DYNAMICS OF EXCHANGE RATE FLUCTUATION OF YEN WITH DOLLAR

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Abstract

Fluctuations of yen exchange rate to US-dollar are analyzed by a method of classical mechanics. The fluctuations are explained by a linear inhomogeneous differential equation of the second order with a constant coefficient. The inhomogeneous term is an external force. Using a method of the least squares, we determine a polynomial expression of the external force. It is known that the depreciation of yen is caused by decreasing of the external force toward its minimum.

JEL classification: C51
Keywords: Exchange rate fluctuation

1. Introduction

There are several theories to analyze the fluctuation of foreign exchange rates. Among all, the AR and MA methods which are said to be the time series analyses are simple and easy to handle the fluctuation. They are also practical for the data fitting. But they do not have dynamics in their frameworks. In order to supplement the luck of dynamics, the ARCH model was invented. See a textbook of Hamilton (1994) for the detailed references in this field.

The dynamical analyses of the exchange rates are developing with some aspects. Firstly, the fluctuations were explained by economic indexes such as the purchasing power parity. This was originated from work of Cassel (1972). Recent developments of this theory are given, for example, in the articles of Cheung and Lai (2001), and Coakley and Fuertes (2001). Secondly, Frenkel (1976) discussed the fluctuations in terms of the monetary indexes such as interest rates. The methods of Cassel and Frenkel are essentially on the line of a

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regression analysis of several variables. For recent developments along this line of research, see an article of So (2001), for example. Thirdly, it is commonly known that the no-linearity ideas such as dynamical systems, chaos, and fractals are essential for the analysis of the dynamics of the fluctuations. See, for example, a textbook of Mentegna and Stanley (2000). In a computer based dealing of currencies, the time series that the dealing produces may have properties of the above mentioned non-linearity. But the exchange rate summarized daily will be within the reach of a linear analysis.

In a previous paper, Obara (2003) showed that a Langevin equation was useful to explain a phenomenon named the anomalous fluctuation of the variance in the exchange rate time series. In this paper, we analyze the exchange rate fluctuation within the framework of classical mechanics: the Langevin equation will be specified in terms of a linear differential equation with constant coefficients. We will get around the complexity coming from the non-linearity, if it exists in the time series, by a technique of data smoothing.

It is said that there is no differential equation in economics corresponding to an equation of motion in classical mechanics. In the section 2, we will construct an equation of motion for the exchange rate under two assumptions: the exchange rate fluctuation is harmonic without external force and the external force is normalized by a principle of minimal work. In the section 3, a characteristic feature of the fluctuation in a time interval in which the external force is periodic is examined. An interesting character of the external force is revealed. In this paper, it is shown that macroscopic behavior of the exchange rate can be explained in terms of classical mechanics.

2. Determination of differential equation

In this section, we will find a differential equation with constant coefficients which explains the fluctuation of the exchange rate. By analogy with physics, a stochastic differential equation for the

independent variable $x(t)$ will be

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = f(t) + \varepsilon(t) \qquad (1)$$

The order of differentiation is at most second. This will be plausible from the fact that an AR(1) or AR(2) process has a good goodness of fit. The second term of (1) represents friction proportional to the "velocity." The parameter k is a friction coefficient. The third term produces the harmonic oscillation. The coefficient ω is an angular frequency which the economic system has intrinsically. The term $f(t)$ denotes external force. This contains the whole contributions from all economic variables outside of the system from the macro-economic indexes to the monetary indexes. The variable $\varepsilon(t)$ is white noise, whose expectation value is $<\varepsilon(t)>=0$.

The system we are going to analyze is that of yen exchange rate to US-dollar. The time series of it is daily.* There are two unknown parameters and one unknown external force in (1). In order to determine the two parameters and the external force from the data, we introduce the following two plausible assumptions: 1) When there is no external force, the system has the harmonic oscillation. 2) The external force acts so that the work W done by the force may be zero.

The harmonic oscillation is the most fundamental oscillation in nature. The second assumption comes from the principle of he virtual displacement in mechanics. From the assumption 1, we have k =0. The assumption 2 means that:

$$W = \int_{(0)}^{(T)} f(t)dx = 0 \qquad (2)$$

where a range of the data is $[0, T]$, and (0) and (T) denote the

* Source is http://pacific.commerce.ubc.ca/xr/, period: January 2, 2001 to Jane 3, 2003.

positions of $x(t)$ at $t = 0$ and $t = T$, respectively. In economics there is not such concept as "work," but this assumption should be interpreted as a way of normalization of the force.

The equation (1) is not suitable for an explanation of the daily exchange rate. Firstly, this is because the fluctuation of the data is too hard. Secondly, although a solution of (1) may have a periodic component, the data do not look periodic. Therefore we will try to transform the raw data $x(t)$ so that the equation (1) makes sense.

For this purpose, smoothing technique is useful. The simplest one is a moving average:

$$y(t) = \frac{1}{N}\sum_{n=0}^{N-1} x(t-n) \equiv < x(t) > \qquad (3)$$

where the order N may vary for the different purposes of the analysis. In our case N =100 is suitable. A plot of the smoothed data is given in Graph 1. Even though it is difficult to find dynamics behind the raw data, it will be easy to find a law behind the smoothed data. Our equation of motion for the smoothed series is

$$\frac{d^2 y(t)}{dt^2} + \omega^2 y(t) = f(t). \qquad (4)$$

The smoothed data will be approximated by polynomials:

$$y(t) = \sum_{n=0}^{\ell} c_n t^n. \qquad (5)$$

A method of the ordinary least squares (OLS) is applied to determine the coefficients. The optimal order is $\mathrm{l} = 10$. The results of OLS are in Table 1.

Graph 1. Plot of smoothed yen rate to US dollar

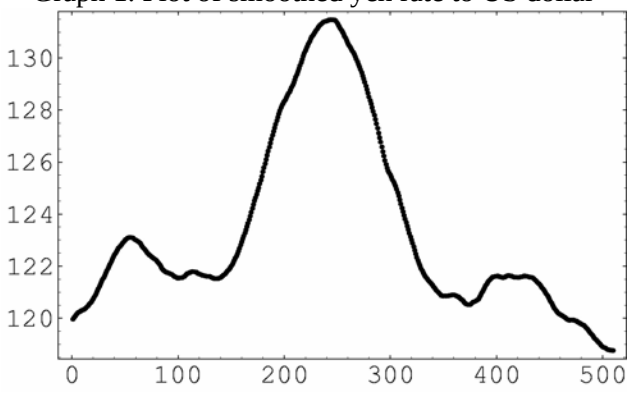
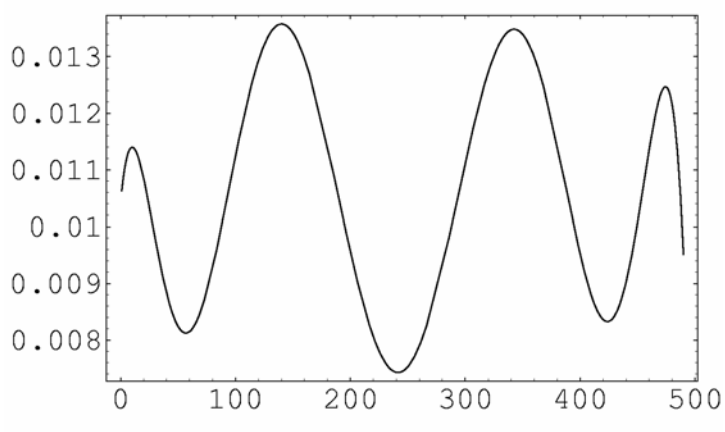


Table 1. Polynomial fitting: $y(t) = \sum_{n=0}^{10} c_n t^n$.

Number of observations	510
Adjusted R squared	0.9963
Durbin-Watson statistic	0.043
Akaike information criteria	-56.92
F-statistic	14017

	Estimated value	Standard error	t-statistic	p-value
c_0	1.19E+02	1.15E-01	1.08E+03	0.00
c_1	4.97E-02	1.47E-02	3.36E+00	0.00
c_2	1.37E-04	6.46E-04	2.13E-01	0.83
c_3	3.31E-05	1.32E-05	2.50E+00	0.01
c_4	-1.20E-06	1.49E-07	-8.06E+00	0.00
c_5	1.46E-08	1.01E-09	1.44E+01	0.00
c_6	-8.75E-11	4.28E-12	-2.04E+01	0.00
c_7	2.90E-13	1.14E-14	2.53E+01	0.00
c_8	-5.47E-16	1.85E-17	-2.92E+01	0.00
c_9	5.48E-19	1.71E-20	3.20E+01	0.00
c_{10}	-2.27E-22	6.69E-24	-3.39E+01	0.00

Graph 2. Plot of external force $f(t)$



Now that we know the coefficients of (5), we get $y(t)$ and its second derivative analytically.

Substituting them into (4), we have the external force including the unknown w^2:

$$f(t) = \sum_{n=0}^{10} d_n t^n$$
$$d_n = \begin{cases} (n+1)(n+2)c_{n+2} + \omega^2 c_n & (n = 0, 1, 2, \cdots, 8) \\ \omega^2 c_n & (n = 9, 10) \end{cases} \qquad (6)$$

In order to determine w, we use the assumption 2. We have an equation

$$W = \int_0^{t_3} f(t)\dot{y}dy = 0 \quad (t_3 = 510). \qquad (7)$$

After a simple calculation, we have

$$\omega^2 = 8.65 \times 10^{-5} \quad (\omega = 9.3 \times 10^{-3}). \qquad (8)$$

From this we determine the external force in terms of a polynomial of time t of degree 10. The shape of $f(t)$ is given in

Graph 2.

There is a minimum of $f(t)$ around $t = 250$, corresponding to which there is a maximum of $y(t)$. The system (4) has an intrinsic oscillation caused by ω and an enforced one caused by $f(t)$. The maximum of $y(t)$ is explained by superposition of those two waves. A more detailed discussion of this phenomenon is given in the next section.

3. Periodic external force

From the Graph 2 it seems that $f(t)$ is periodic in the range $[t_1, t_2]$, $t_1 = 80$, $t_2 = 400$. To be more definite, we put

$$f(t) = a + b\sin(\varphi t + \delta), \quad t_1 \le t \le t_2. \qquad (9)$$

Using a method of non-linear fitting, we have

$a = 0.0104$, $b = -0.00306$, $\varphi = 0.00314$, $\delta = 0.269$.

Substituting (9) into (4), we have a solution of (4):

$$\begin{aligned} y(t) = {} & \frac{a}{\omega^2} + r\sin(\omega t + \theta) - \frac{a}{\omega^2}\cos\omega(t - t_1) \\ & + \frac{b}{\varphi^2 - \omega^2}\frac{\rho}{\omega}\sin(\omega(t - t_1) + \phi) - \frac{b}{\varphi^2 - \omega^2}\sin(\varphi t + \delta) \end{aligned} \qquad (10)$$

where r and θ are integration constants and other constants have the following relations:

$$\rho\sin\phi = \omega\sin(\varphi t_1 + \delta)$$
$$\rho\cos\phi = \varphi\cos(\varphi t_1 + \delta), \quad \rho > 0.$$

The first term of (10) is a known constant. The second and third terms represent the intrinsic oscillation. The fourth and fifth terms are the interference with waves of angular frequencies ω and φ.

Let this amplitude be A:

$$A = \frac{b}{\varphi^2 - \omega^2}\frac{\rho}{\omega}\sin(\omega(t - t_1) + \phi) - \frac{b}{\varphi^2 - \omega^2}\sin(\varphi t + \delta)\,. \quad (11)$$

We evaluate the amplitude as

$$|A| \le \left|\frac{b}{\varphi^2 - \omega^2}(2 + \frac{\varphi}{\omega})\right|. \quad (12)$$

Here let $\eta = \varphi / \omega$, and the right-hand side of (12) be $h(\eta)$. Let a ratio of $h(\eta)$ to $h(0)$ be $g(\eta)$:

$$g(\eta) = \frac{h(\eta)}{h(0)} = \frac{1}{2}\left|\frac{\eta + 2}{\eta^2 - 1}\right|. \quad (13)$$

The function $g(\eta)$ has a singularity at $\eta = 1$. If η is close to 1 that means $\omega \approx \varphi$, $g(\eta)$ becomes large. This phenomenon is said to be resonance. In our case, $\eta = 3.376$ 1, then $g(\eta)$=0.258 1. There is no evidence of resonance in $[t_1, t_2]$. The contribution of φ to the amplitude A is small. Then the maximum of $y(t)$ near $t = 250$ is interpreted as an ordinary superposition of the two oscillations. Lastly, we will calculate the work in $[t_1, t_2]$:

$$W = \int_{t_1}^{t_2} f(t)\dot{y}dt = -0.0113\,.$$

This is a very interesting phenomenon. The work is negative in this range. The external force has work done: it absorbs "energy" from the system. Since the whole work is normalized to zero, outside this range, the external force does positive work: it emits energy. We are apt to think that the big fluctuation of an exchange rate will be caused by the work done by the strong external force. In our model, the external force acts the other way round. The contribution of ω, the intrinsic oscillation of the economic system, to the oscillations of the amplitude is rather strong, and the external force acts to reduce the amplitude. The minimum of $f(t)$ produces the maximum $y(t)$. Economically it is said that depreciation of yen is explained by

decreasing of the external force toward its minimum.

4.-Conclusion

The time series of yen exchange rate $x(t)$ is transformed to $y(t)$ by the method of moving average, whose order is 100. The time series $y(t)$ becomes an object that a method of classical mechanics is applicable to. By this method, the dynamics in the fluctuations of the exchange rate was revealed. The two assumptions play the essential role to get a dynamical structure of the fluctuations: 1) An exchange rate oscillates harmonically with an angular frequency ω, if there is no external force. 2) Work that the external force does is zero. What the external force means are all effective economic variables that influence the fluctuation of $x(t)$.

The main results of our analysis are as follows:

1. The smoothed data $y(t)$ satisfy a linear inhomogeneous second order differential equation with a constant coefficient ω.

2. Using the two assumptions, we determined the coefficient ω and the inhomogeneous term of the differential equation that represents the external force.

3. The external force is periodic with frequency φ in some region of time. In this region, the force has work done. The force absorbs energy from the intrinsic oscillation. Since $\varphi / \omega \square 1$, there is no evidence of occurring resonance in this system, therefore the maximum of $x(t)$ is a result of the superposition of two waves made by ω and φ. The depreciation of yen is explained by decreasing of the external force toward its minimum.

4. Finally, and in summary, the maximum of the exchange rate $y(t)$ is not simply caused by a specific behavior of the external force. In an interval of time prior to the occurrence of the maximum of $y(t)$,

the economic system prepared the appearance of it by the interaction between the intrinsic oscillation and the external force. After reaching the maximum, $y(t)$ is ready to go back to the normal oscillation under the interaction with the external force.

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Journal published by the Euro-American Association of Economic Development. http://www.usc.es/economet/eaa.htm