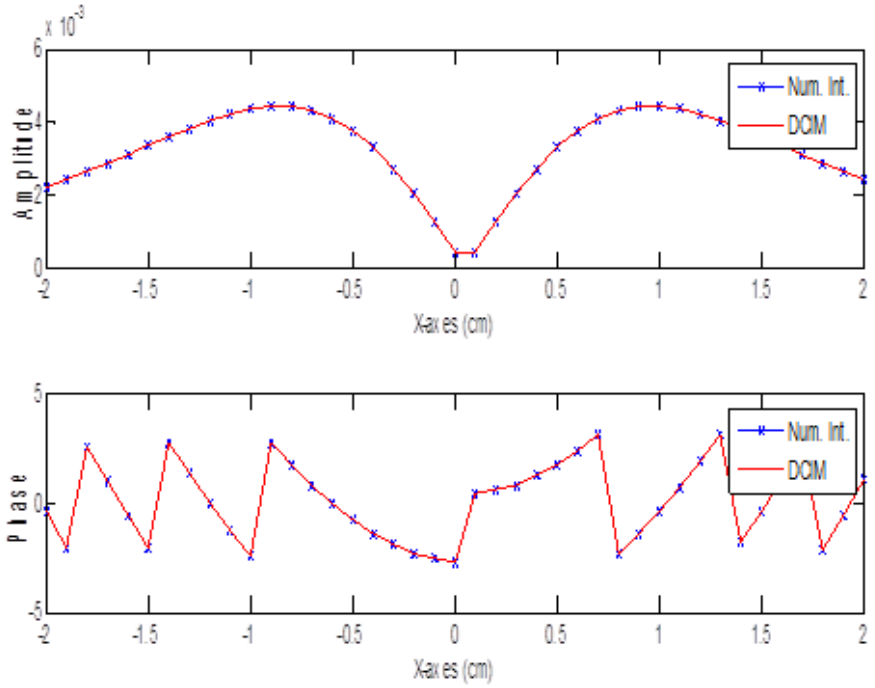


Figure 4.8: Gzx component of Dyadic Green's Function.

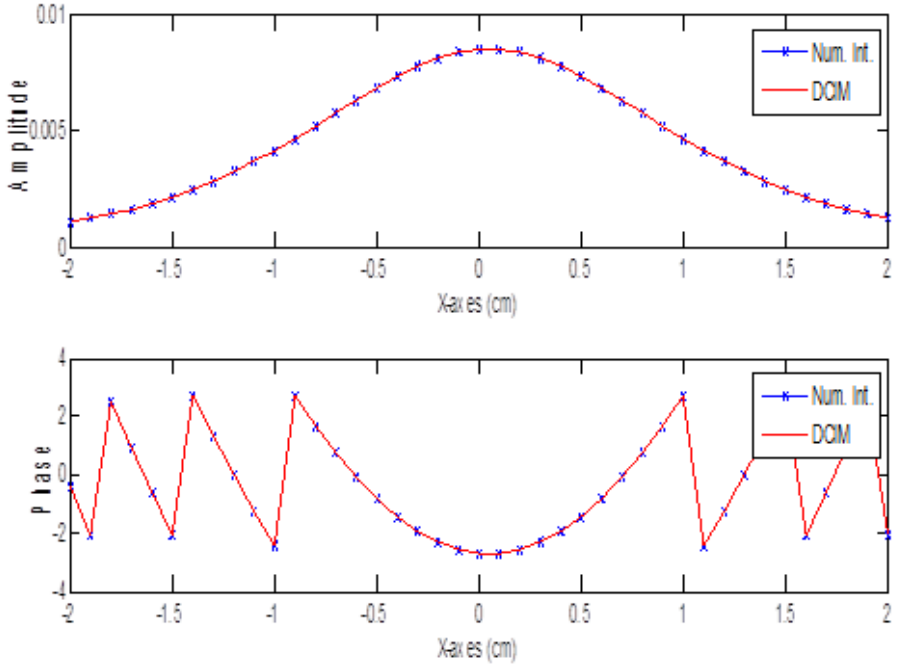


Figure 4.9: Gzy component of Dyadic Green's Function.

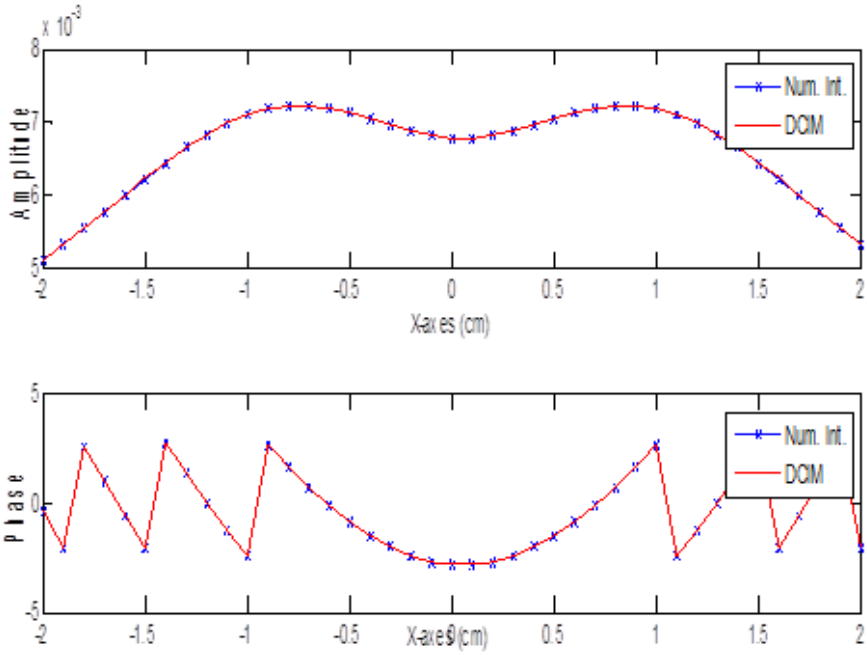


Figure 4.10: Gzz component of Dyadic Green's Function.

In Figures 4.2-4.10 the numerical integration results and DCIM results of some component are shown. As it is shown, the results match each other. The time consumed for numerical integration is 33.586915 seconds while for DCIM is 1.166365 second, the benefit of usage of DCIM in terms of computational time is unavoidable. In Figure 4.3 and 4.5 the results do not match each other, this may be due to the oscillations of the functions.

4.2 Analysis of Fast Evaluation of Volumetric Integrals of Electric Field Green's Functions

The operating Frequency is 1GHz and electric permittivity and magnetic permeability for the upper medium are 1 and 1 respectively and for the lower medium are 10 and 1 respectively. The conductivity for upper medium is 0 and for lower medium is 0.2. The loactions of source is (0, 0, 0.0005) and observation points are and (0.503, 0.3, 0.029) - (0.53, 0.3, 0.029).

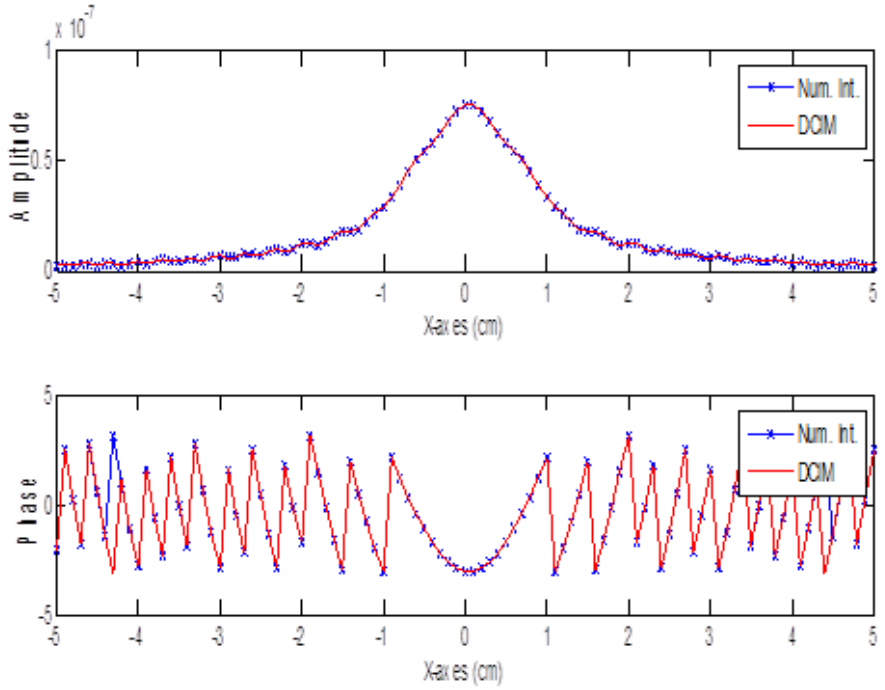


Figure 4.11: Volumetric integral of Gxx component of Dyadic Green's Function.

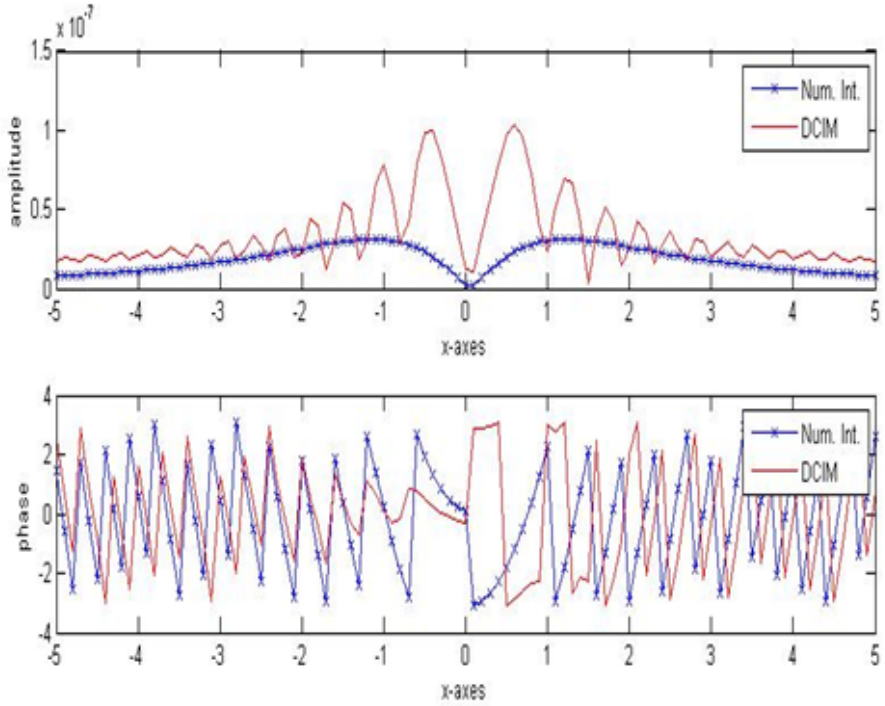


Figure 4.12: Volumetric integral of Gxy component of Dyadic Green's Function.

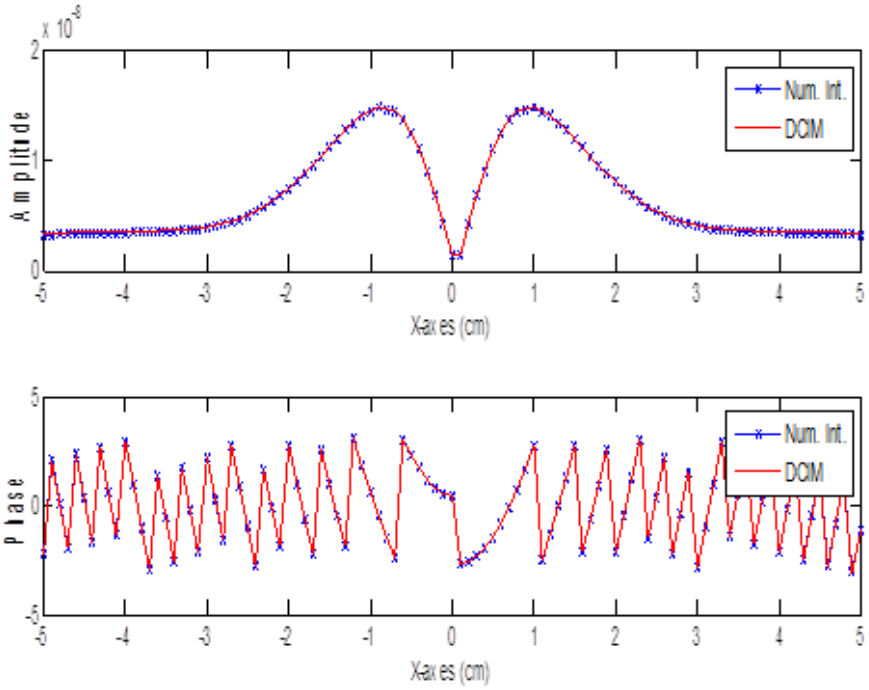


Figure 4.13: Volumetric integral of Gxz component of Dyadic Green's Function.

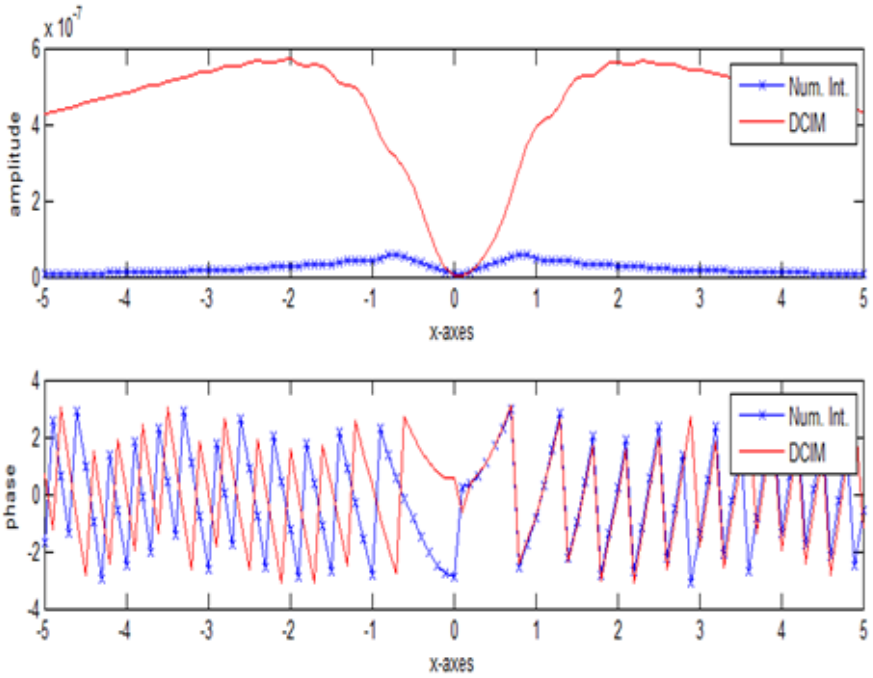


Figure 4.14: Volumetric integral of Gyx component of Dyadic Green's Function.

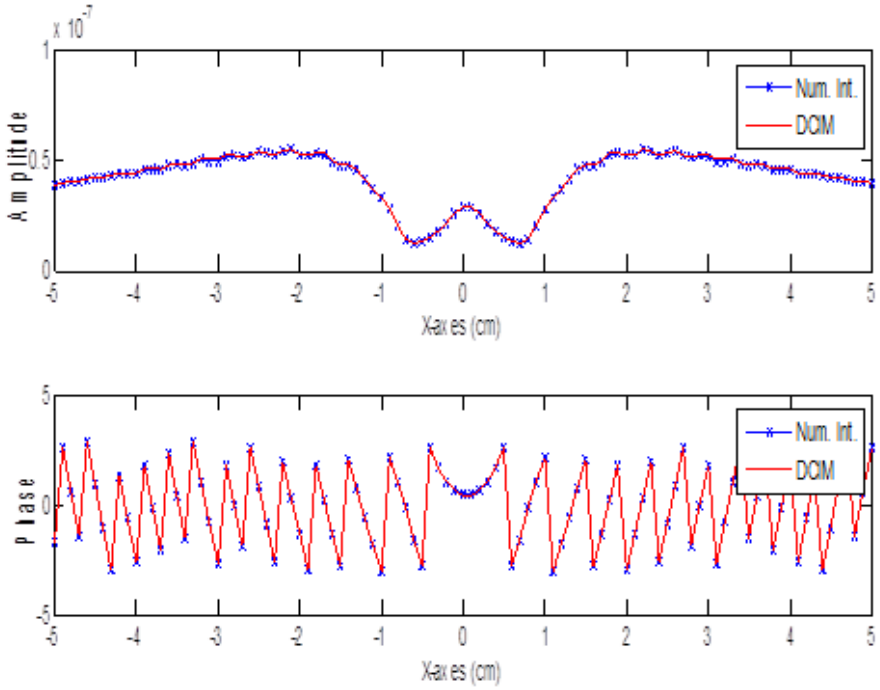


Figure 4.15: Volumetric integral of Gyy component of Dyadic Green's Function.

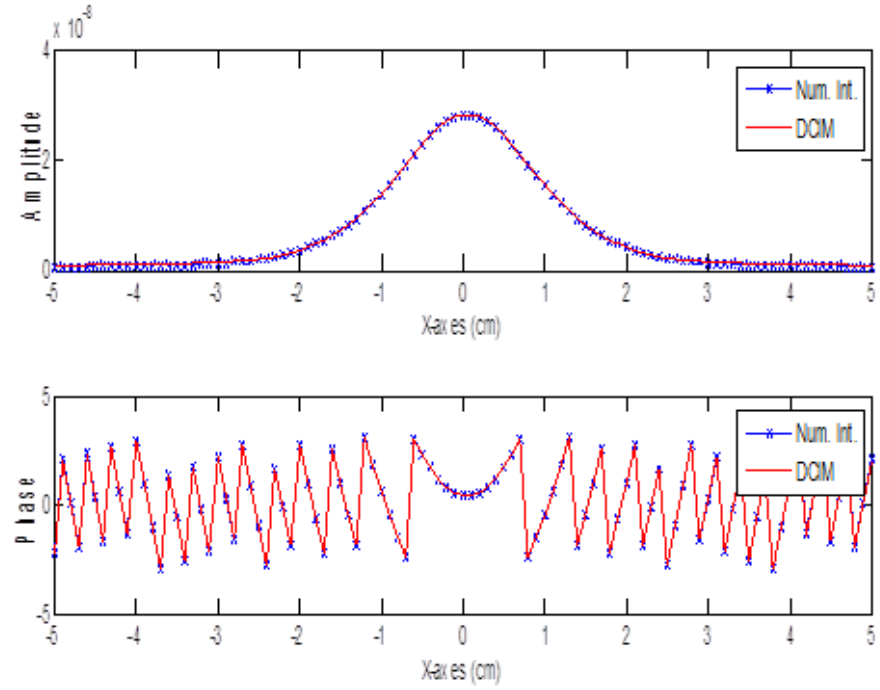


Figure 4.16: Volumetric integral of Gyz component of Dyadic Green's Function.

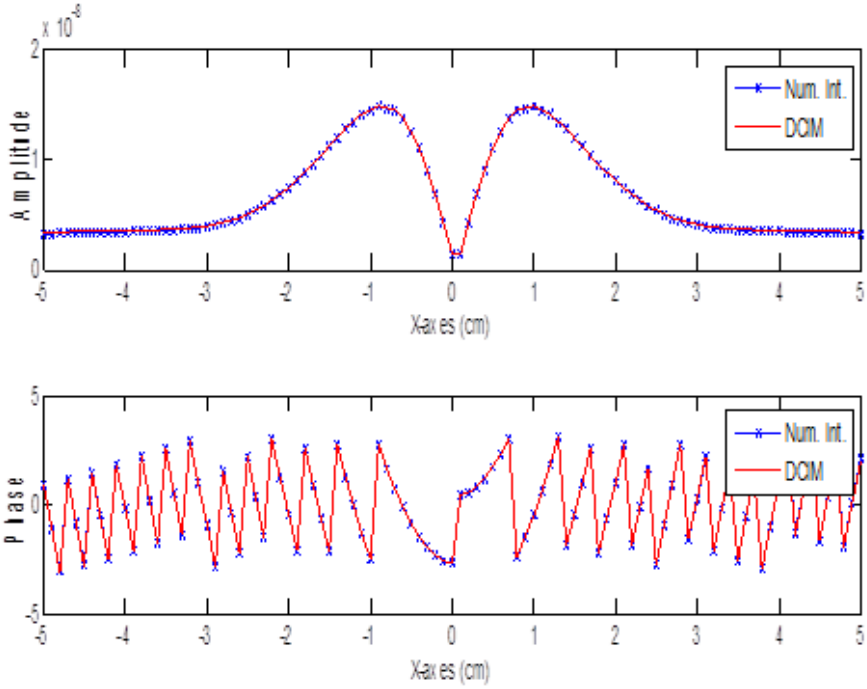


Figure 4.17: Volumetric integral of Gzx component of Dyadic Green's Function.

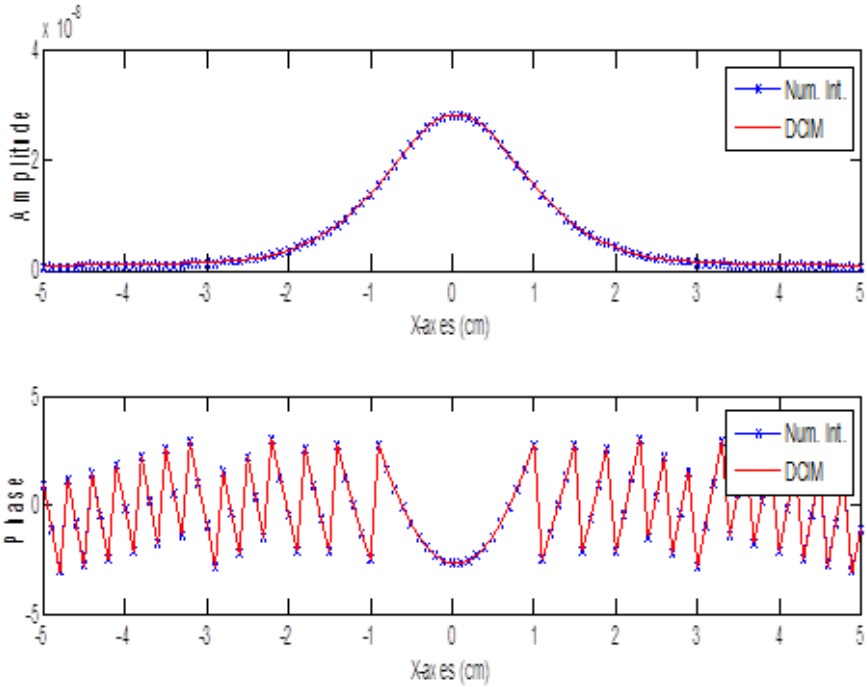


Figure 4.18: Volumetric integral of Gzy component of Dyadic Green's Function.

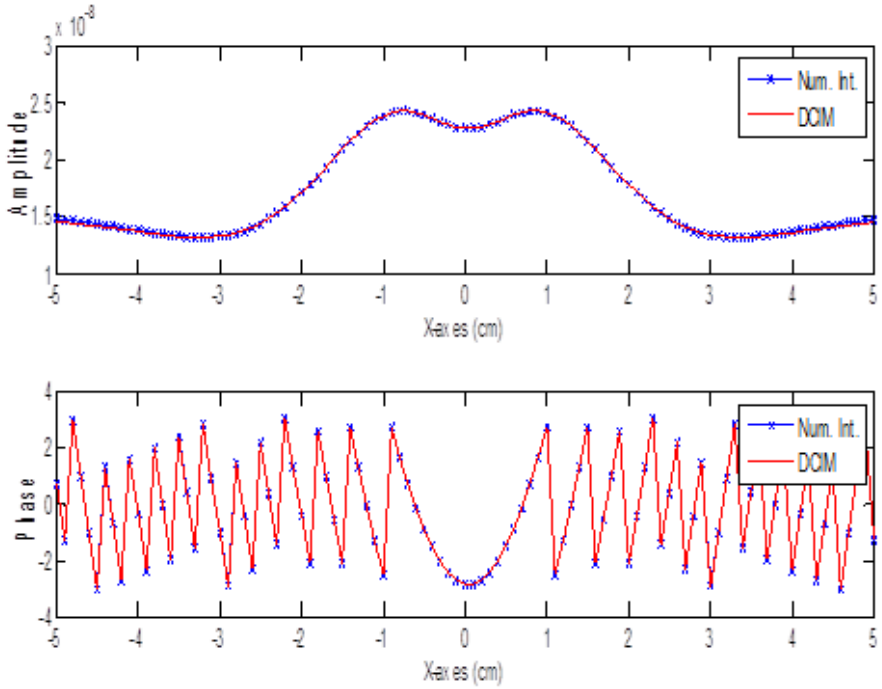


Figure 4.19: Volumetric integral of Gzz component of Dyadic Green's Function.

In Figures 4.11-4.19 are shown the numerical integration results and DCIM results of some component. As it is shown, the results match each other. The time consumed for the evaluation of volumetric integrals of Green's functions by numerical integration is 28.480989 seconds while by DCIM is 0.674168 second, the benefit of usage of DCIM in terms of computational time is clear and also it is obvious that there is not any additional computational burden caused by volumetric integrals. In Figure 4.12 and 4.14 the results do not match each other, since the results of only Sommerfeld integrations did not match neither.

5. CONCLUSIONS AND RECOMMENDATIONS

In this thesis, electric field Green's functions of layered media and their volumetric integrals were obtained. The results obtained by the DCIM were verified with the results obtained by direct numerical integration.

The study started with the general derivation of Green's functions for free space and layered media. Then, the volumetric integrals of Green's functions over cuboid cells were calculated. In the following chapter DCIM was described in details. Several numerical examples of spatial domain Green's functions were presented. Numerical examples of volumetric integrals of Green's functions were illustrated as well. It was shown that the results obtained by numerical integration and the DCIM approach do overlap with each other. The computational time for the examples was recorded which proves the contribution of DCIM approach and it is concluded that the evaluation of volumetric integrals do not cause additional computational burden.

Even though the 2-level DCIM gives very good results in a variety of different cases, in some cases it cannot approximate the Green's function sufficiently. Due to this deficiency, as the future work 3-level DCIM is suggested, since it succeeds in capturing branch points contribution more then 2-level DCIM thus leads to better approximation.

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CURRICULUM VITAE

Name Surname: Fjolla SHEHU

Place and Date of Birth: 12.03.1987 Prizren / KOSOVO

E-Mail: shehu@webmail.itu.edu.tr

B.Sc.: Telecommunication Engineering in Istanbul Technical University

List of Publications and Patents:

▪ Suer C., Dizdaroğlu E., **Shehu F.**, Tunçay A. H. and Canlı G. A., 2011: Microwave Non-Destructive Testing of Deformations in A Composite Material. *13th International Symposium on Microwave and Optical Technology*, June 20 – 23, 2011 Prague, Czech Republic.

PUBLICATIONS/PRESENTATIONS ON THE THESIS

▪ **Shehu F.**, Cayoren M., Yapar A. and Akduman I., 2011: Fast evaluation of electric field Green's functions and their volumetric integrals for near field imaging applications. *5th International Conference on electromagnetic Near-field Characterization and Imaging*, Nov. 30-Dec. 2, 2011 Rouen, France.